Modelling and Simulation of Dynamic Systems
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Lecture - 16
System models - Linearity and non linearity in systems

In this lecture linearity and nonlinearity in systems and this is a part of our course that is modeling and simulation of dynamic system. When we talk about modeling of a system it is very important to understand whether your system is linear or it is nonlinear. Truly speaking in nature most of the things are nonlinear. But the problems with the nonlinear systems are that if you want to analyze it is difficult and there is no generalized method to will deal with the nonlinear system.

So there the actual problem comes so hear in this lecture we will be see lo ing at how can we linearize these non linear system and we will take up some of the examples. So previous few lectures we have been seen the individual building blocks which comprises of systems in mechanical electrical pneumatic hydraulic and thermal now the real physical system will be combination of any one of this.

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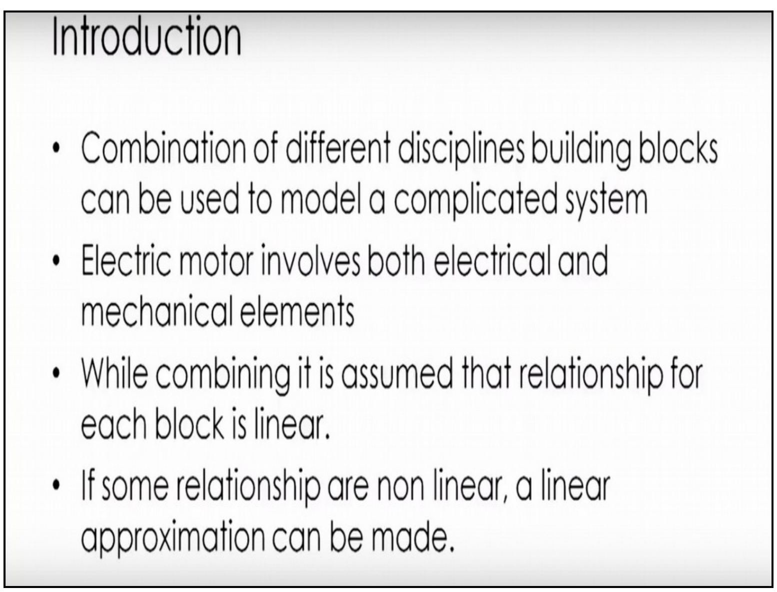

So we will be coming across combination of different discipline building block. And these different discipline building blocks can be used to model a complicated system. If I take an example of electric motor then here you see that involves both electrical and mechanical elements. The electrical elements are armature Induct ends of armature resistance of armature and the mechanical element could be the load which is going to drive so that is going to be there.

Now when we combine these two different systems mechanical and electrical it assumes that the relationship for each block is linear. And with this assumption only we will be able to combine these different blocks. If some relationships are no Linear A linear approximation can be made for such type of cases. Now let us see the characteristic curves of various type of nonlinearities.

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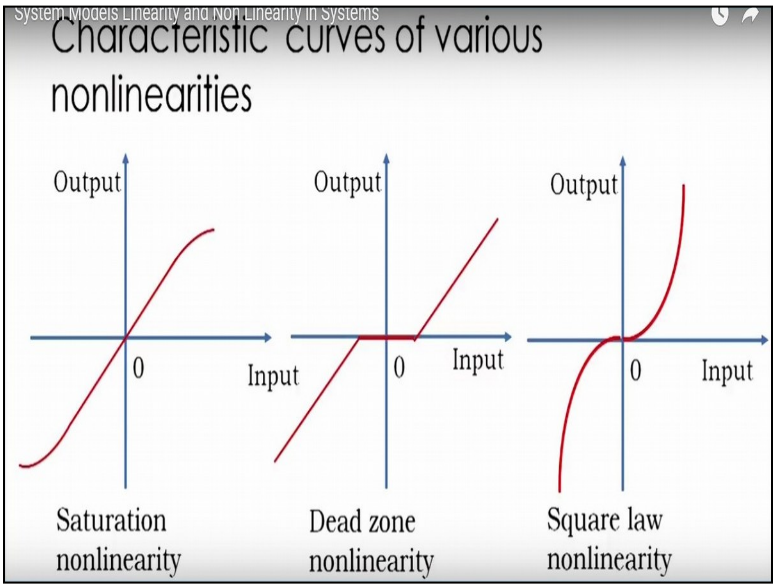

The first one you can see that with increase in input the output also increases. But after a certain value of input there, output remains almost constant and this type of behavior is call it as the saturation nonlinearity. Other type of nonlinearity could be the dead zone nonlinearity. If we lo at this graph now here in see that for a certain value of input there is no response for the output we don't have any output response and of course after sometime output changes with increase in input.

So these types of nonlinearity call it as dead zone nonlinearity. That is there is no response or there is no output for the given input. And other type of nonlinearity could be the square law nonlinearity that is here if you see the input and output then you can see the output varies almost with the square way of the input. So if I lo at some of the reasons, causes or examples for these types of nonlinearities the saturation as I said

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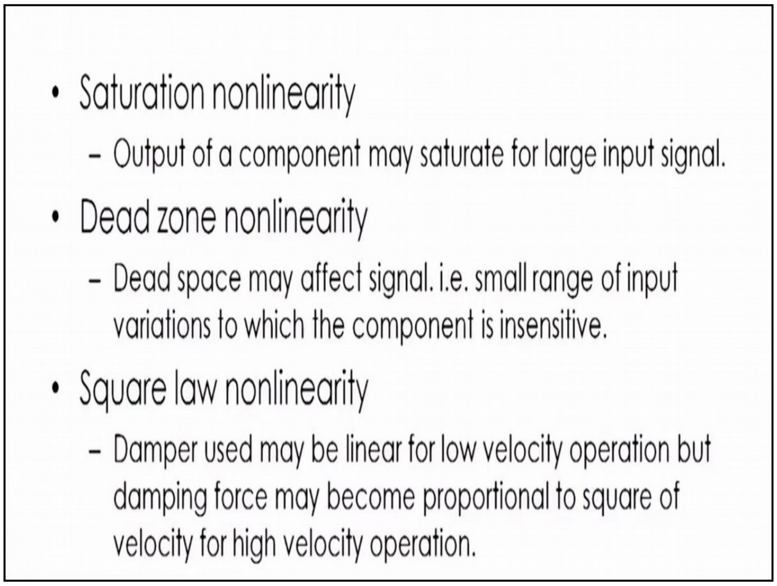

Saturation nonlinearity output of a component may saturate for large input signal. So if the input signal is large then the output may saturate it may not vary at all for the given input. So this type of nonlinearity as I said we call it as the saturation nonlinearity.

Then the dead zone nonlinearity as I said dead space may affect signals. That is small range of input variations to which your component becomes insensitive and you don't have any output forming and square law nonlinearity. If I want to give an example, we all know that the damping force for a mechanical damper is given by damping coefficient into velocity $F=C\ V$ now here you can see that F is proportional to V.

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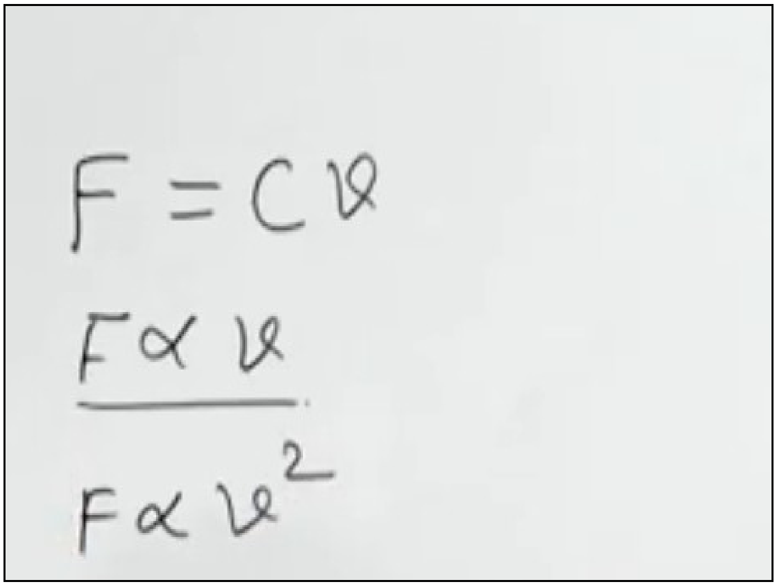

So this is basically the linear behavior. But if it may happen that the damping force may become proportional to square of velocity for high velocity operation. Then this equation will be becoming like f proportional to v square and this type of nonlinearity what we call it as the square nonlinearity. So there are many other ways of nonlinearity appearing in the system. For example A if your system equation has got some sin and cosine terms trigonometrically terms.

They also make your system in nonlinear. Now let's lo at how do we decide whether a system is linear or non linear. So, to tell that next take a simple example of the spring we are all have aware of I have discussed a lot about a spring. It is a mechanical system element and we have seen this. When we are discussing about the mechanical building blocks.

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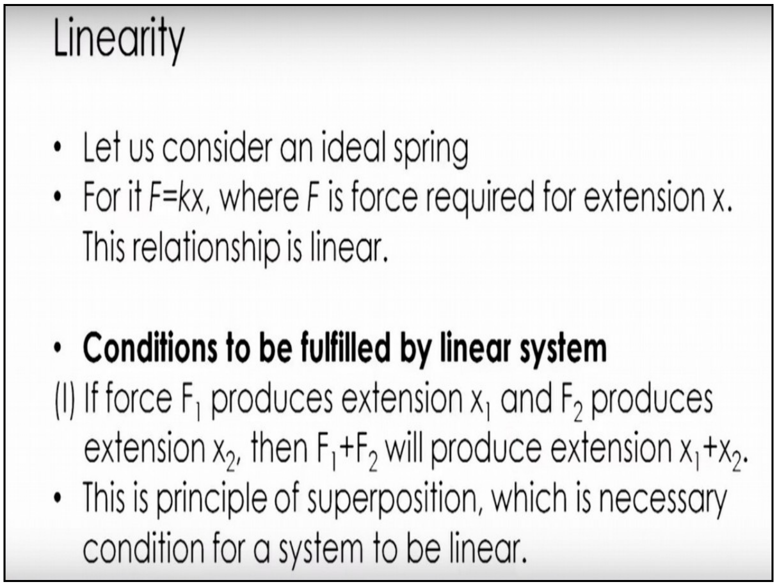

Now if I take an ideal spring then for ideal spring. We know that force is given by k into x where k is the stiffness and access the extension in the spring or what we can that the force is proportional to the extension x. And this relational ship is what we call it as linear. Now there is characteristic of a linear system and that characteristic is that they fulfill.

The principal of super position and that position of super position is for a given input if you have some output. And then for given another input, we have some another output then you can have the combinations possible. And this is what we call it as the principal of super position. I will explain it in detail whatever spring example. I was taking if a force F1 produces extension x1 and another force F2 produces extension of x2.

Then F and applying a force F1+F2 then it will Produce an extension x1+x2. And this principle as I said is what we call it as the principle of superposition to proportion and this is remember a necessary condition for the system to be linear. Another condition which we can that is the another property which is exhibited by the linear system is this one that is if force F1 produces an extension x1. Then, some multiplication of F1, c f 1 is going to produce an extension cx1 and where c is a constant multiplier.

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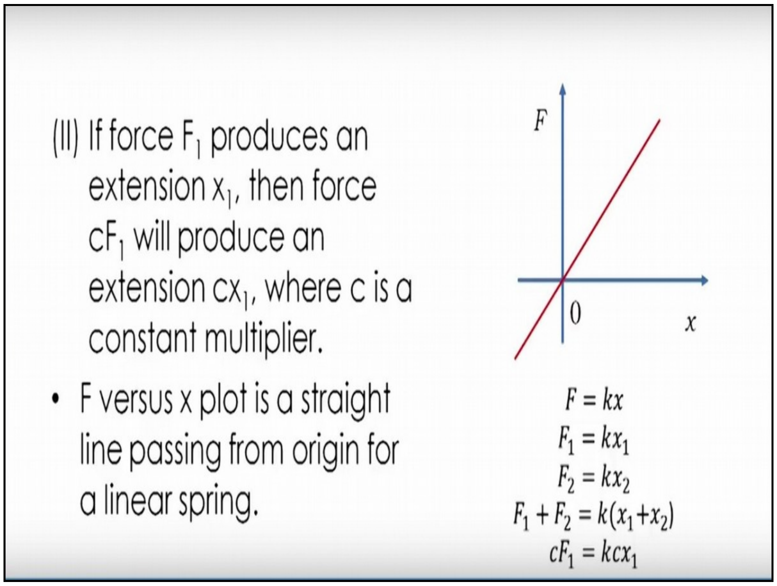

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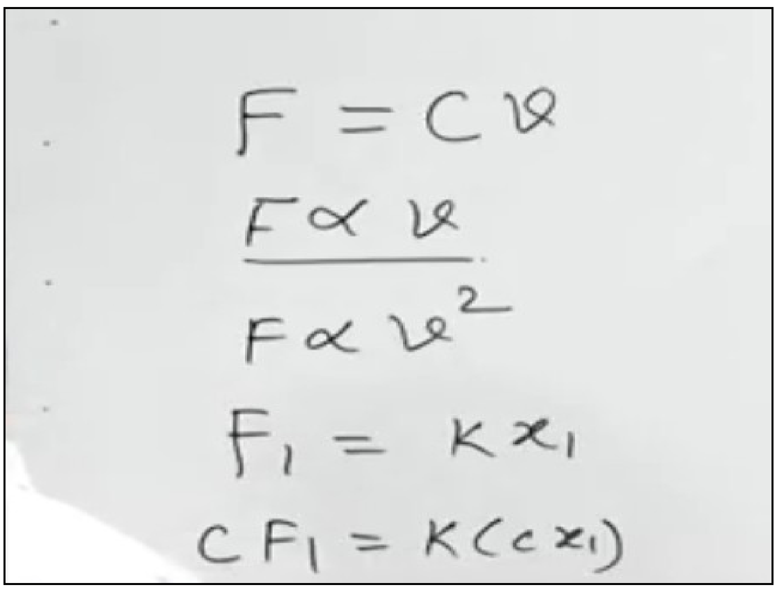

So, the thing is that if you have F1=k x1, and then if I multiplying here so cf1 will be basically k times cx1. This is what I mean by so these two conditions are fulfilled by a linear system. So let us see graphically this suppose we have a spring linear is spring and if I plot the force verses is x here then you can see that F= k x which is the characteristics for the spring and F1=k x 1 and f2=k x 2 then if i combine the f1+f2 i should get k(x1+x2) likewise if I put cf1 should get c times kx1.

So, if this behavior this type of behavior is exhibited by any system then we calls the system to be a linear system. Now the thing is that as I told to you in my opening remarks for this chapter that in nature most of the things are nonlinear then the problem is how to analyze those systems.

So, the real spring is a nonlinear spring as you can see in the graph here if i plot F verses x for a real spring then you can see that there is certain portion here up to which its behavior is linear. But after that the nonlinearity is coming. So, how do we analyze this so what actually is done that we assume linearity for some range of operation.

And of course we ensure that our spring operates in that particular range so if we do this then our linear assumption is going to be valid. Now here as I said in this figure if we assume that spring operates the central zone, then linearity approximation is going to be valid in this case.

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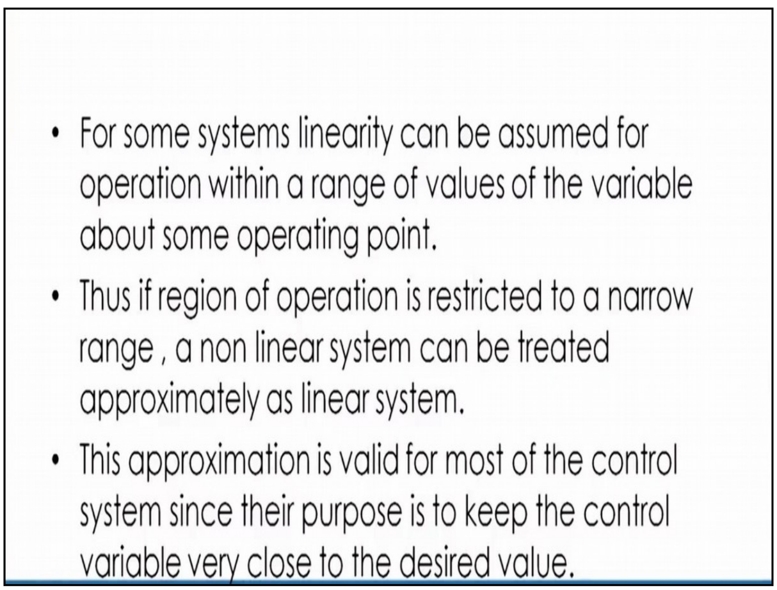

Now, for some systems linearity can be assumed for operation within a range of values of the variables about some operating point. If this case we assume linearity about said in the central region. But it is not always necessary that thing should be linear in the central region only. So it could be linear anywhere. So we can assume operation within the range of values of the variable, about some operating point.

Thus, if region is region of operation is restricted to a narrow range a nonlinear system can be treated approximately as a linear system. If i am assuming that my operation is going to be there in that is small region where the system is linear. This need not be in the central band as i said. Now, this approximation is valid for most of the control system. Why because, control system purpose is to keep the control variable very close to the desired value.

So the control system will always try to keep your variable within a small range of desired value. So as i said within a small range, if i am talking about then naturally in that a small range my system is going to be linear. And this is one of the very reasons for assuming the linear behavior of a control system.

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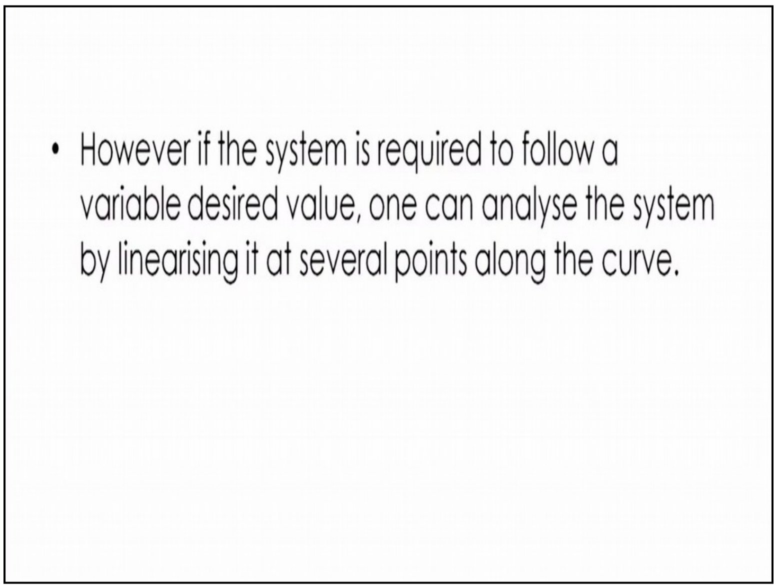

However, if the system is required to follow a variable desired value. In that case, one can analyze the system by liberalizing it at several points along the curve. Now so what should be procedure for liberalizing a nonlinear system? So let’s see actually we do that. How actually we linearism a nonlinear system. So, this is what i was talking to you that is for a nonlinear system to linearize, what we can do the following.

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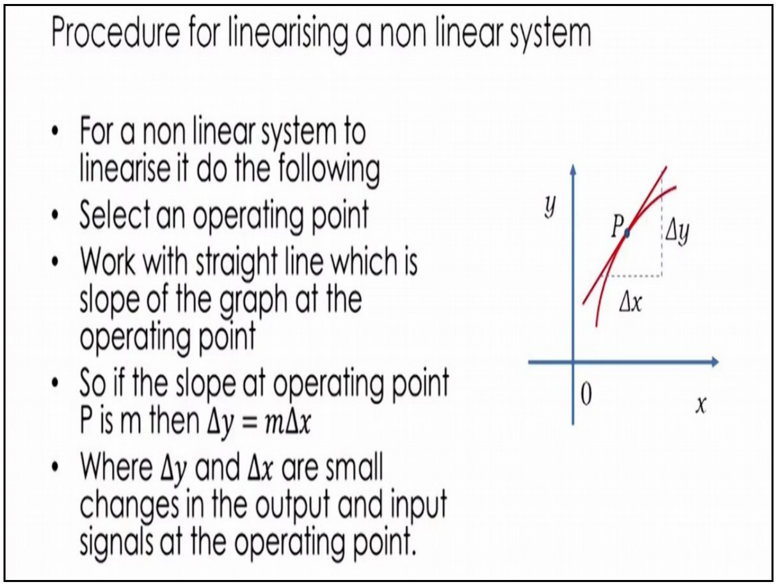

That is here suppose this is by output versus input curve for a system. And as you can see the behavior of this curve its a nonlinear curve. So, to analyze this or linearize this actually is done is that we select an operating point. Here, i am selecting the P as the operating point.

And then work with straight line which is slop of the graph at that operating point. So i draw a tangent at this operating point P. And then i work with this tangent. Work with this straight line and which is the slope of the graph at this operating point. And you know as if this element if his segment of length delta y and this segment is of delta x then i can always write delta y=m delta x. Which of course comes from the definition of the slope. That is tan theta.

Delta y by delta x that is tan theta. And which we represent as an m. So here what actually we have done that that we have represented this curve, by this straight line and the expression for the straight line is delta y=m delta x. And this is how i am able to linearize this curve at this particular operating point P. And as i said here delta y and delta x are small changes in the output and input signals at the operating point. Now we can see this thing this delta y=m delta x mathematically also.

So if i lo at the curve again. For a general element with input x t and output y t the input output relationship as i said is shown here which is actually a nonlinear relation. Here the relationship is nonlinear but of course i am going to assume that it is continuous.

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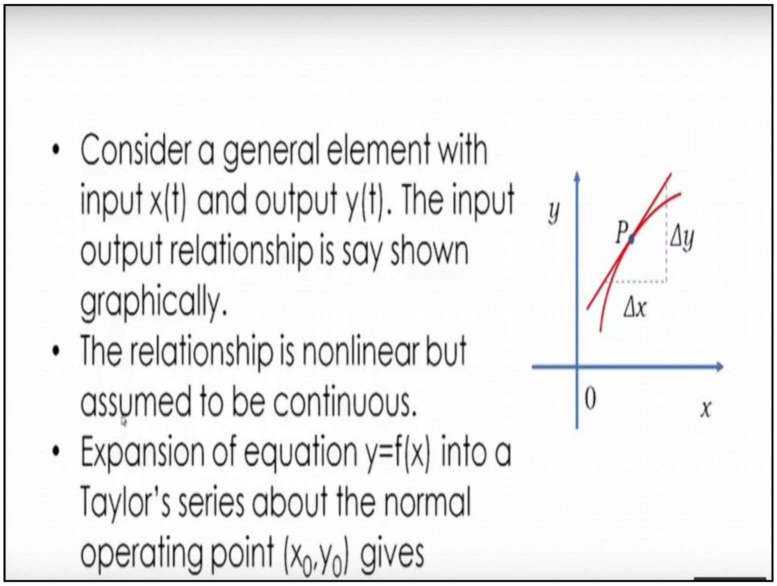

Now what we can do is that we can expand by the equation y=fx into a Taylor's series about the normal operating point x0 y0. Now if i do that this is what i am going to get the Taylor’s series expansion.

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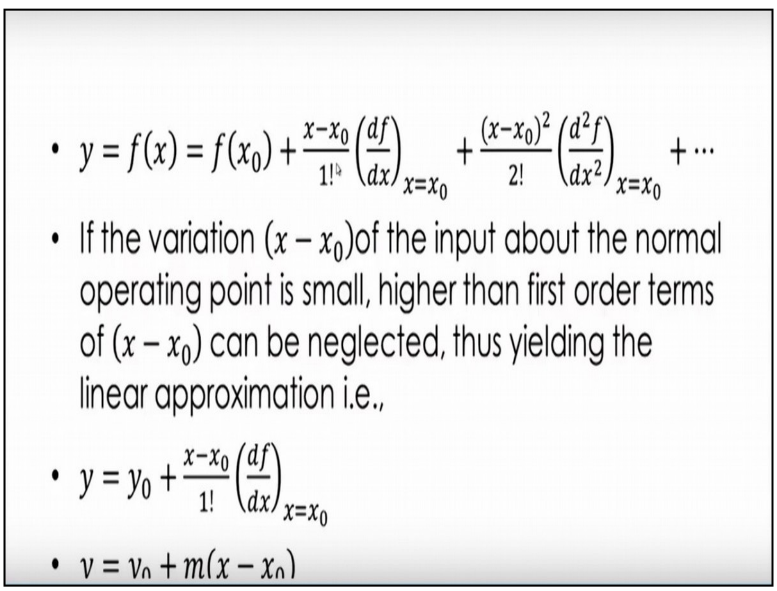

So y = fx and this is f which is fx0 where x0 is your operating point x - x0 by factorial one and evaluation of del f and del x at x=x0. Like wise you have the higher order terms Now if i assume the variation x - x0 of the input about the normal operating point to Be the small then i can neglate this higher order terms. And if i do that this is what i get y=y0+x - x0 by factorial 1 at x = x0. Or here what i have y=y0. If i represent this as m or the slope which talk to you in my previous slide so here it is m into x - x0.

Where m is as i said del df by dx at x=x0 is the slow point the operating point.

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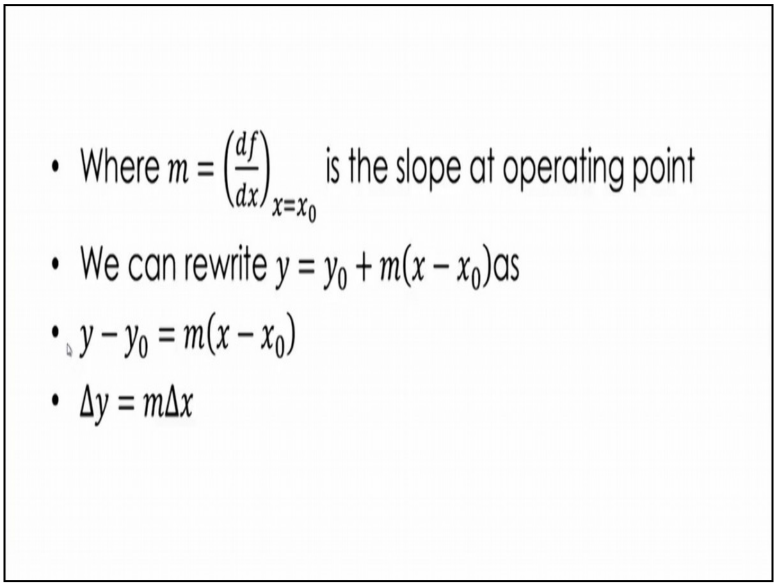

And then i can write this equation as y - y0=x - x0 or i can further write this as delta y=m delta x So this is the same equation which i showed you graphically. And here i just wanted to tell you that what happens mathematically. So basically mathematically we get the same result if we assume the x - x0 variations to be small and we can negate the higher order terms of the Taylors series expansion. Now let us take few examples so here i am going to take two examples. The first is the flow through and orifice and the second i will be taking about the thermostat. Now here you can we have a orifice.

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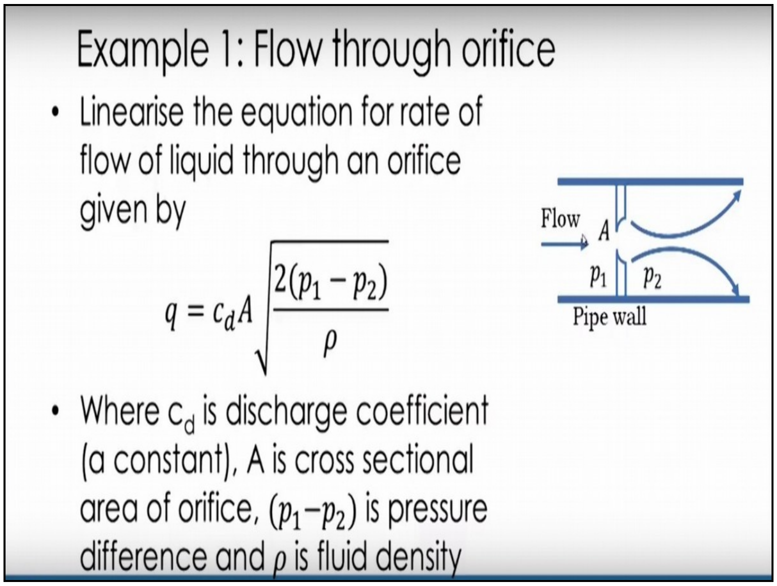

And there is a flow q taking place through this orifice and the causations area here at the orifice A and the pressure at up streams and down streamsi side are p1 and p2. Now in this case discharge equation through orifice q is given by cda under root 2 p1 - p2 by row. Where this cd is the coefficient of discharge a is the sectional area as i said and here here row is the density of the liquid. And of course p1 - p2 is the pressure difference as i said that is the pressure difference between the upstream and downstream side.

Now the question is you can see that this equation equation is the nonlinear equation. Why nonlinear because this equation involves the square root of p1 - p2 term. This equation is not a linear equation because you cannot apply the principle of super position here. Alright so our job is to linearism this equation. Lets see how we can do that.

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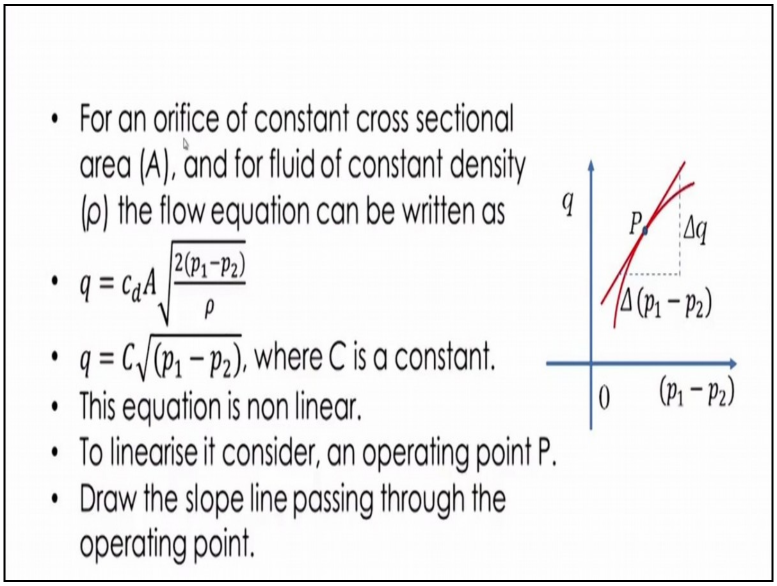

So as i said this was our basic equation and suppose I assuming this basic equation that cross sectional area of orifice is constant as well as the density of the liquid is the constant, then i can write the constant the constant terms together as I see this equation is q=c under root p1 - p2. And as i said this equation is nonlinear. Now to linearize this equation again we have to consider a certain operating point so if i lo at this plot this plot is basically between input and output so i have input p1 - p2 output q here. And if i plot this then this is my nonlinear plot.

Nonlinear variation of q with p1 - p2. Now to analyze this i take an operating point p to linearize i take the operating point P. And what we do next we a tangent a slop line passing through this operating point. Then the slop of the line can be given as you can see i can write the slop as delta q upon delta p1 - p2. But this was our basic equation so i can write dq by d p1 - p2 as this one i can differentiate this equation. So dq will be c by 2 under root p1 - p2 and p of p1 - p2.

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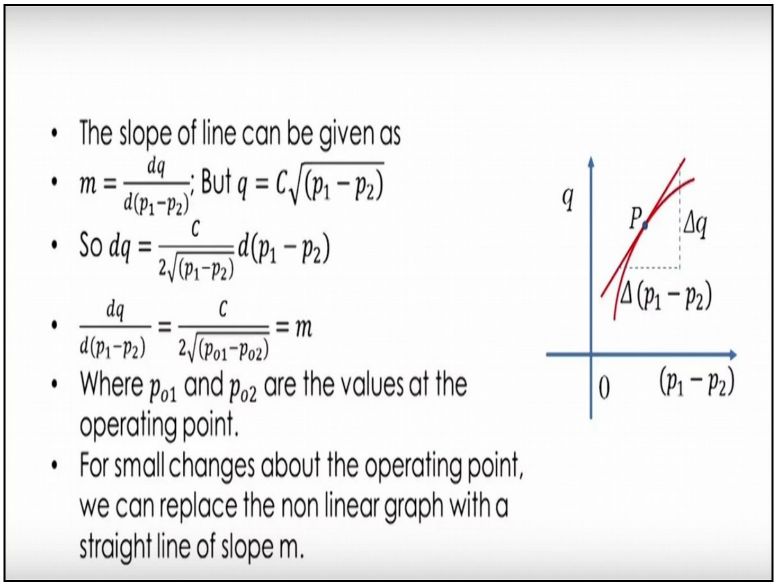

And form here i can write this value that is dq by d p1 - p2=c by 2 p01 - p02. Why p01 because i am talking about this operating point so slop is basically a constant c which comes from here. And this value at the operating point. That is in this case the value at this operating point now for a small changes about the operating point we can replace the nonlinear graph with a straight line of slope m. So this is what we have done here. The slope of the straight line can be given by delta q by delta p1 - p2.

And thus the linearize equation can be written as here del q = m del p1 - p2. And this equation in this equation m can be determine for by the values at operating point p.\

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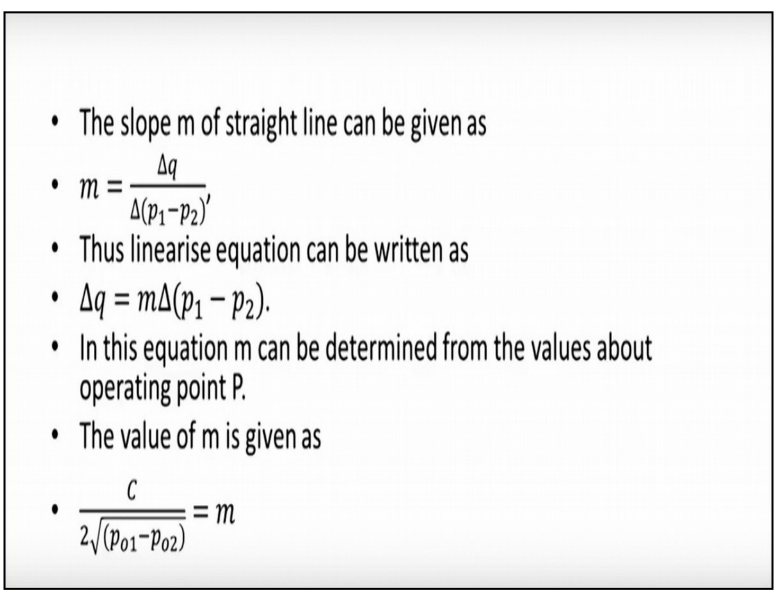

And the value of m as i said that we can get here Is equal to c by this value so i can get the one here and when i substitute this m value I get the linearize equation.

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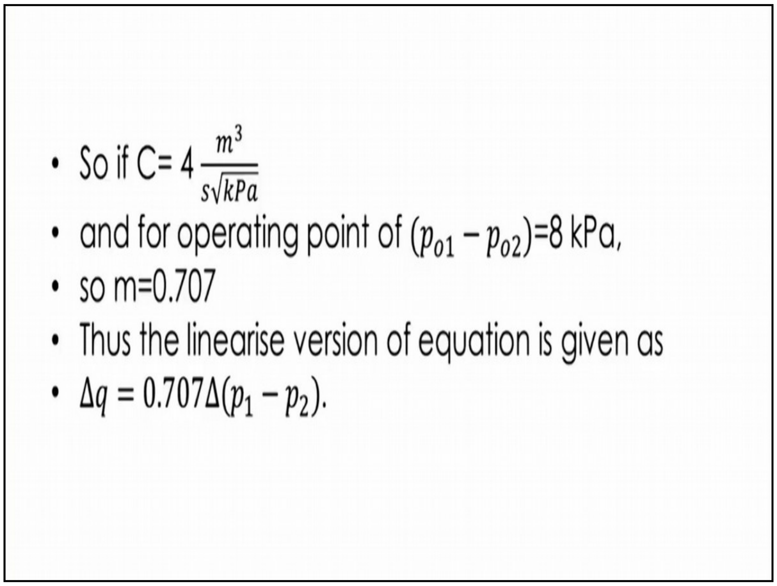

So lets lo at it, suppose if i am given a c value like this that is what actually I do is that for a given p1 - p2 here I find out what is q. And i from there I can get the value of c. If my c is this value that is 4 meter cube power second under root kilopascal and the corresponding this pressure draft well will loosea eight kilo pascal and if i substitute this i get value of m is this one and i can substitute this value of m here and this is what i get my linearized equation.

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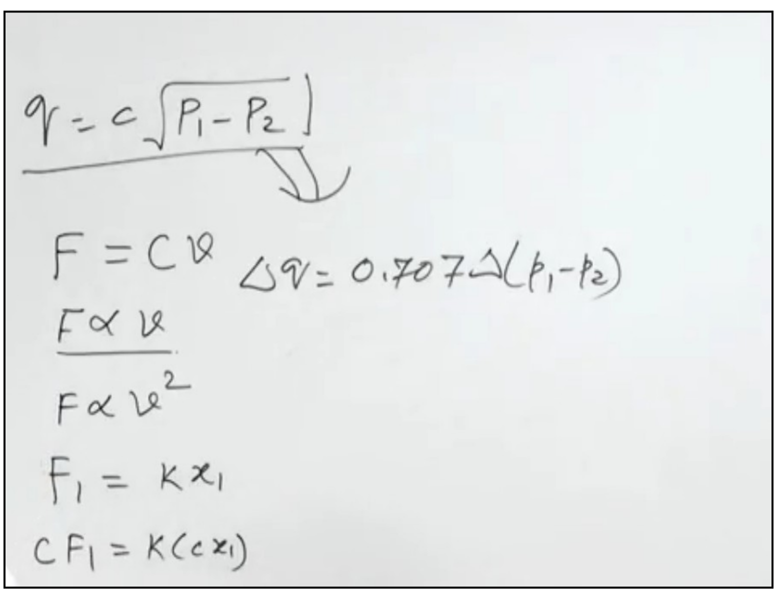

So this way i can get the linearized equation for our basic equation which you remember it was q=c under root p1 - p2. So this was the nonlinear equation and this nonlinear equation has been linearize into del q = 0.707 del p1 - p2.

So this is the linear version of it now let us take another example a thermostat. Now this thermistor as we know is used to measure the temperature. Now the relationship between there resister r of the thermistor and the temperature t is given by this equation.

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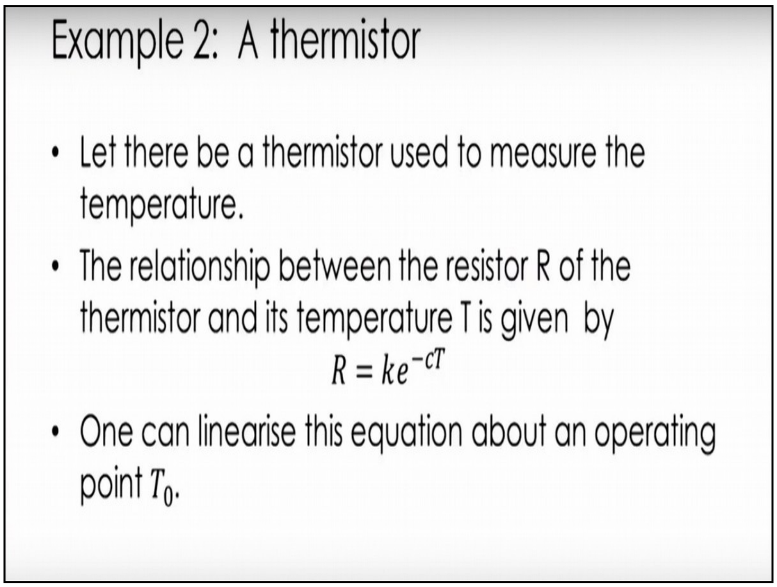

You can see that there is a exponential variation here you k and c are the constant and t is the temperature. Now I can linearizes an equation about an operating point t 0. So what happens, then the slope of the graph of r verses t at the operating point can be given by this one. That is I just differentiate this previous equation I differentiate this equation and I find out the d r by d t are the slope and this is the slope.

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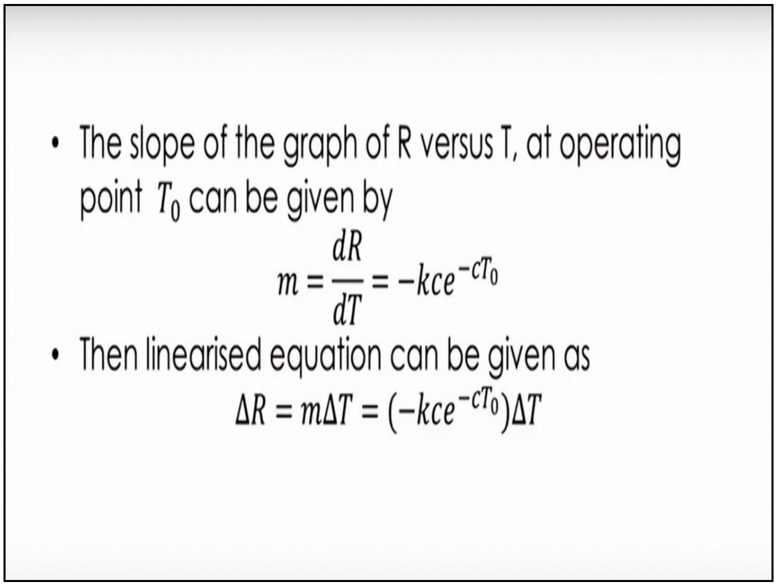

Then the linearize equation can be given by del r=m del t as we have seen previously and i can substitute the value of m from here. And here you see i know the constant k i know the constant c i know this t0 also because i know my operating point. And thus i get this equation delta r=this constant into delta t.

And this is what we call it as the linearize equation for this nonlinear equation. So the this was the small concept given to you to linearize the nonlinear equation. Now why do we want to linearize a nonlinear equation.

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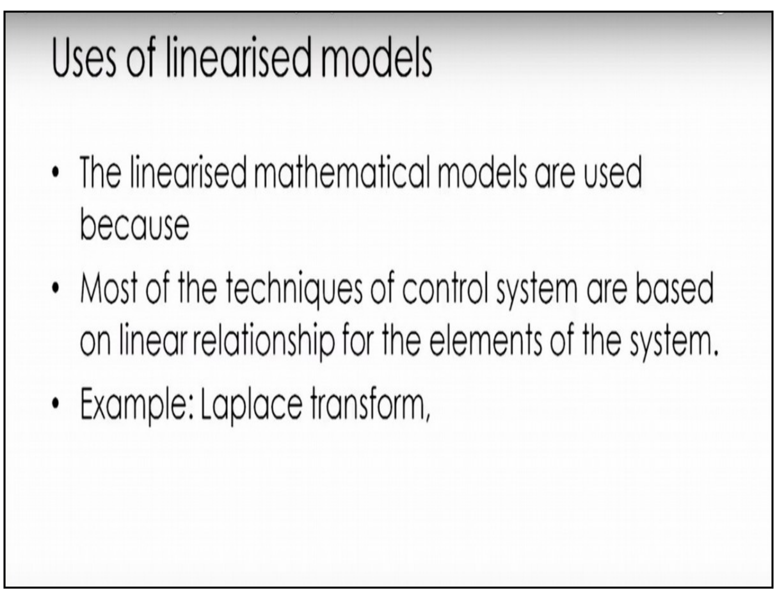

So the reason for doing this that there are ample amount of tools available for the linear systems. And once if you have linearize your nonlinear equation you can always use those tools to study the varies characteristic of your system be it is stability or be it as its frequency response or any other behavior if you want to study about that system there are many tools available. Once such tool have given the Laplace transform which all of you know that it used to convert the differential equation into an algebra equation because we are more comfortable in dealing with the algebra equation.

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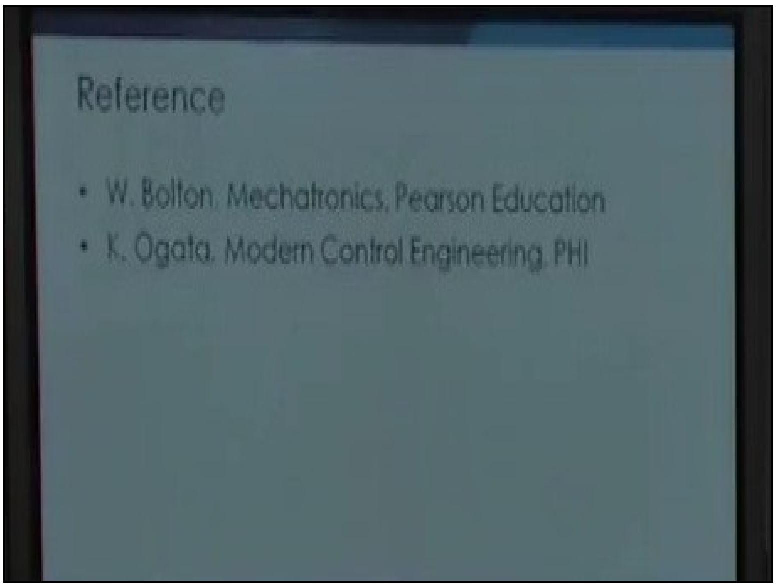

So with this I will close this talk. These are two references which you can refer if you want to read it further. Thank you