Modelling and Simulation of Dynamic Systems
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Lecture - 15
Basic systems models - Thermal Systems

In this lecture on basic system models thermal systems and of course this sub models for the course on modeling in simulation of dynamic systems. So, in this model we will be discussing about the basic building blocks required to model a thermal system, and initially we will be looking at those building blocks and then we will take up those building block examples building blocks expressions in order to model a thermal system.

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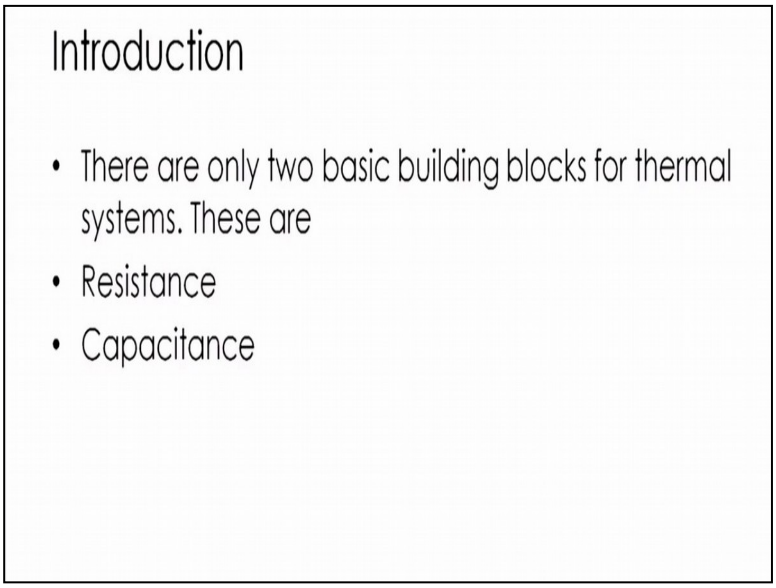

So, in thermal system there are only two basic building blocks which are used to model the thermal system. And these building blocks are the resistance and the capacitance. So, let us see one by one,

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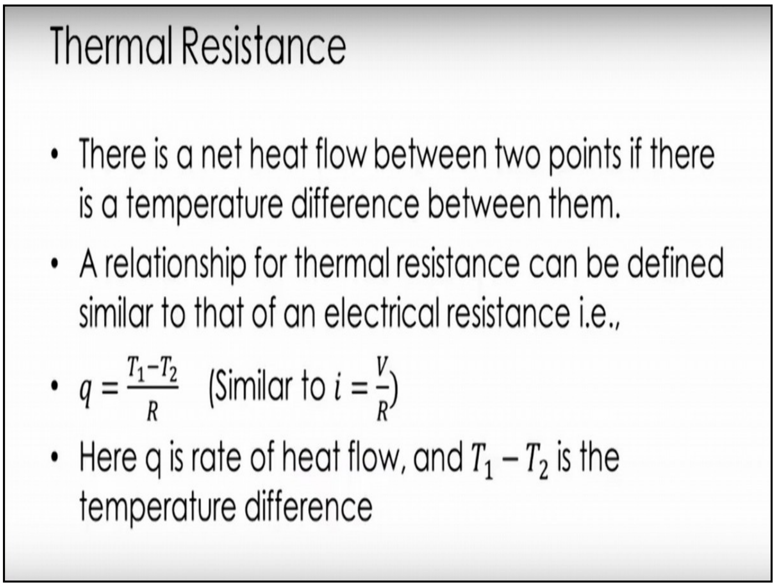

First, we take up the thermal resistance building block now a you see that there is a net heat flow between two points if there is a temperature difference between them and a relationship for thermal resistance can be defined similar to that of the electrical resistance.

That is q is equal to T 1–T 2 upon R where the T 1-T 2 these are basically the temperature difference and R is your thermal resistance and q is the rate of heat flow. So you can see that this expression is very similar to i is equal to v by r. So what does this means, this means that the temperature difference in the thermal system corresponds to the voltage drop in the electrical system and rate of heat flow in the thermal system corresponds to the current in the electrical system , and of course here we have the thermal resistance and here we have the electrical resistance.

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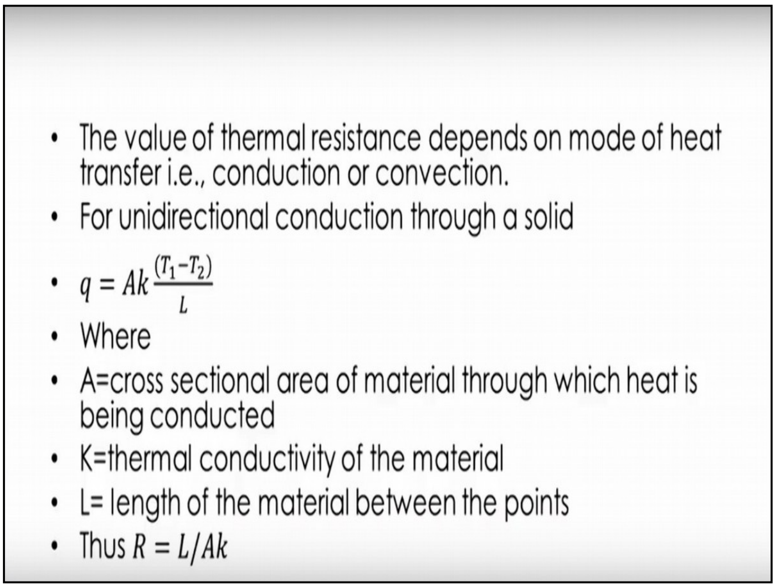

Now, the value of thermal resistance depends on the mode of heat transfer that is whether the heat transfer is taking place through connection or through conduction. Now for unidirectional conduction through a solid we know that q is equal to ak T 1-T 2 upon l. So where here you know a is the cross sectional area of material through which heat is being conducted and k is the thermal conductivity of the material and l is the length of the material between the points.

Thus if I tried to write this expression in the form of the definition which we have given for the thermal resistance this l upon ak basically belongs to the thermal resistance. So you can say that the thermal resistance for the conduction is l upon ak. That is what I was telling you that our general expression is q is equal to T 1-T 2 upon R and for conduction this q is equal to your ak and T 1-T 2 upon l or this I can write as T 1-T 2 divided by l upon ak.

So, this term corresponds to this thermal resistance alright. So this is for the conduction for conduction. So for convectional heat transfer that is liquid and gases, heat transfer to liquid and gases the relationship is q is equal to ha T 1-T 2 so this is for conduction and for convection q is equal to ha T 1-T 2 or I can write this as T 1-T 2 divided by 1 by ha. So, this basically corresponds to your thermal resistance.

So, this way here we define the thermal resistance for case of the convection now here a is what is the surface area a cross with there is a temperature difference and h is the coefficient of heat transfer and so from here r is equal to 1 by ha. Next building block is the thermal capacitance.

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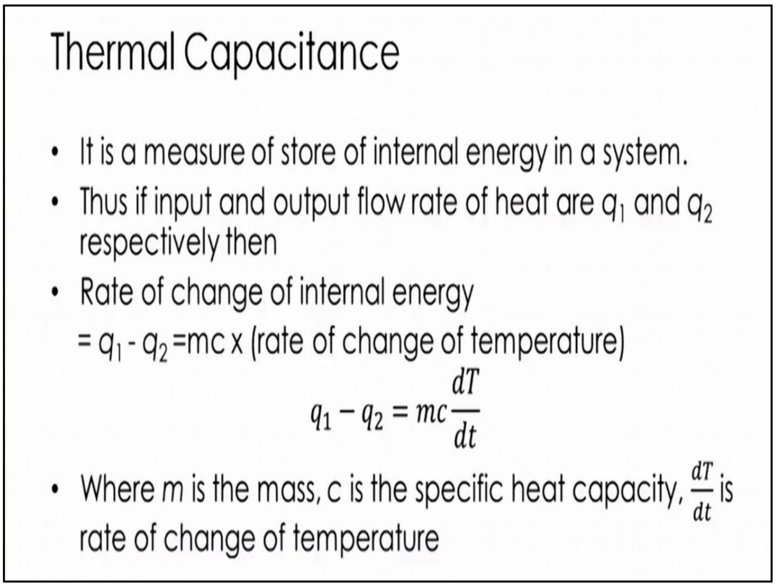

Now the thermal capacitance is a measure of a store of internal energy in a system. So if input and output flow rate of heat are q1 and q2 then the rate of change of internal energy is equal to q 1-q 2 and what is this, this is basically equal to mc into rate of change of temperature, where what is m - m is the mass and c is the specific heat capacity. So I can write this as mc into dt by dt and this dt by dt is the rate of change of temperature.

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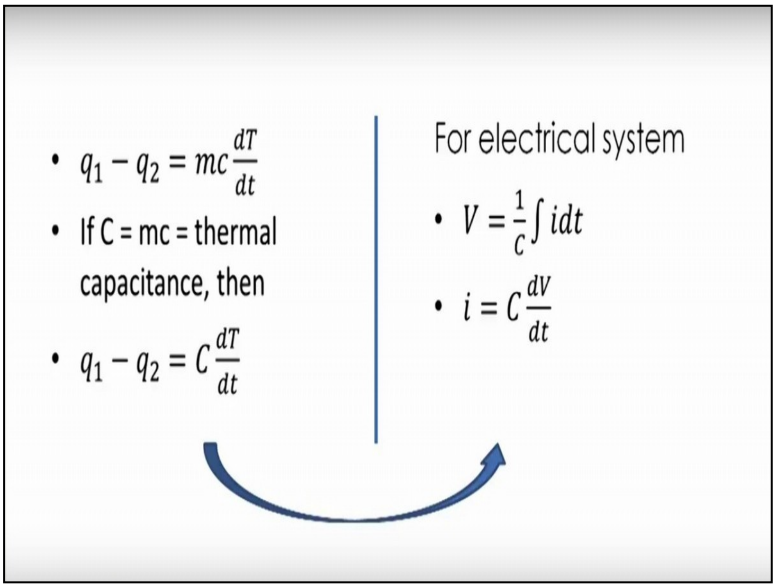

So this way we define this and this q 1-q 2 is equal to mc dt by dt I can give a single notation for this m into c which we call it as the thermal capacitance so this is what we get q 1-q 2 is equal to c dt by dt and if I compare it with the electrical system then my v is equal to 1 by c basically this is the q term that is charge across the capacitor plate and when expressed in terms of current it is integral idt.

So form here I can write i is equal to c dv by dt. Now if I compare these two expressions that is of electrical system and that of the thermal system then you can see that i corresponds to q 1-q 2 here and v corresponds to t here and then that we can define the thermal capacitance using this equation . Now using these two expressions for the thermal capacitance and thermal resistance we can model any thermal system so let us see.

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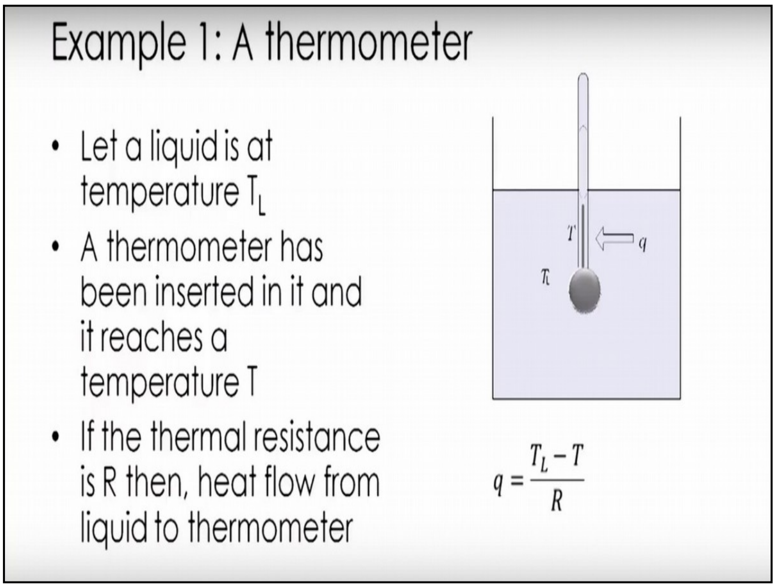

First of all, we take up an example of a thermometer alright so suppose this is a container and in this container said there is a liquid whose temperature is Tl I insert a thermometer in this liquid so happens there is a heat transfer form liquid to the thermometer and because of that say the temperature are the mercury level indicates the temperature t there by the thermometer . So here as I just said the liquid at temperature tl and thermometer has been inserted and it reaches a temperature t which is indicated by the height of the mercury column in the thermometer.

Now, if the thermal resistance is r then by heat flow form liquid to the thermometer. This can be given by q is equal to tl -t by r. So this equation we got form the definition of the thermal resistance so here the r is the thermal resistance q is the heat flow from the liquid to thermometer and tl -t is the temperature drop .

Now, after defining the thermal resistance we can define the thermal capacitance and then we combine both of these expressions that is the expression of thermal capacitance and the thermal resistance.

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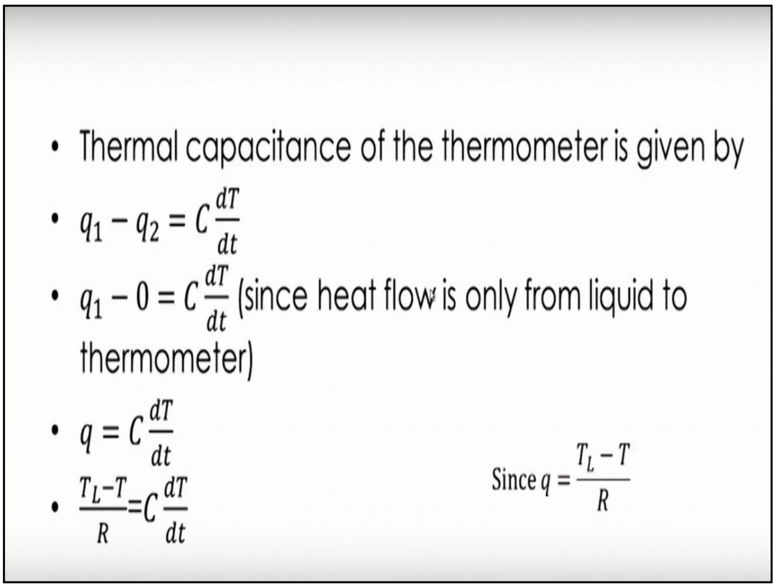

So, the thermal capacitance of the thermometer is given by q 1-q 2 is equal to c dt by dt. We have just seen from the definition of this thermal capacitance building block. Now here q2 is 0 since heat flow is only from liquid to the thermometer. So I take this q2 is equal to 0 so what I have q1 let me just write it is q here is equal to c dt by dt .

This q I can replace from the definition of the thermal resistance that is tl - t by r. So if I do that, this is what I get tl -t by r is equal to c dt by dt, or I can write this expression further as rc dt by dt + t is equal to tl. So, here you see what is our input the input here is the liquid temperature tl and what is my output is the temperature of the thermometer or the temperature displayed by the thermometer.

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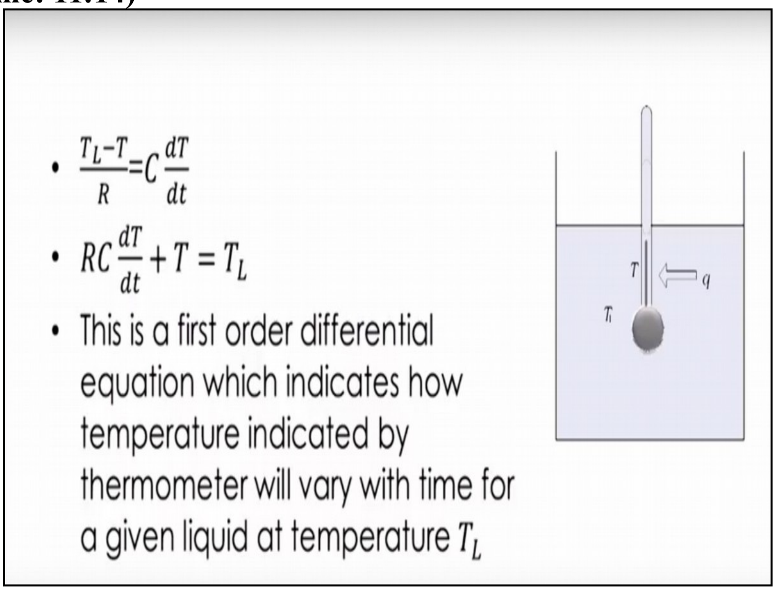

So, this expression is basically the first order differential equation and this states that basically how the temperature of the thermometer or the temperature indicated by the thermometer is going to change with time for the given input tl for the given input tl here

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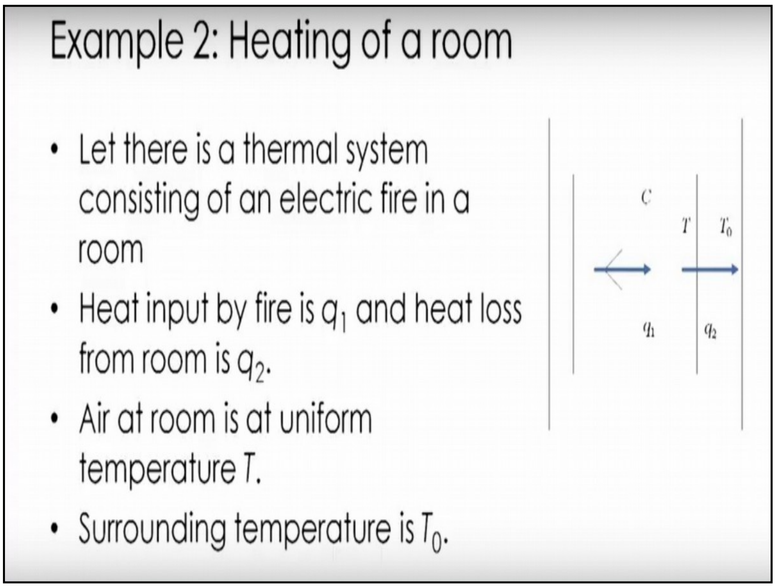

Now, let us take another example this is an example of heating of a room. So suppose this is my boundary of the room alright and here there is a heating coil. This symbol basically indicates the coil, heating coil so we had the heating coil and heating coil actually gives a heat flow of q.

Because of the heating coil say there is heat flow of q1 and because of that there is a room temperature of say t and the thermal capacitance here is say c. Now beyond the wall or say from this room to the other side through the wall there is heat transfer q2 and say thus surrounding temperature is t 0. Then as I said, we can write the expressions here.

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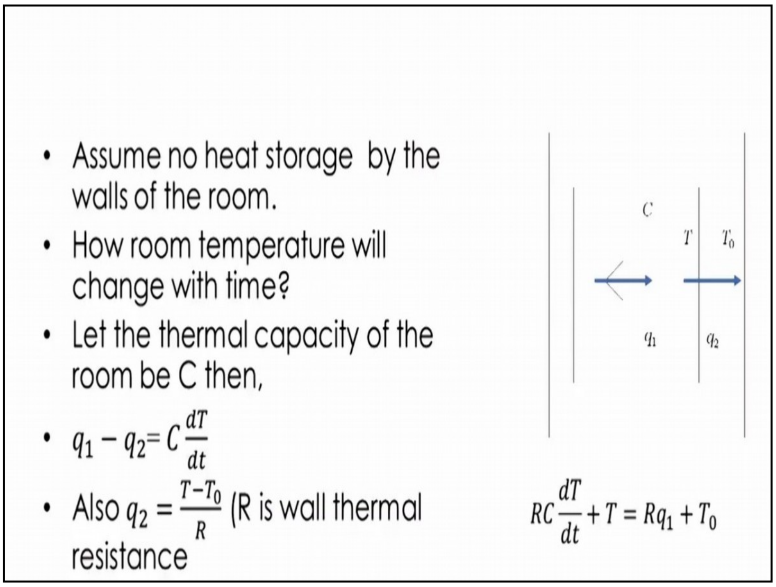

Alright then, I just said here we are talking about heating by electric fire and heat input by the fire q1 and heat loss from the room is q2 at the room uniform temperature of the room t and the surrounding temperature t0 .So let say we assume that there is no heat storage by the walls of the room . That is the walls of room do not store any heat then the question is that how the room temperature will change with time so let the thermal capacity of the room be c.

Then we can defined q 1- q 2 is equal to c dt by dt. Here q2 what will be q2 basically t-t 0 by r so temperature this side is t this side t0 so t- t 0 by r this is what is going to be q2 where r is the wall thermal resistance .so what exactly I am doing is I am defining the thermal capacity of the room and I am defining the wall thermal resistance and now I can combine both of these equations .so if I combine that is if I substitute for q2 here this is what I get. So if I put q2 t-t0 by r I cross multiply by a so I get rc dt by dt + t is equal to rq1+ t 0.

Now, here the input here the input is my q1 and here again this expression is the first order differential equation and you can see that this expression tells us how the temperature of room is going to change with time .So this way for a given q1 or given input of q1 we can find out how the temperature of the room is going to change with time . So next let us take a little further complicated system that is thermal system involving two compartments.

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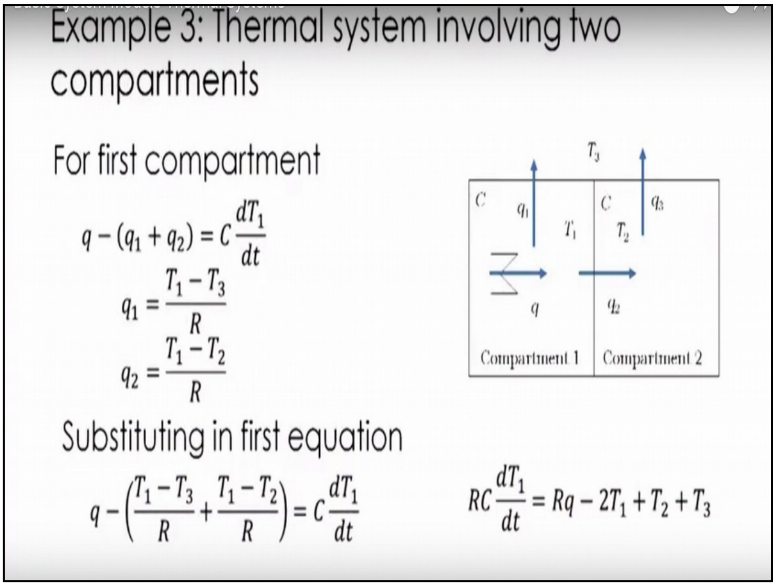

So suppose this is my compartment 1 and this is my compartment 2 Now here say there is a heating coil which is giving a heat of q to the room the room temperature is say t1 its capacitance is c and there is a q1 heat transfer taking place between room and surrounding and this surrounding is the temperature t3. So there is adjacent room or say compartment q 2 or compartment 2. Now the temperature at the compartment 2 is said T 2 .

So you are going to have a heat transfer from compartment 1 to compartment 2 this is q 2 as well you are going to have heat transfer between compartment 2 and surrounding and say this is q3. And we assume that the thermal capacitance of both thermal compartments is saY c. Then I can write the expressions for 1st compartment alright and remember we are going to use the two basic building blocks here for the thermal system and these are the thermal resistance and the thermal capacitance.

So let us go for the 1st building block so thermal capacitance So, what is my net heat in so q is my input and q1 and q2 are output from the 1st compartment. So my net heat influx is basically q-q 1+q 2 and this is equal to c dt1 given by dt where t1 is the first room temperature .now what is q1 I can write this q1 as T 1-T 3 by r. Where this r is the thermal resistance of the wall and T 1-T 3 is the temperature of the compartment1 and the surrounding.

So, this is how my q1 is going to be so this q1 is T 1-T 3 by r likewise I can write values for this q2 this q2 is T 1-T 2 by r. of course I am assuming that the thermal resistance is same for the wall and it is same at all sides say here fine .and I am not a assuming that any heat transfer taking from this to this one and similarly from here to here.

Now what we can do is that remember our q is the input and the output is T 1 that is I want to know how the room temperature T 1 varies with time so I substitute for q1 and q2 here in this equation and this is what I get q-T 1-T 3 by r+T 1-T 2 by r is equal to c dt1 by dt or I as I said my i interest is to know about dt1 by dt.

So, I can write this is rc dt1 by dt is equal to rq- 2T1+ t2+ t3 here this t1+ t1 this 2 times getting added up they will be giving a 2t1 term and of course t2 and t3 term .so this way we this expression basically tells you how the temperature of the 1st compartment is going to change with time next we can look at how the temperature of the second compartment is going to change with time. So let see how do we do that. So here this is my second compartment

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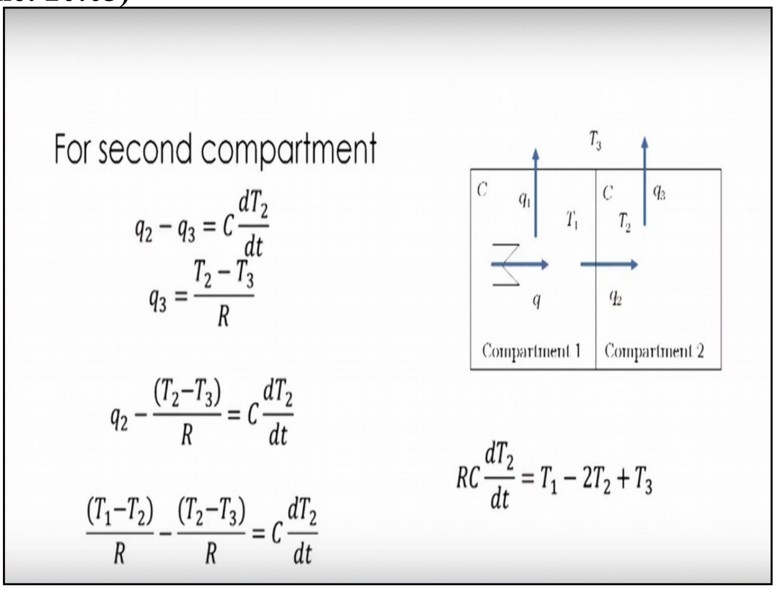

Fine, now in this compartment what is the net heat flow so you see that in this compartment the heat in is q2 in this heat out is q3. So this is q2 and this is q3 so the net heat influx is basically q2-q3 and this q2-q3 is equal to c dt2 by dt.

So, this expression basically which comes from the thermal capacitance and I can write the expression for q3 here by defining the thermal resistance of the wall . So this q3 is T 2-T 3 divided by r and I can substitute this q3 value from this expression into this one so what I have is q2-T 2-T 3 by r is equal to c dt2 by dt and this q2 value I get from the first expression for the 1st compartment here this one so this is my q2 that is T 1-T 2 by r.

So, this is how we defined the thermal resistance for this one so that I am putting here q2 is equal to T 1-T 2 by r and this is my q3 that is T 2-T 3 by r is equal to c dt2 by dt and this way I can find out rc dt2 by dt is equal we have the t1 term here and then we have minus 2 t2 term and then we have to a t3 term so this expression actually tells you how the temperature of the second compartment is going to change with time. Thank you