Modelling and Simulation of Dynamic Systems

Dr. Pushparaj Mani Pathak
Department of Mechanical and Industrial Engineering
Indian Institute of Technology - Roorkee

Lecture - 14
Basic systems models - Pneumatic systems

I welcome you all, in these model of the course modeling in simulation of dynamic system. Today we will talk about modeling of pneumatic systems. Pneumatic systems are very popular system in industry as they are complaint in natural light in weight. So because of this characteristics they are very much perform are they are not pneumatic very accurate.

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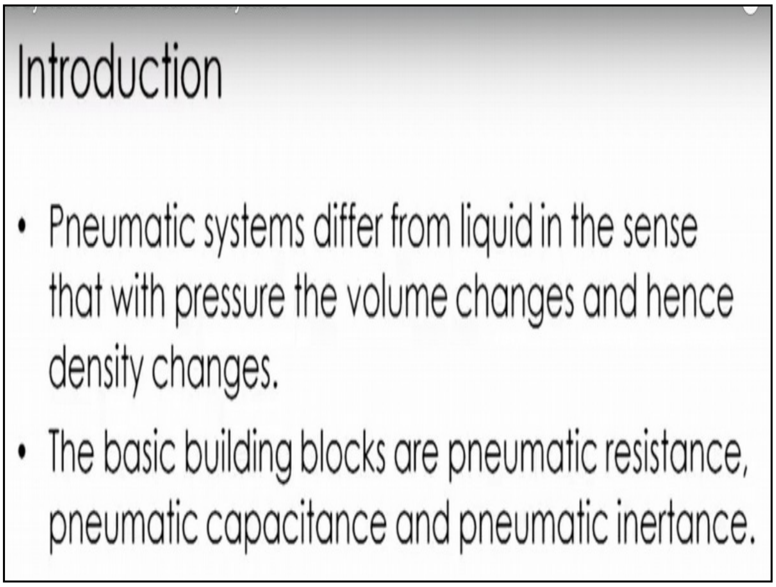

These pneumatic system differs from liquid in the sense that with pressure the volume changes and hence the density changes. If u remember in our previous lecture we consider hydrological systems and there we assume that density of the liquid to be of the constant but in pneumatic system we assume that density varies as with other system models such as mechanical electrical or hydraulic.

Here also we have the three basic building blocks and these all the pneumatic resistance, pneumatic capacitance and the pneumatic inheritance. So, initially will be looking at the definition of all these see building blocks there are governing equations and then will take can compare nation taken example of the pneumatic system which I am taking here the case of blower and will try to model that blower. So let us begin with basic models basic building blocks first we take pneumatic resistance.

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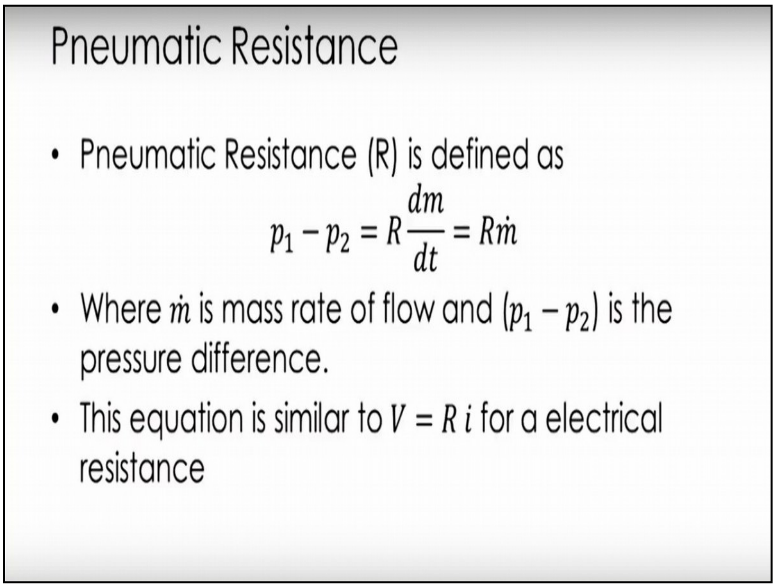

The pneumatic resistance r is defined as the p one - p two is equal to r dm by d t clear this b m by d t is the mass flow rate. Here I set where m derivative or m dot is mass flow mass rate of flow and p1 - p2 is the pressure difference are the pressure drop and here r is the pneumatic rise the now if we look at this equation is very much similar to what we have for electrical resistance that is v equal to I into r so what that is mean that the v corresponds to the v voltage of electrical system co response to pressure drop in the pneumatic system.

The current in electrical system response to low in the pneumatic system and of course or the electrical raise system response to here the pneumatic raise system and corresponds pneumatic system these way we describe the pneumatic raise with the help of relational ship that is p1 - p2 equal to r m dot or m derivative.

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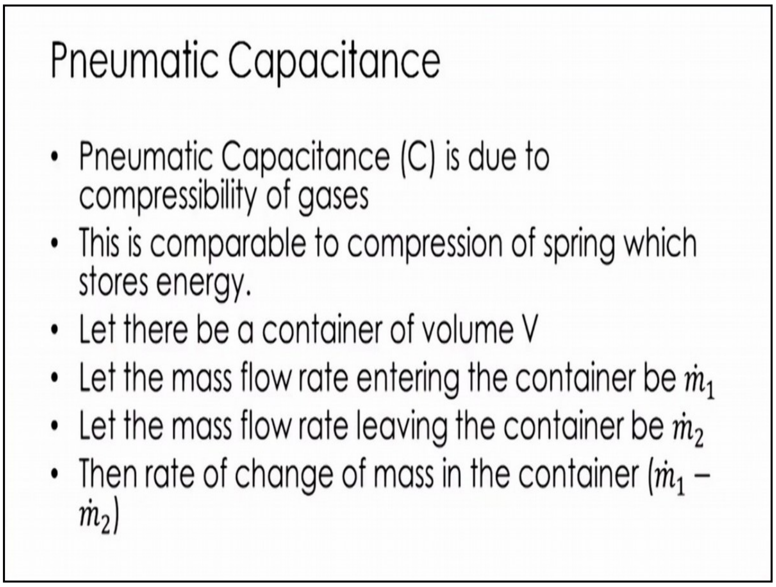

Next basic building blocks are pneumatic capacitance. Now pneumatic capacitance is due to the compressibility of the gases and this is comparable to the compression of spring which stores energy in a mechanical system. Now let there be a container of   volume v and there is a mass flow rate entering the to the container which will be m1 derivative and the mass flow rate leaving the container will be m 2 derivative then the rate of change of mass in container is going to be m1 derivative - m 2 derivative.

Now as I said there is going to be the variation in the density here in gas of pneumatic system.

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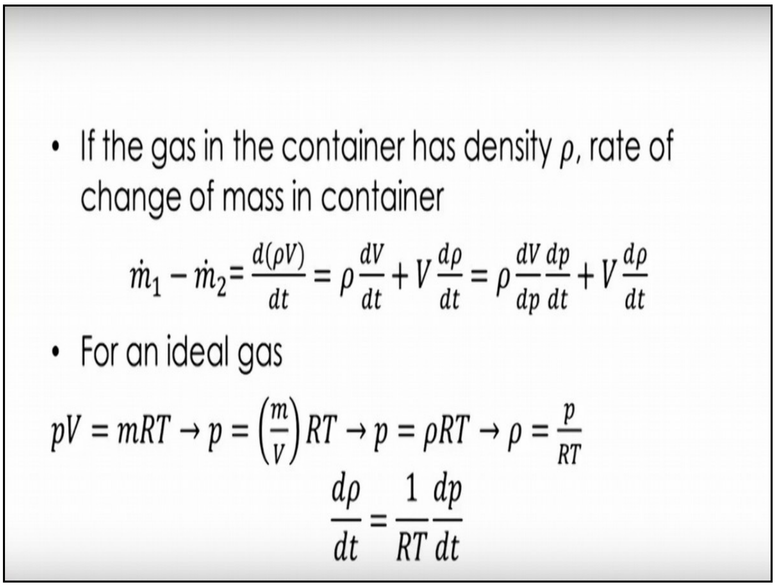

So, if the gas in a container has density row the rate of changes of mass in the container. We can give as m1 dot - m 2 dot equal to d by d t of rho v. Now, here you can see that is got the units of kg per meter cube and v has got the united of meter cube per seconds we have the united as the kg per second now here a set rho and v both are variable so when we differentiate carry out the differentiation by par t so we have rho d v by d t pls v 0 by d t.

Now, this d v by d t I write using champ rho as d v by d p into d p by d t + of course I have v d row by d t now for an ideal gas we know the gas law p v equal to m r t so form here of I can define p as my v by v into r t r this is rho r t r from here i can write rho as p by r t. So from here I can define d rho by d t equal to 1 by r t into d p by d t so basically am trying to define this term here in this equation.

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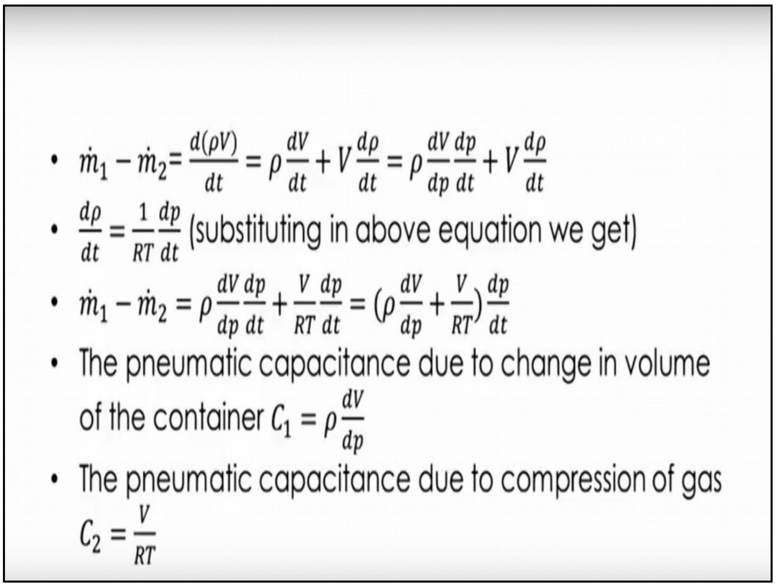

Now, let us substitute for this do by d t in this term what we get m1 - m1 dot - m 2 dot equal to d by d t row v and this is row v by d t + v d rho by d t equal to rho d v by d p + d p by d t + v d rho by d t as I and this is for what we got in the now previous slide and if is substitute this d row by d t here this is what we get. So, I substitute for d row by d t as 1 by r t d p d t here so what I have v by r t d t and if by take this d p by d t outside this is what I get row d p by d t + v by r t now you can see that here we have the two terms are available so the pneumatic capacitance due to change in volume of the container can be defined as c 1 equal to row d v by d p.

Similarly, the pneumatic capacitance due to compression of the gas this is define v by r t so here in case of pneumatic system. the capacitance this due to the changing volume of the container as wells due to the compressibility of the gas so for both we have going to the have a capacitance and total capacitance of the pneumatic system is going to be submission of this c 1 and c 2 here. So let us substitute that if I do that this is what get c 1 + c 2 d p by d t.

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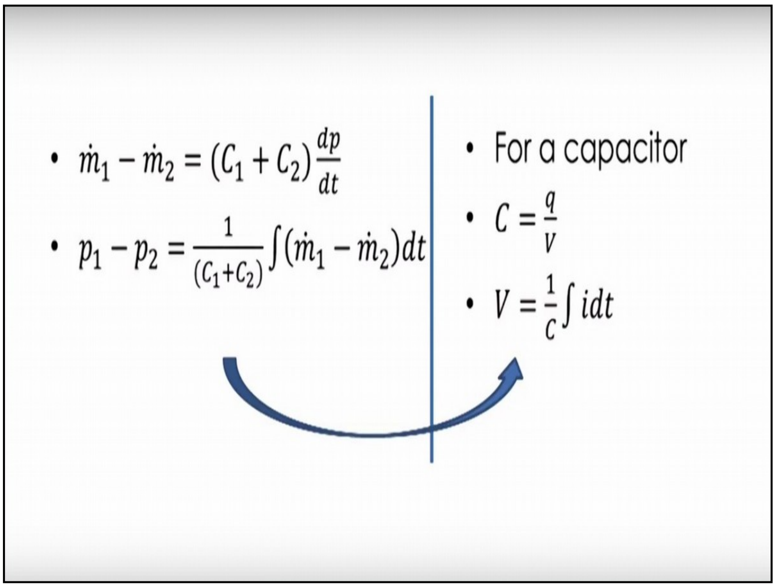

I can write in terms of p1 - p2 that is integrate the equation this is what I get 1 by c 1 + c 2 integral of m1 dot - m 2 dot d t. Now if I recall for a capacitor for a capacitor c equal to q by v where v sent the voltage across the plates and q I can write in the terms of current that is integral of i d t . Now if I compare this expression with these one then you can see that this capacitance or the term associated the capacitance in the capacitor is system that is 1 c 1 + c 2 so for a pneumatic

System c 1 + c 2 calico the pneumatic capacitance and this things I have already explain to you where the pressure drop is basically equivalent to the voltage in a electrical system and the mass flow rate corresponds to the current in the electrical system. So we these is about the pneumatic capacitance.

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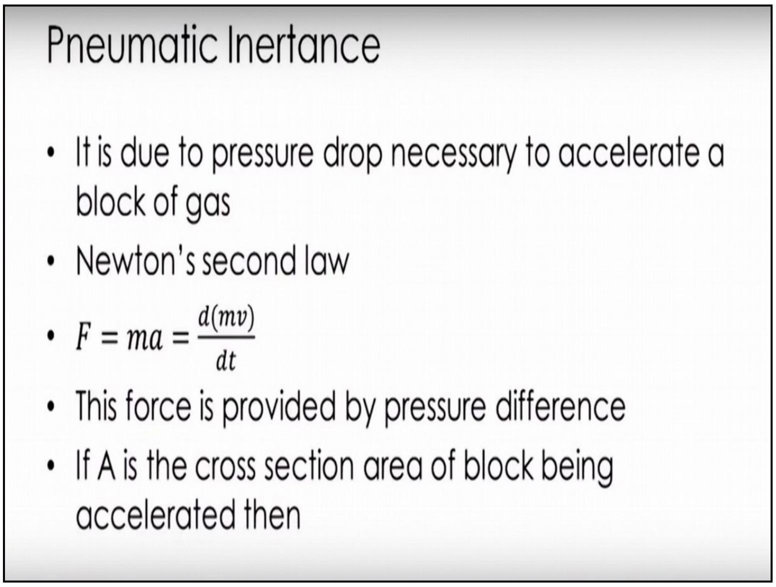

Now, we look at the pneumatic inertance. It is due to pressure drop necessary to accelerate a block of the gas now we have a block of the gas. Now for this block of the gas how much pressure drop is necessary to accelerate that if we look it at we can get the definition of the pneumatic inertance and to arrive at the Newton's second law that is f equal to mass insulation of this is d by d t of m v of that is get by changes momentum. Now this f I am talking about this f is actually provided because of the pressure drop.

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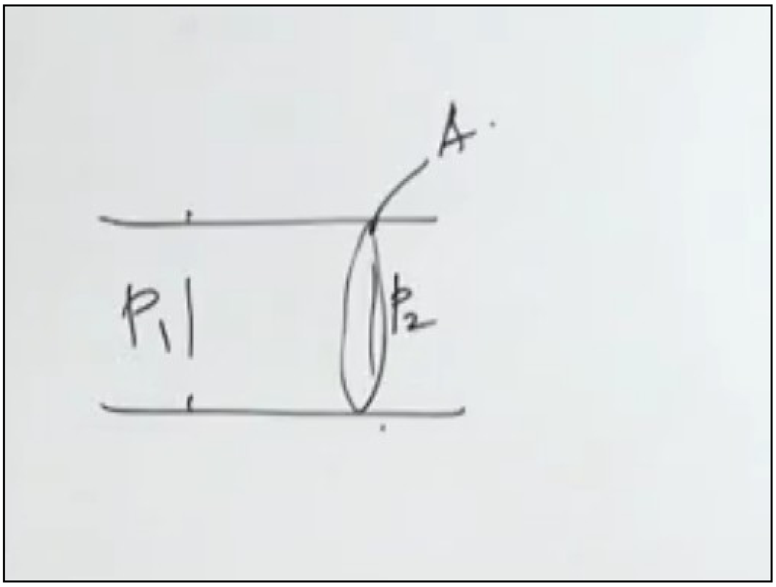

So, this force is provided by the pressure difference and if A is the cross section area of block being accelerated that is if I have got I block here like this is by block pressure here is p1 and this one pressure here is p2 and these cross sectional is basically it a then then my expression is going to be equal to this p1 - p2 into a so this a force and this is by d v by d t of m v.

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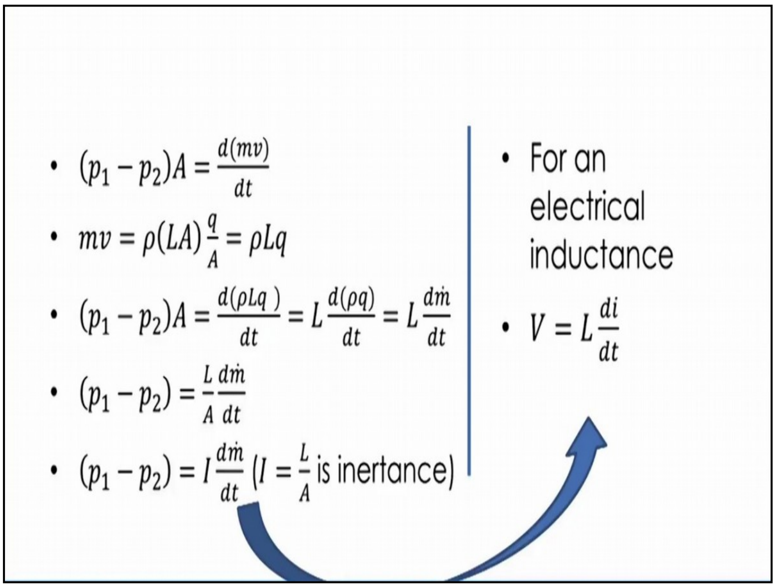

Now, what I do this m actually I write as this is basically momentums of density and rate of flow .so row into l a is basically the length of this and a is this cross sectional area so here it is this l. So basically l a will give you the volume here.

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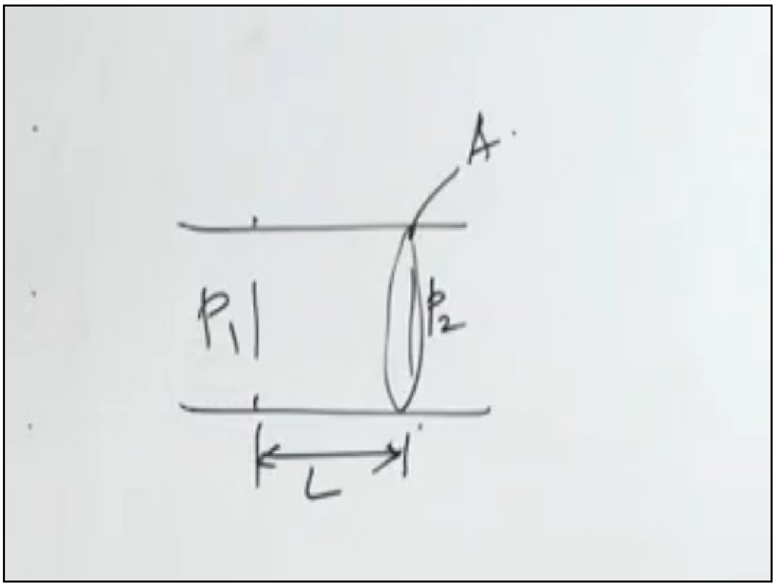

And this volume multiply the density will be giving you the mass and here this is meter cube per second and divided by the meter is square .so that will be giving you meter by seconds so basically here we get the momentum all right and this I can write this a a get consuls I have row l q .now I can substitute in this expression so what I have a p1 - p2 into a equal to d of Ql row q upon d t I can take l which is the content and outside of.

So l is outsides so I have d of row q upon d t and this row q if at loo at units a row is kg per meter cube and this is meter cube per second so basically what we get kg per second so this is d of m dot d by d t all right so form here l can write p1 - p2 its equal to ml by a dm dot by d t are if by write this l by a as i which I define as the inheritance i get the definition of the pneumatic inheritance. I can simply compare this with then electrical inductance here u see for electrical inductance.

Write the expressional v equal to l d i by d t so here you see we corresponded to the pressure dot and I corresponded to the m dot here to l corresponded to the i so this is how we define the pneumatic inheritance. So we have define the pneumatic resistance we have define the pneumatic capacitance and we have define the pneumatic inheritance now let as use this the basic building block two model for pneumatic systems . So, next we take for example pneumatic system lets take a bellow

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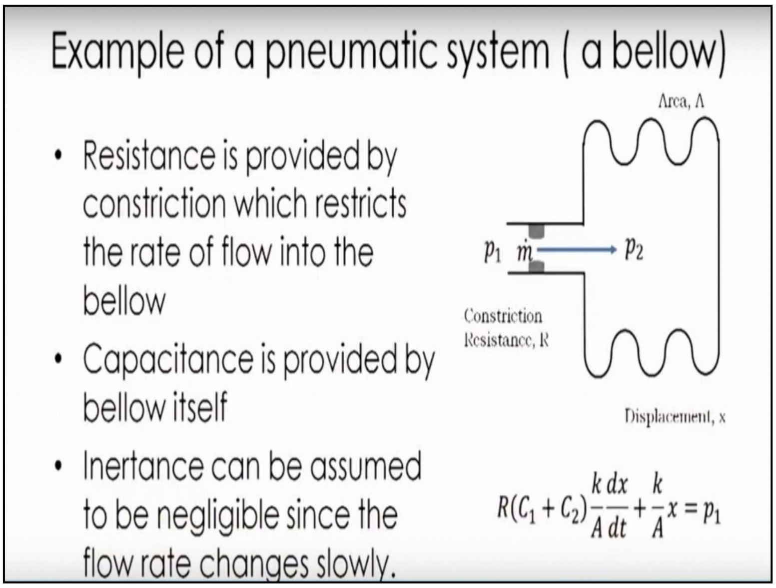

So I will explain you about bellow suppose you are bellow and here in this that is some construction here which essentially provided by the pneumatic resistance all right. So the contrition here the provides pneumatic resistance suppose the pressure drop here is a p1 before the constriction after the constriction is pressure is the p2 and the area of the section of the bellow is a and the displacements in the bellow they are x.

Now, as I the resistance is provided by the constriction which restricts the rate of low into the bellow and capacitance is provided by the bellow itself now all right and we can neglect the inheritance here in this case by assuming that the flow rate changes very slowly. So we can assume that acceleration of the gas is very acceleration of the not they are that is zero.

So we can neglect the inertance and we can just to capacitance and the building blocks to draw the system equation for this bellow and this is how we are going to the ultimate equation.

But, I will explain how to be get this so here you can see that is expression you are getting here p1 input pressure from way here that is the pressure the gas and bellow and of course the output displacement x of the bellow here u can see that this is a first differential equation for the below.

Now, let see how do we get this differential equation so our ultimate aim is here it is known that if my input p1 then what my displacement how displacement going to be are how I displacement is going to change with time for the given input pressure p1 .

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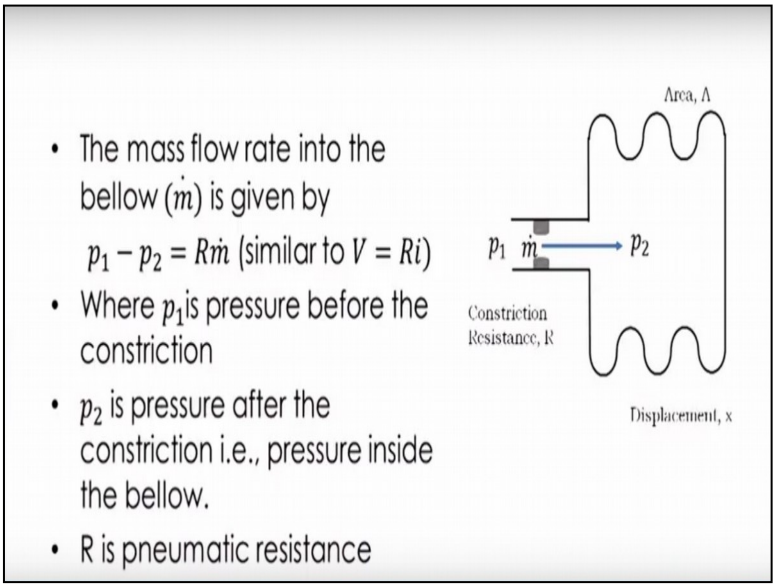

So let us begin with first a all of model for the constrictions here all right so the constriction can be model just like is resistance in electrical circuit. So, here the pressure drop p1 - p2 will be equal to r into m dot so these is the mass flow rate into the below is given by this one as I it is very similar to v equal to ri. So here these is the p1 pressure this the contrition and this the pressure p2 and the m dot is flowing from here so we have this relationship very pressure this are is the pneumatic resistance give to the constrictions here find.

So, as I here pressure p1 is the pressure before that constrictions and p2 is the pressure after the constrictions are we can that the p2 inside the bellow and r is the pneumatic resistance. So what we get be define here for this case the pneumatic resistance and then next will be defining the pneumatic capacitance have already said we other not going to consider the pneumatic inheritance and then will complain the relationship of the pneumatic resistance and the pneumatic capacitance in order to get the system equation.

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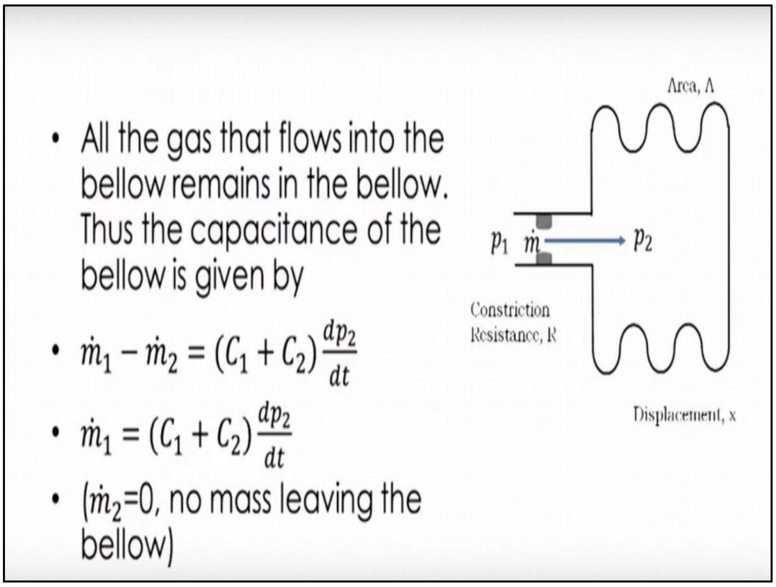

Now, as we have see in all the in this gas all the gas that flows into the bellow remains in the bellow because there is no way for gas to escape out of the bellow thus the capacitance of the bellow is given by this is the relationship which we have a see in for the capacitance of a of a pneumatic system are this is the basic building block the pneumatic capacitance building block definition which we are got m1 dot - m 2 dot equal to c 1 + c 2 d p2 by d t now as I there is no gas which is going to escape from the bellow so what we can do that we can take this m 2 dot equal to 0 .

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- $\dot{m}_1 = (C_1 + C_2)\frac{dp_2}{dt}$
- $p_1 - p_2 = R\dot{m}_1$
- $\frac{(p_1 - p_2)}{R} = (C_1 + C_2)\frac{dp_2}{dt}$
- $R(C_1 + C_2)\frac{dp_2}{dt} + p_2 = p_1$
- This shows how p_2 varies with time for given input pressure p_1

Now if by take this m 2 dot equal to 0 this is what I get m1 dot equal to c 1 + c 2 into d p2 upon d t now ia am taking d p2 because this p2 is the pressure here inside the champ inside the bellow. So, we have m1 dot equal to c 1 + c 2 d p2 by d2 and as I said the from the definition of the constriction in resistance here p1 - p2 equal to r m1 dot now this is the expression.

Which comes from the definition of the constriction the resistance are pneumatic resistance and this is the definition which comes from the pneumatic capacitance. Now let as combine this two equation so how do we combine I substitute for m1 dot this equation into this one.

So if by do that what I get p1 - p2 by r equal to c 1 + c 2 d p2 upon dot all right are I can write this as r c 1 + c 2 d p2 upon dot + p2 equal to p1 so here in this equation is c the input is by p1 the input pressure is p1 and my output pressure is p2 and this expression these are the lilted by as I said this is first order different equation and of course.

we have the terms which tellas the pneumatic resistance as wellas the pneumatic capacitor and this shows how p2 very we times for given input pressure p1 but as I said our interest in low but actually how much bellow moves because of this pressure p1. So that we can find out how much bellow moves because of this pressure p1 so next let as see that how much bellow moves.

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- Bellow behaves as a spring i.e. it expands or contracts as a result of pressure changes inside it. Thus we can write

$$F = kx$$

- Where F is the force considering expansion or contraction and it depends on p_2
- k is spring constant of bellows.
- x is displacement of bellow due to x
- But $F = p_2 A$

Let us assume that the bellow behaves as a spring that is it expands or contracts as a result of pressure charges inside it so the below behavior modeling expiring the mechanical behavior of the below am modeling is a spring I am writing p equal to x where x is the displacement of the bellow and is the is daffiness of the bellow and f is the force considering expansion or contraction and of course this depends on the p2 because p2 is responsible for the expansion or contraction of the bellow and I said is spring construct of below and x is the displacement of below fine.

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- $p_2 A = kx \Rightarrow p_2 = \frac{kx}{A}$
- $R(C_1 + C_2)\frac{dp_2}{dt} + p_2 = p_1$
- $R(C_1 + C_2)\frac{k}{A}\frac{dx}{dt} + \frac{kx}{A} = p_1$
- This equation is a first order differential equation which explains how extension or contraction of bellow (x)changes with time when there is an input of pressure p_1.

Now, but this f this f is equal to p2 into a where a is the area of contraction of the below so let us substitute that so we have the p2 a this is the force and this is equal to or so from here I get p2 as ox upon a and this was my basically equation the dynamic equation for the below and now in this expression I substitute for p2 equal to ox by a. So this is what I get r c 1 + c 2 by a duo by dot + ox by a equal to p1 so this the same equation.

Which I discussed I the begging for the bellow now here you can see that the input this p1 and the output is the x and this is the first order differential equation which explains how the extension of below text place extension or contraction of the bellow changes with the time when there is and input pressure p1 .now the question is how to you evaluate this c 1 and c 2. So, let us see how we can evaluate this c 1 and c 2.

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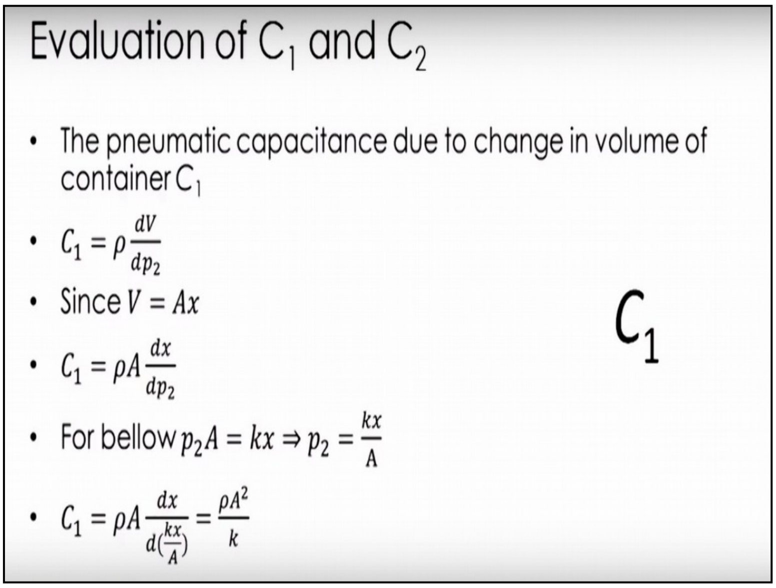

So, evaluation of c 1 and c 2 so I can think of evaluating the c 1 that is the pneumatic capacitance due to change in volume of the container here in this gas was talking about bellow. So c 1 I define row do by d p2 it is actually row do by dip but here the p is basically the p 2 alright so but p is equal to ax so I put it c 1 is row a row a do upon d p2 or for bellow we have also seen p2 a equal to ox that is force equal to the is stiffness construct for bellow and displacement of the below.

So, from here I get p2 as ox by a and define substitute here for p2 this is what I get row is square upon so this is the pneumatic capacitance due to change in volume of the container next the pneumatic capacitance due to the compressibility of the air.

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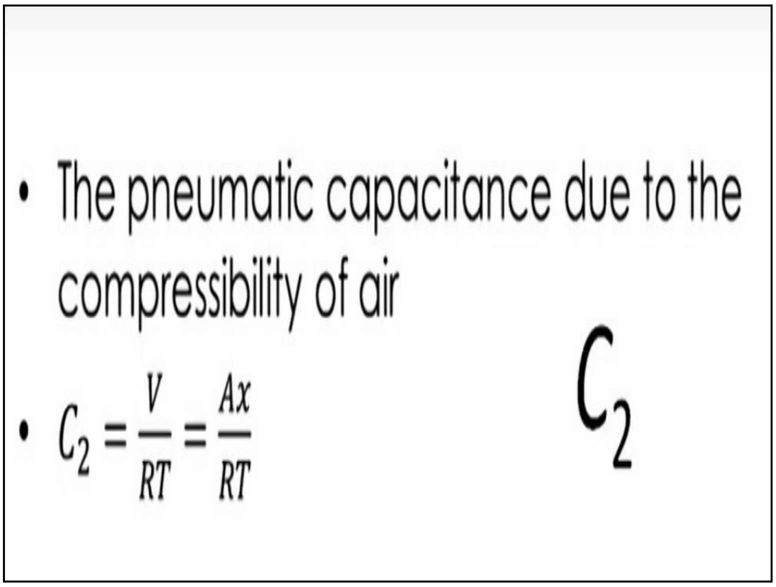

So, from here c 2 we can define as v by rat and this volume of I can take as ax and divided by r t. So this way I can evaluate the value of c 2. So we can summarize the basic building block definition inheritance electrical system.

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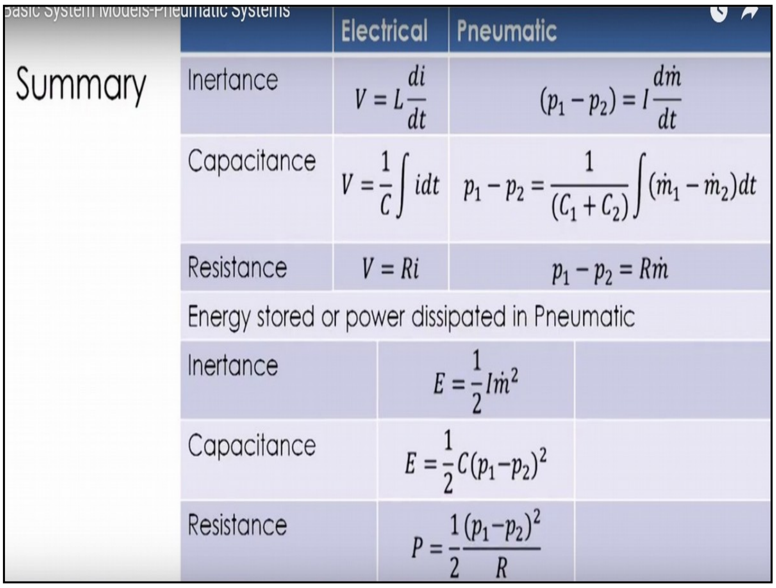

We have v equal to l did by dot so here we have p1 - p2 equal to 1 dm dot by dot capacitance v equal to 1 by c integral I dot so here we have p1 - p2 equal to half c 1 + c 2 integral m1 dot - m 2 dot and resistance v equal to r,1 so here we have p1 - p2 equal to r m dot and energy stored or power dissipated you know that for in case of inheritance and capacitance the energy is being is stored and incase of resistance the power is being dissipated .

So, for inheritance we can just write as half lid square as for the electrical system so here l emplace of l I have I and emplace of. So, capital I and emplace of correct I have m dot I m dot is square for capacitance this is half c o square for capacitor the cap energy is stored in capacitor is half co square so here this half c where this c is basically c 1 + c 2 into p1 - o2 hole is square and for resistance the power dissipated is v square by r 2r v square by 2r so here is 1 by 2 p1 - p2 hole is square by r .

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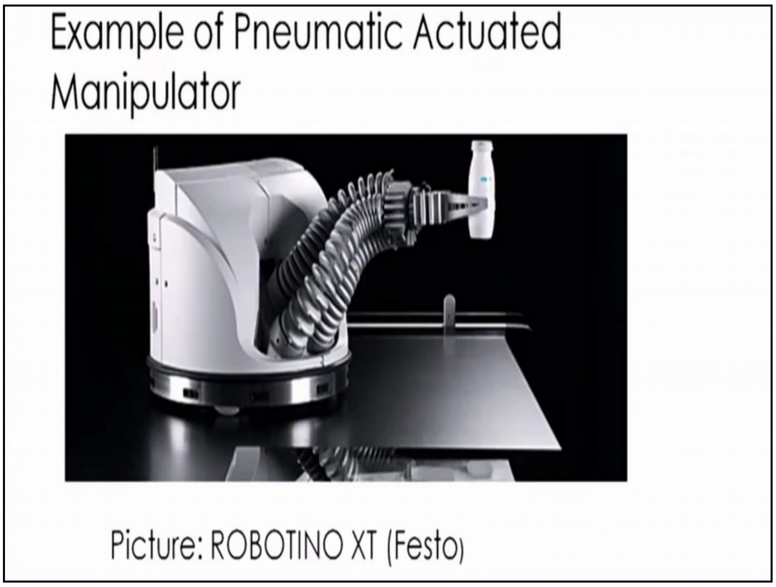

Now, using this concept you can think of modeling pneumatic actuated manipulators. So this is the image of robotino xt where basically there are 6 tubes see are basically which are pneumatically actuated.

So, you can thing of modeling of these is segment of the tube using the two basic building blocks which we have used for modeling of the below. So, those concept we can used for modeling of these 6 tubes connected here and of course from there you can write and find out the position of the pneumatic manipulate not only this.

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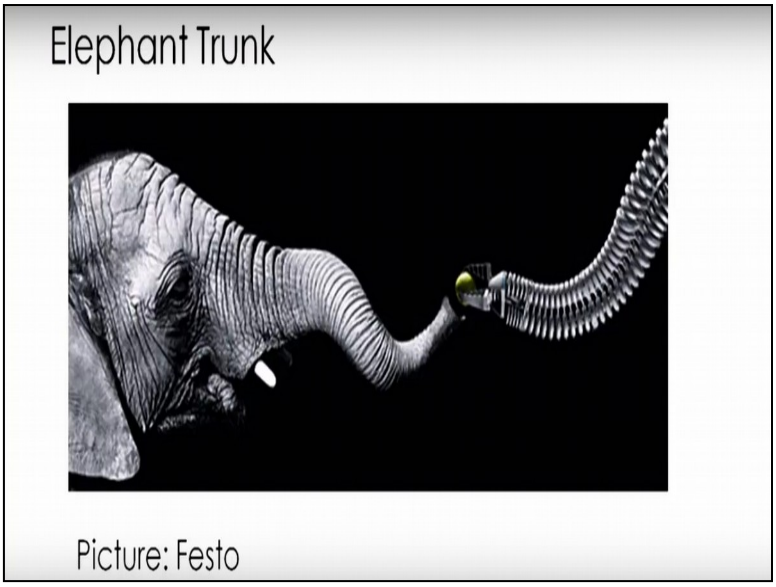

We can think of modeling of the elephant trunk are elephant trunk likes structure like this again which is pneumatically actuated. So this am not describing in detail it just I e a thought for you think about that how can we model this type of system using this basic building blocks.

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This is the reference you can refer very popular book by professor Bolton Mechatronic Pearson if you want to read it for that. Thank you.