Modelling and Simulation of Dynamic Systems

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Lecture No -17
System Model of Combined Rotary and Translatory Systems

In this lecture on system model of combined rotary and translations system. In this course on modeling and simulation of dynamic systems. So previously we have seen the various types of Systems. I mean independent systems such as mechanical system electrical system pneumatic hydraulic and thermal system. And we have seen the building blacks for modeling of these systems.

Now from this lecture onwards in few lectures we will be taking the combination of various types of these systems. So in todays this lecture is going to deal with combination of translatory and rotary systems. So basically it's a part of mechanical or we can have combination of mechanical and electrical because rotary motion or usually coming through motor. So I will be taking up one example were we have motor and motor destroying do rotate something and then some disk and from that disk we have trying to pull ever are something like that.

So here combination of translatory and rotary systems. So there are many mechanisms which converts rotary motion to translational motion or vice versa. I can name a few which are use for this type of conversation rack and pinion arrangement a very popular a way of converting the rotary motion into the translatory motion.

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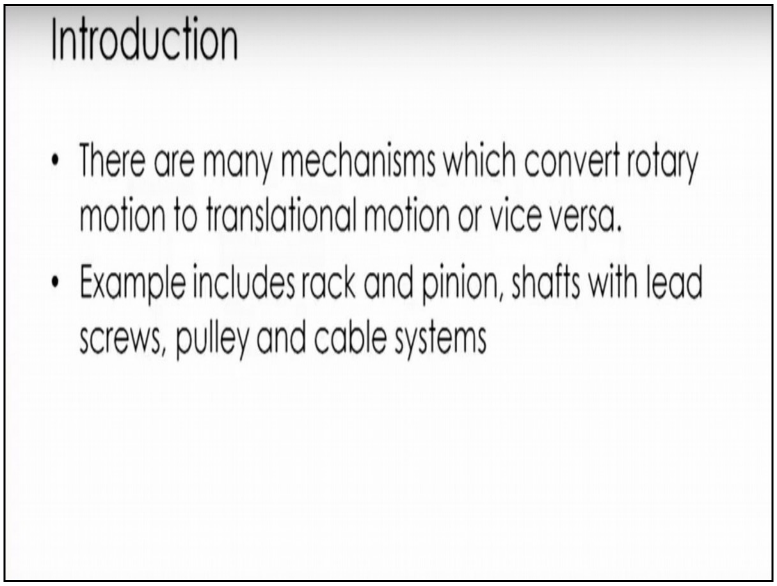

Are in fact many of the linear actuated or basically based on this rack and pinion arrangement where we have a motor and on the motor shop a pinion is mounted and that pinion actually exist a rack. And this way we can get the rotary motion in to a translatory motion. There could be other examples soft with lead screws all of us know the example of the lead and screws which is used in lath and we are basically we want to have translatory motion of the tool pose.

Fine then there are could be systems such as pulley and cable system so sorry a pulley mounted on a shaft and through that pulley as a cable bounded and unbounded and that i system can be considered like this. Other example could be pa camp followers systems also although they are the motion are not continues a motion could be intermittent motion.

But again there also we have camp which usually has rotary motion and the follower has got a translatory motion. So that type of combination also comes in to the picture. So here we will be just looking at the basic two models here that is the first model the rack and pinion system how to model rack and pinion system and how to model the pulley and cable system.

And here I would like one to you to emphasize that since we have already covered the bond graph modeling of dynamic systems. So try co-relate whatever am going to discuss here with the respective bond graph model and the that will give you better feel of system modeling.

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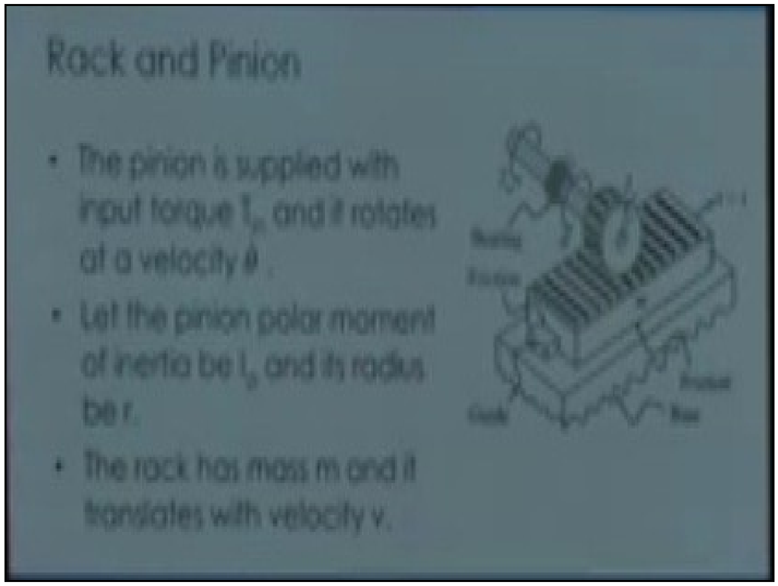

So let us a look at this rack and pinion arrangement. So here we have got it pinion there is a bearing here and there is a torque been acting on this pinion. t in and the torque which is applied at the periphery through periphery on this rack basically is t out. And because of this torque this torque generates attendance force here and that attendancial force is responsible for the motion of the rack.

And there is a guide way here provided for the guidance of the rack and assume that there is some friction here between guide and the rack that also we can model and everything is fixed on it's the ten base. So I just said the pinion is applied with input torque t in and it rotates at velocity theta dot. So this soft rotates at velocity theta dot.

Next let the pinion polar moment of inertia be ip as it indicated in figure and its radius be are. And the rack has mass am and it translate with a velocity v .so we are consisting a mass m of the rack and of course there is a velocity v are x derivative in this in this direction for the rack. Now as I said that we assume that there is a friction between guide and the rack.

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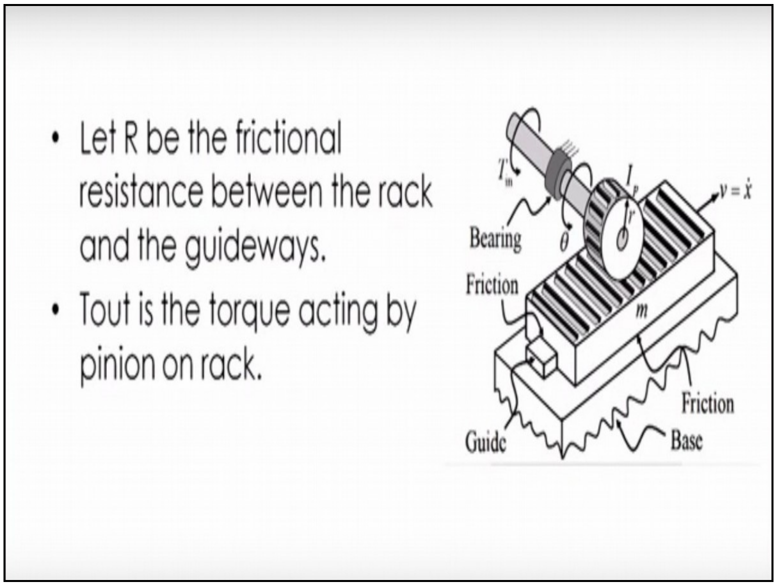

So let r be the friction between the rack and the guide ways. So we try to model that friction also and t out is the torque action by pinion on the rack that is there. So with this definition of various parameters we can right system equations for the pinion and system equations for the rack separately and then we can combine this to system equations.

So this will give as better understanding of how the rotary motions are the dynamics of rotary motions are related with the dynamics of the translatory motion. So the next torque acting here on the pinion is t in - t out and this torque net torque is actually responsible for the angular actualization of the pinion.

So this net torque will be equated to i into the angular acceleration were this i is nothing but its ip that is the moment of inertia of the pinion .so this is basically i into alpha were alpha is the angular acceleration of the pinion. So this way we can right these equations. Now as you know that the rotation of pinion we result in transition translational velocity v of the rack.

And this translatory velocity v can we give in a by r into omega were r is the radius of the pinion and omega is the angular velocity of the pinion. So I can write this equation that is torque in - torque out is equal to I can substitute for omega here omega in terms of v and r.

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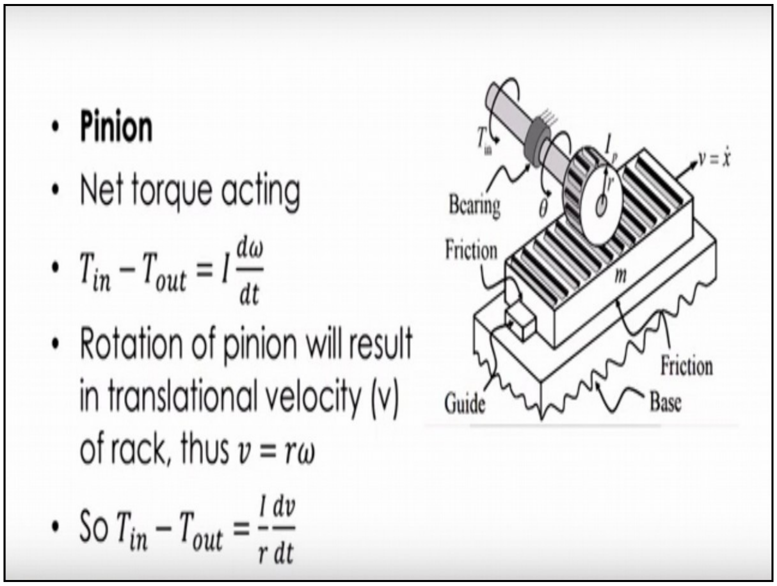

So here I have i by r d v by d t. So this is how I can write the equation for the pinion. Next let as to the look at equation for the rack .now we can find out the force acting on the rack. I just said t out is the torque which is being acting here so I can find out the force acting on the rack as t out divided by the r. So I divide the output torque of the pinion by radius so I get tangential force which is acting on the rack and I can I assume the frictional force here to be just like that is the force acting on the damper.

That is which is the force which is proportional to the velocity so here the frictional force is model as r into the v that is the velocity of the rack. Then I can right the equation of motion of the rack.

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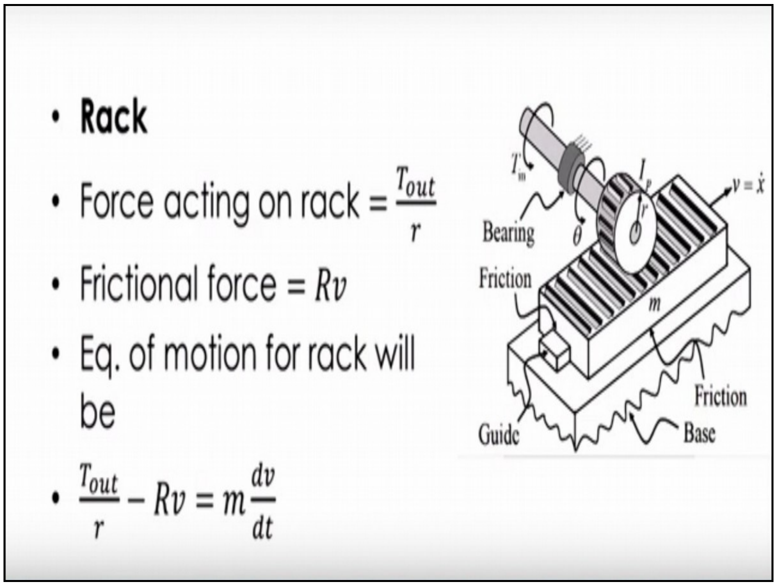

That is the net resulted force which is acting in the direction of motion here is t out by r - r v and this net force is going to be responsible for the acceleration of the rack. So this net forces t out by r - r v I am equating to mass into acceleration of the rack. So this is t out by r - r v is equal to mass into d v y d t. This how I could write it now I can combine both of this equation that is equation of the pinion and equation of the rack ab in other to right the system equation.

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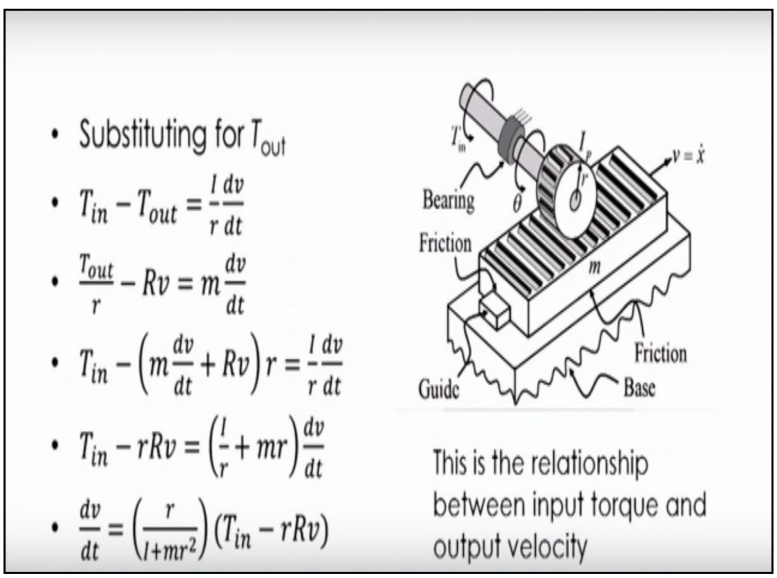

So substituting for substituting for the t out substituting for t out here this was our basic equation for the pinion t in - t out is equal to i by r d v by d t and this was equation for the rack that is t out by r - r v is equal to m d v by d t. So if I substitute for t out here from this equation into this equation this is what I get t in - this will be basically m d v y d t + r v into r so m d v y d t r v + into r is equal to i by r d v y d t.

So this expression is by combine expression. I hope you understand this so this expression is combined expression which I got by eliminating this t out from the first and second equation. So now I can further right are simplify this equation in terms of the input and output and this how I do it t in - r r v is equal to here i by r is here I can take this d v y d t common. So d v y d t am taking common here so here i by r and + mr this is term which am getting. And as my aim is to know the velocity.

So am writing this is the friction as d v y d t is equal to now I can write t in - r r v into r by i + mr is squared. So this is my system equation for this rack and pinion arrangement. And you can that this is the relationship between what my input that is t in is and what my output that is the velocity is.

And it's first other differential equation alright so this way I can model the combination of rotary and translatory system. And here whatever I have explained to you I have followed the approach of the free body diagram. So I have draw in the free body for pinion first and then I have draw in the free body for the rack and I have written the forces acting on this pinion and rack and then those forces I have equated to equal to the corresponding initial term.

And this how I got this equation. We can get the same equation in fact using the bond graph so I just want to remain you of the bond graph. We have because we have already studied this bond graph modeling so here I can just draw bond graph here that torque in is here.

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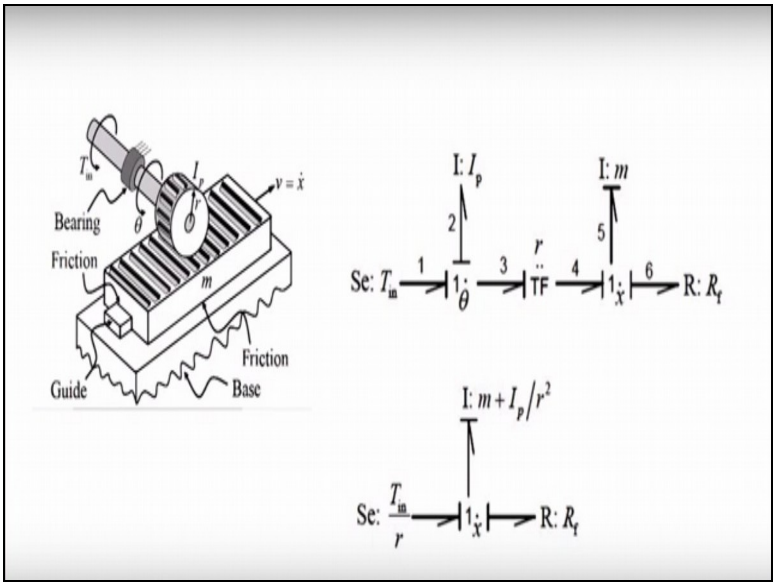

So I just write a source of effect element is in here. And this pulley as that moment of the intarsia got inertia puller moment of inertia ip here so I Attach here at one junction and then your rotary motion is being converted into that translatory motion. So that I am able to do by am applying an a transformer element here of modulus are and so I reach the velocity of the rack here and at this point I attach the inertia of the rack that is am here the mass are the rack and friction between the rack and the guide.

And that modeling here by r is equal to rf and I can causal here. So you can see if you causal it here then you can see the oilmen comes under differential causality and this is usually that case why because a two inertias or being related so you have differential causality.

Here but I can simplify this bond graph model into this form we are I can write are the force acting as t in by r and the combined inertia of the rack and pinion I can write as m + ip f on r is squared and of course this is my the friction at the guide way. And then of course you can write the equation for these that is t in what are the net forces coming here.

So t in by r is the input and these are the two r output. So if you take a force balance here you will be able to get the same differential equation this one. So I leave it as an exercise to you here you can try to work it that out through the bond graph modeling to derive the system equation for this system.

And those who you are interested in this for that and if you find the difficulties in doing this you can look at the intelligent mechatronics system book written by me and my colleagues. And there as soon the details derivations of the derivations of the system equations using bond graph which I am not discussed here.

Next let us I take the example as I promise here I am taking the example of the pulley and cable system. Now in this pulley and cable system you can see that there is a pulley .which is mounted on the ideal bearing and there is a mass less rigid shaft.

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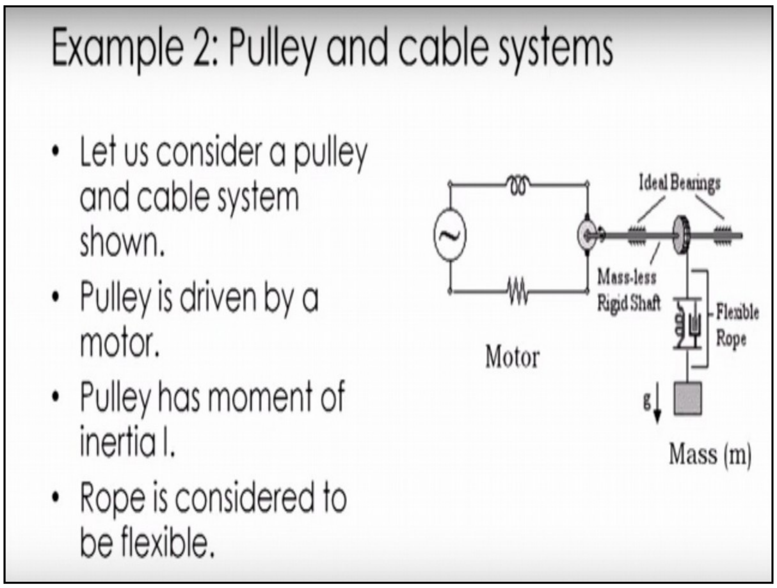

Alright and in this pulley the wire is wound .and through this wire the mass is hanged. And for rotary motion of the pulley is the pulley is turned by that is the pulley is mounted on the shaft and the shaft is turned by a motor. So here this circuit which you have seen that is basically the circuit of the armature of the motor where there is a voltage source and there is an inductor and there is a resistor here.

And of course here the when we are going to model the electric source will be having back emf. Which will be coming here coming on acting on the motor? So let us see the here how can we model this rotary and translatory system. So as I said I will be following the approach of the free body diagram here and I will give you a clue for the bond graph modeling of this and I will suggest to you derive the system equation yourself and tally the results.

Then with the by doing so you will appreciate the power of bond graph .so as I said in this pulley and cable system the pulley is driven by the motor and the pulley is got a moment of inertia I and I considering rope to be flexible. That is this rope is not a resist rope I am considering this rope to be a flexible rope.

So first let us model the motor. So modeling of motor here we can use the Kirchhoff's voltage law. That is what is a kirchoffs voltage law that is in a loop the voltage a across the all the samisen of the voltages across all the elements is going to equal to zero.

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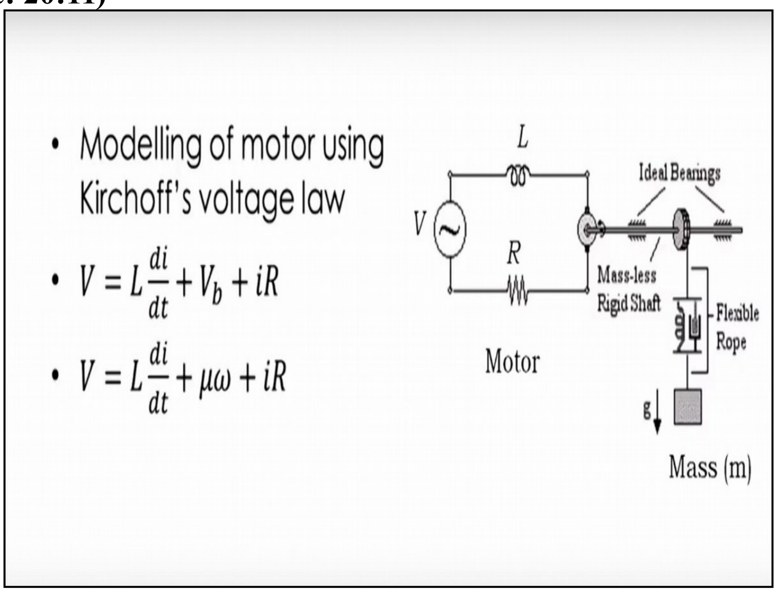

So here this voltage v which is being supplied this is going to be equal to what we have l d i by d t that is the voltage across the inductor + vb that is the back emf + the voltage across the resistor. So this is how it is going to be and I can for the write for the expression v is equal to l d i by d t + and the vb is basically I am writing as mu into w. Where the mu is the torque concept of the motor.

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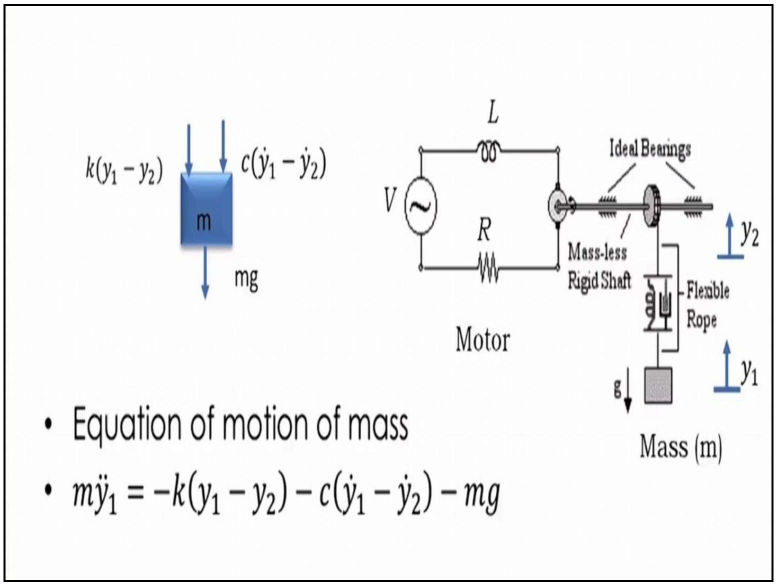

And omega is the angular speed of the motor Shaft. So this completes the modeling of this electrical circuit. Next let us consider the modeling of this block. So here in order to model it you can see that I just take a coordinate system y one here and y two here so this is nothing but spring of damper system type of situation.

So let us us draw the free body diagram for this mass now in this mass .the way to set is here mg. And for this as the positive direction of the motion here the force applied by that is spring and damper are going to be this one.

The force did to spring is K(Y1 - Y2) this is the Y1 and Y2 this is the direction so the relative displacement will be Y1 - Y2 so this is K(y1 - Y2) and the damping force will be C (y1 dot - Y2 dot)where C is the damping coefficient. That is the wire which I am modeling as spring damper system. So for the damping I am repressing ting the damping coefficient of the wire at C. Now here we can find out the net force or the total force acting in the direction of the y1 in this mass.

So the total force acting in this direction is going to be - of this K(y1 - y2) - of C(Y1 dot - Y2) and - mg why I am putting - because this force direction is opposite to my assumed positive sense direction that is y1. And this complete I am equating to m y1 double dot. So these are the total forces here and these forces are going to be responsible for acceleration of this mass and so that is why I am equating it to m y1 double dot. Now let us see the equating of motion for the pulley.

Now for pulley for pulley actually here there is little mistake this I am should not be there. This l is should not be there so now the forces which are going to act on pulley these are this is my positive senses direction for of y2. So for this positive sense direction the spring I am damping forces are k y2 - y1 here and see y2dot - y1dot here for this one.

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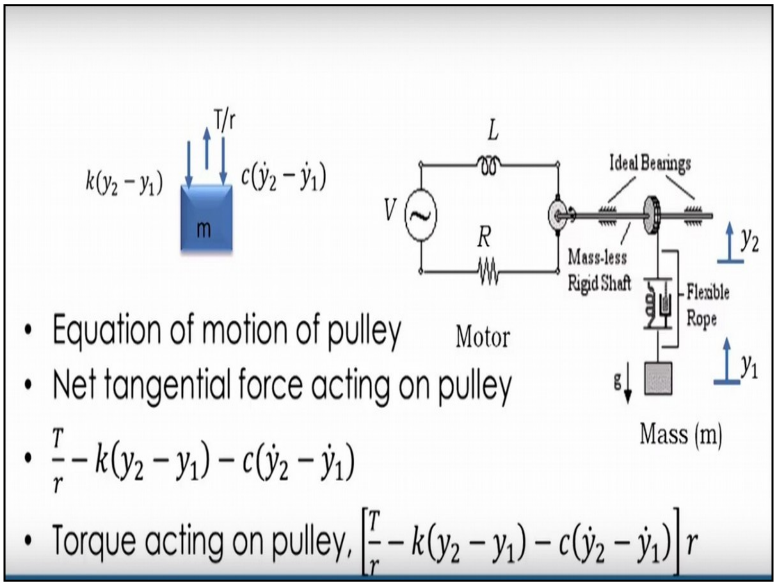

And the force tangential force acting and the pulley will be that torque which is acting on the shaft divided by the radius of the pulley. And this is going to acting in the positive direction upward direction .so to write the equation of motion of pulley I find out the net tangential force of the pulley. So net tangential force acting on the pulley will be t by r - k y 2 – y 1 - c y 2 dot – y 1dot.

So this is my net tangential force acting on the pulley. And then I can final out the torque acting on the pulley. So what I do this net tangential force I multiplied with the radius. So I get the torque acting on the pulley and now this torque which I have got I equate to I omega. Dot where I am the pulley movement of inertia of the pulley and omega dot is the rotary acceleration of the pulley.

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- $I\dot{\omega} = \left[\frac{T}{r} - k(y_2 - y_1) - c(\dot{y}_2 - \dot{y}_1)\right] r$
- $I\dot{\omega} = T - kr(y_2 - y_1) - cr(\dot{y}_2 - \dot{y}_1)$
- Summary
- $V = L\frac{di}{dt} + \mu\omega + iR$
- $m\ddot{y}_1 = -k(y_1 - y_2) - c(\dot{y}_1 - r\omega) - mg$
- $I\dot{\omega} = \mu i - kr(y_2 - y_1) - cr(r\omega - \dot{y}_1)$

So this I can further simplify I omega dot is equal to I multiply this are inside. So what I get t - k are y 2 – y 1 – c r y 2 dot – y 1 dot and this way. I get the equation of motion for the pulley. So if I summaries the first equation is basically the equation for the motor. So this is the v is the input voltage supplied to the motor this is the voltage drop because of the inductor and here this is the back emf and this term is basically voltage drop across the resistor.

And this equation is for my mass. The equation for the mass that is the translatory of the mass and this expression is for the rotary motion of the pulley. Now here further what I have done is that the y 2 I have represented as are omega. Where omega is the rotary that is the angular velocity of the pulley. So that omega is there I multiplied by are so I get the velocity form and that is same as that of y 2 dot.

So that I have done and this torque .I have replaced by I into me because torque generated by motor is a given by motor torque constant into I so now in this way we can write the system equation for combination of that translator and rotary system and in this case also for the electoral circuit.

And those who have interested in bond graph I can just draw the bond graph model for this system and you can write out the derivation of system equation using this band graph. So first I model the electrical portion where I have the voltage source I have the inductor I have the resistor here. Remember in the bond graph the back emf them will be coming over here. So I model the model as motor and gy with the torque constant as mu which is represented by the modules of the generator.

Then I came here so I have the rotary inertia of the pulley so here this is by I of the pulley remember this is the angular velocity port in the band graph terminology. And after this rotary inertia is being converted sorry rotary motion is being converted in translator motion so I put it transformer with modules as are so here I have breezed up to this velocity.

And between this velocity and this velocity of the mass I can will be decided by the inertia. That is the mass here and between these two velocities I have got the spring mass damper. System which is nothing but which represents the damping and the stiffness presents in the wire. So this way I can draw the band graph model for this system.

I can causality also like this way t and this way here and them I can call the i here flow so effort here so flow so effort here how can model the inter system entire and then you can draw the system equation. This part is basically here motor and here. Using this are I am converting the rotary motion into the translator motion.

So this is how it could be done and this I can write the stiffness and this is my c which I have used in Terminology it deriving the system equation and here is the torque and angular velocity of the motor. And here this is back emf which have been talking about and here this is the current.

So these are the effort and flow variable so for me year you can see is back emf vb is mu times omega and this torque is mu times

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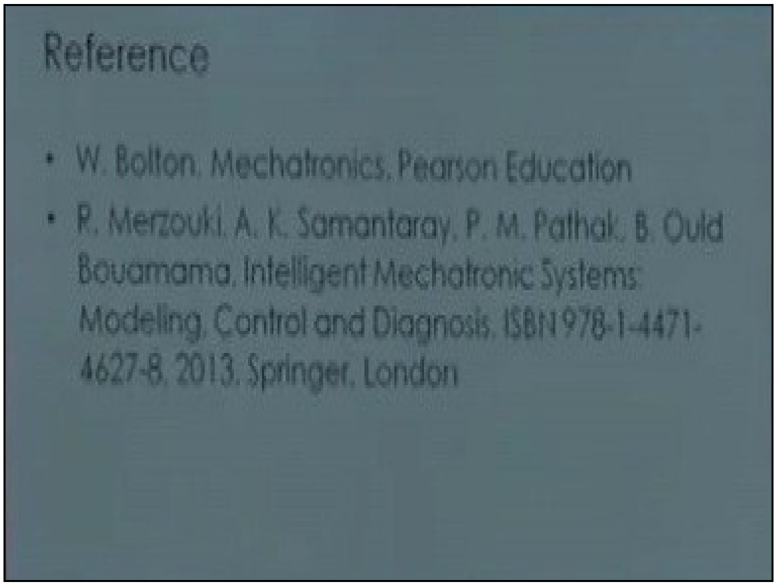

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