Modelling and Simulation of Dynamic Systems
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Lecture - 13
Basic System models - Hydraulic systems

Welcome to this lecture, on basic system module hydraulic system which is the sub module for modeling and simulation of the random system course. As the title indicates in this lecture we will be understating what are the basic buildings blocks in order to model a hydraulic systems and after learning those basic building blocks w will see how we can model will be taking basic some two examples for modeling the hydraulic systems so as we have see that hydraulic system and mechanical system as well as hydraulic system.

As we have see that for the mechanical and electrical system. Where we have the three basic building blocks and with the help of these three basic building blocks we were able to model those systems

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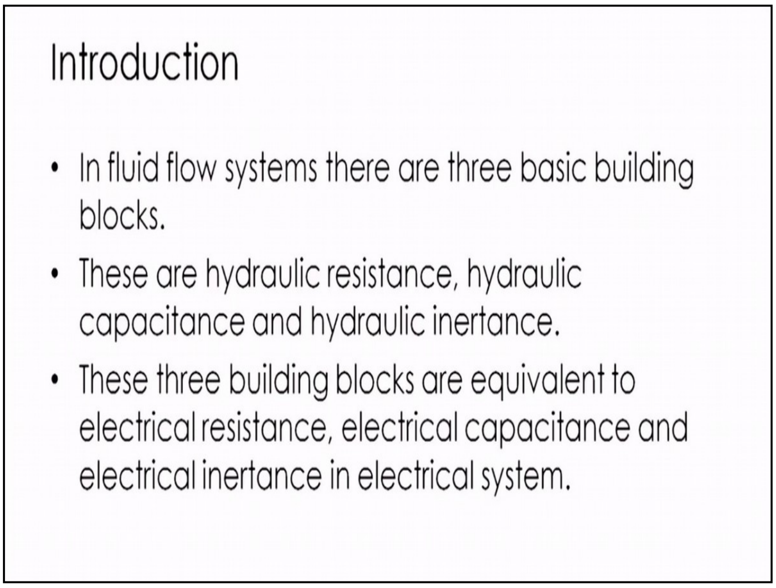

Likewise, here in the hydraulic system also, we have the three basic building blocks and these basic building are hydraulic resistance hydraulic capacitance and hydraulic inheritance .and in fact these are equivalent to what we have seen in case the electrical system that is electrical resistance electrical capacitance and electrical inheritance in the electrical systems. So we can have 1 to 1 analogy between this.

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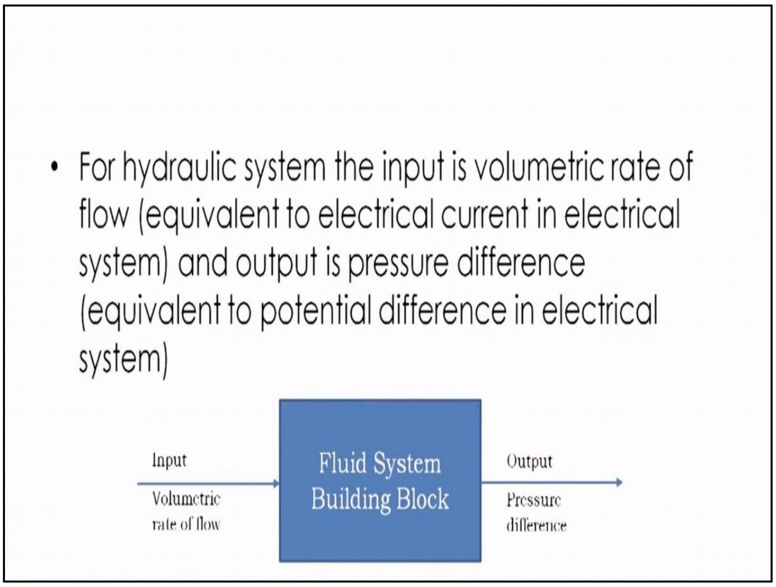

Now in case of hydraulic system the input is volumetric rate of flow which is equivalent to current electrical system and the output is pressure difference which is equivalent to the potential difference in an electhere and for a land for these fluid system blocks input is the volumetric rate of flow and the output is the pressure difference.

So, will be writing expressions. For this building blocks and other example of hydraulic systems in terms of these input and output as far as the fluid systems. are concerned the fluid system can be classified into 2 types these 2 types are the hydraulic systems and pneumatic systems.

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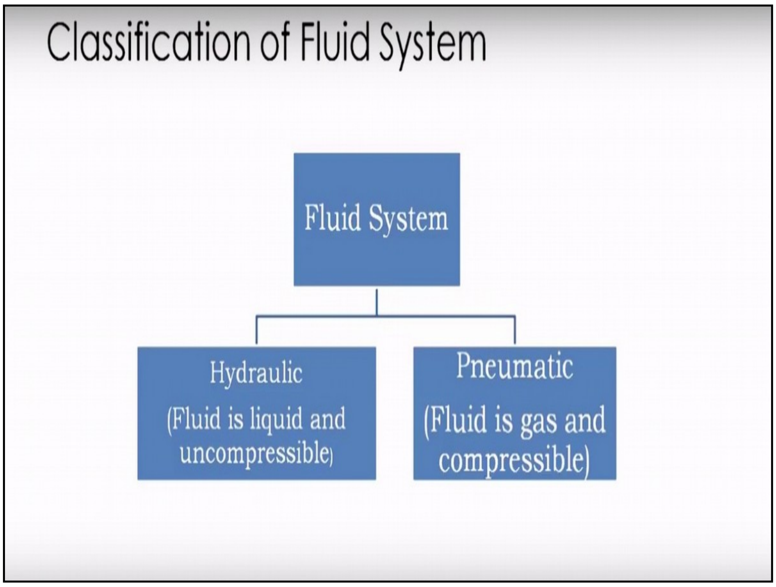

In case of hydraulic system, the fluid is liquid and uncompressible. So, the fluid having this behavior we classify as part of the hydraulic system and when the fluid is gas uncompressible these things are treated under the pneumatic category. So in fact the next lecture which I will be taking that will be on the modeling of the pneumatic systems.

Now, as I said we have the three basic building blocks for the hydraulic system, and these building blocks are let as see that first 1 the hydraulic resistance in the electrical systems. Now these type of situation comes there are many examples for these type of situation. It could be the operation of the valve. Now when the valve operates you know here in this figure we see that if I move this x l up and down basically what I am doing is that I am trying to open or close this opening.

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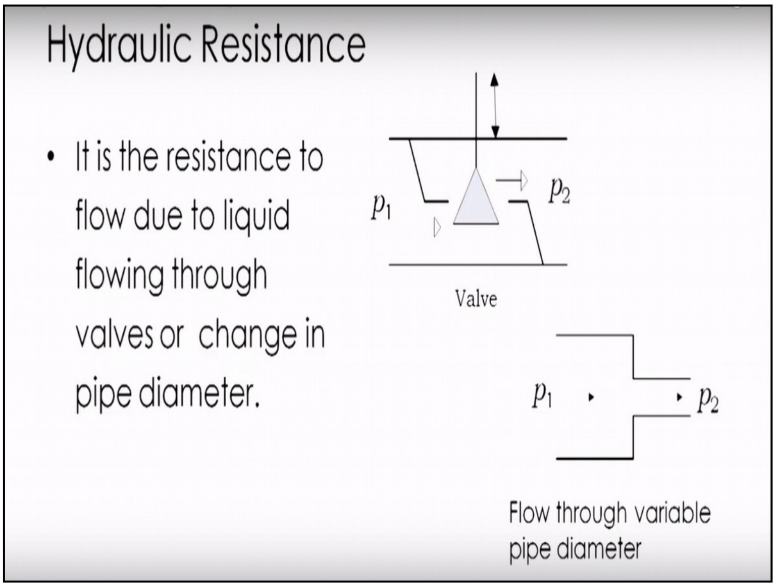

So, this is how valve operations takes place or when we have the flow through pipe variable cross section or variable diameter then also the hydraulic resistance or resistance for the flow comes in to the picture. Now it is the resistance to flow due to the liquid flowing through to the valves or changes in the pipe diameter. So whenever this type of situation comes those situations can be model us the hydraulic resistance.

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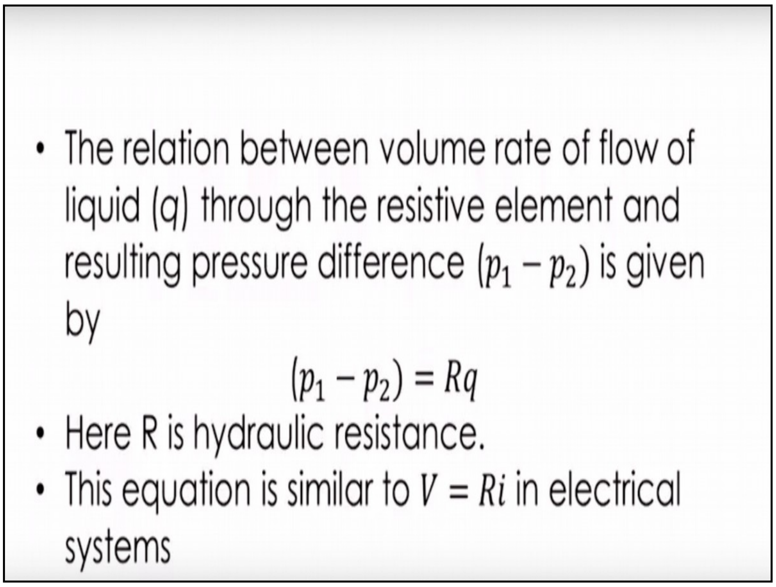

The relationship between volume rate of flow of liquid threw the resistive element and the resulting pressure difference is given by this equation. so here the p1 minus p2 equal to r into q here q is the volume rate of flow of liquid and when that liquid passes through either the valve or through the pipe of variable cross section. This is the pressure drop precaution. this pressure drop is equal to r in to q.

What does this means basically ?, this means that as the hydraulic resistance increases your pressure drop will also increase for a given amount of the input flow for a given amount of volume rate of flow of liquid. So this is what it means it and as I said here are the hydraulic resistance and if you can see look at this equation basically this is same us v equal to ri in electrical systems and we can see the matching here that is i matches to q r of course same and v matches to p1 - p2 here we call it us the electrical resistance and here we call it us the hydraulic resistance. So this how it is there.

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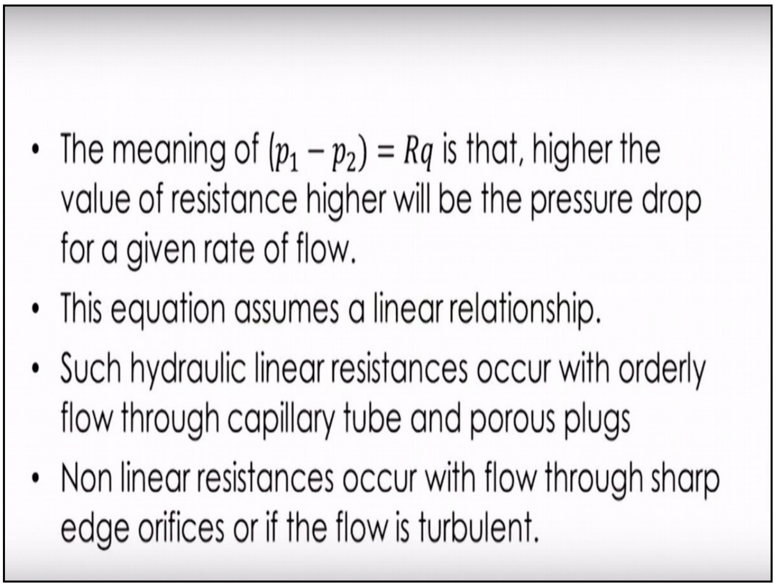

Next, this is what I told you the meaning of it. The meaning of it as i said p1 - p2 equal to rq is that higher the value of the resistance higher will be the pressure drop for the given rate of flow. This equation of course as you can see it from here it assumes a linear relationship, and such hydraulic linear relationship occurs with orderly flow through either the capillary tube and porous plugs and the non linear resistance occurs. When the flow is through sharp edges orifices or if the flow is turbulent flow.

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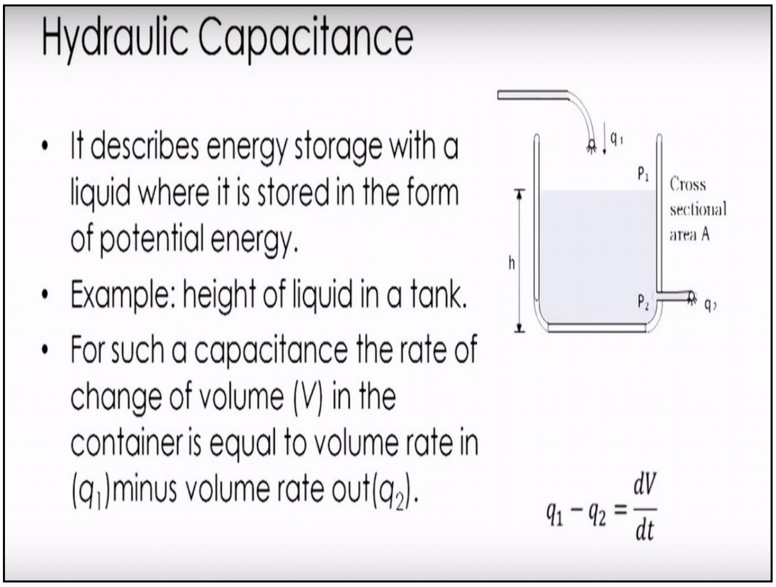

So, next building block is hydraulic capacitance. Basically, this describes energy is storage in the liquid when it is stored in the form of potential energy. So here you can see that we have a tank basically a container you can see whose cross sectional area is a hand in this container we have a liquid of a volume rate flow of which q1 is coming in and the q2 is going out and here the height of liquid is h.

So as I said this basically describes what amount of potential energy stored is there and this stored what example as I said height of liquid in a tank. Now, for such capacitance is the rate of change of volume in the container that is equal to the volume rate in - volume rate out.

Now for such a capacitance the rate of change of volume v is basically equal to whatever flow coming in minus whatever flow going out. So we have q1 minus q2 equal to dv by dt so this is the rate at the volume changes here this flow in and this is flow out. So what we have q1 minus q2 is equal to dv by dt.

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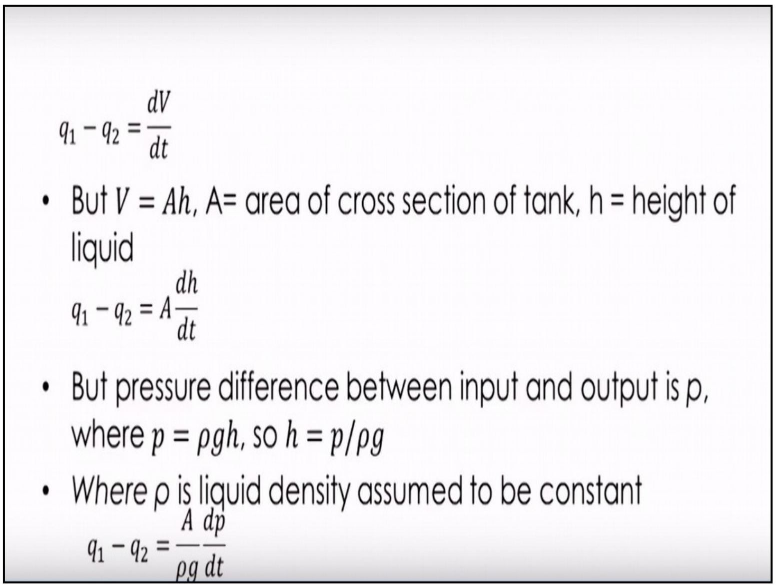

Now, this volume I can write into h where is the cross sectional area of the container and h is the height of the liquid. Now, if I do so what i get here basically a we take it us a constant So i take out so we have q1 minus q2 equal to adh by dt but the pressure difference between the input and output. is p.

Where we can define p equal to gh. orIi can write hsp by rho g now i can substitute for this h from here into this equation and what i get q1 - q2 is a by rho g dp by dt now this term which you are seeing which this is what is called as the hydraulic capacitance .and of course as i said we are taking about ab the liquid here.

So, we assume that the density is constant. of course the density is constant that with that assumption only i can take this rho outside. So this is my equation q 1 minus q 2 is a by rho g and dp by dt so and here as I said that,

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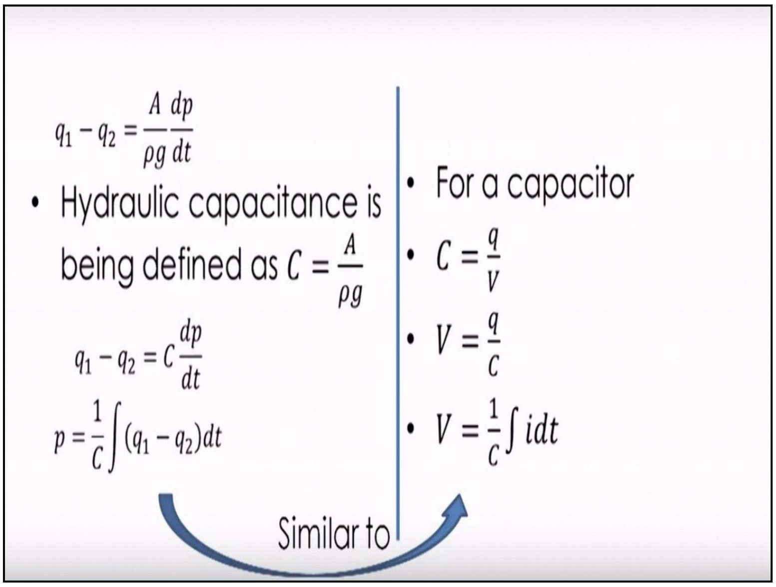

Hydraulic capacitance is as being define c equal to a by row g. I can write this expression in the another form also here that is q1 minus q2 i write this side c dp by dr or I can write p equal to 1 by integral of q1 - q2 into dt now if we look at this expressions and if we see electrical system for a capacitor.

Then for the capacitor building blocks expression are c equal q by v or I can write v equal to q by c or q I can write as integral of idt. So here we can see that we know that i current in electrical systems correspond to the volume flow rate here and voltage corresponds to the pressure and this of course if you compare this then capacitor of electrical system can be compare to the capacitance or the hydraulic capacitance.

So this way, we have 1 to 1 relationship. In order to explain the similarity between the electrical system and the hydraulic system next accelerate the example of hydraulic inheritance.

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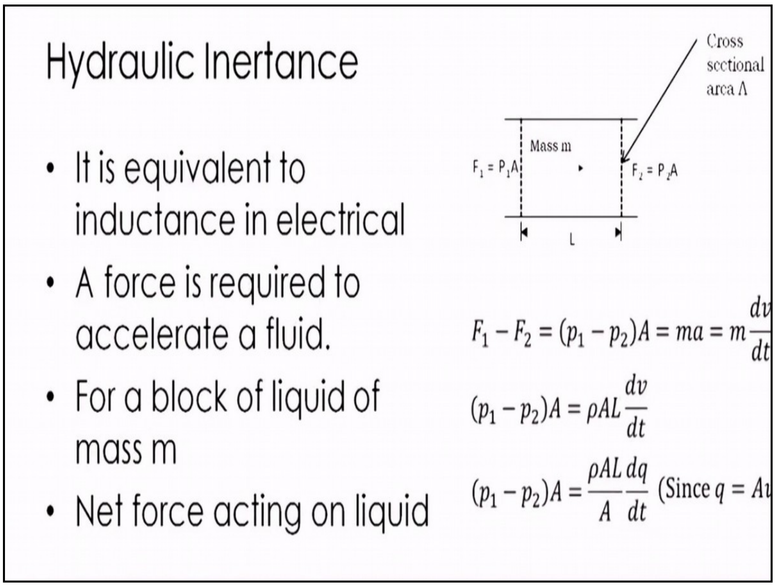

Now, to it is basically equivalent to inductance in the electrical systems, and we know that of course I consider a segment of the fluid system whose mass is m and the length of this element is l and it has got cross sectional area a then the force acting as the left hand side is going to be pressure p1 into a and at the right hand side is going to be f2 equal to p2 into a where p1 and p2 are the pressure at the left and right hand side.

Now you see that we know from the basic mechanics that force is required to accelerate a fluid. and for a block of a liquid mass m the net force acting on this is going to be what f1 minus f2 basically if I substitute for in terms of pressure and cross sectional area this is what I m going to get and this I can equate to the mass of the fluid here. In this segment into the acceleration alright and this mass acceleration i can write as dv by dt or this expression p1 minus p2 into a now mass.

I can write in term form of volume into density. so the volume of the liquid is a into l and density is rho so rho al dv by dt fine or further from here since the volume rate of flow is area of cross section into velocity I can substitute for this velocity and I can write here dq by a so this is what expression I get.

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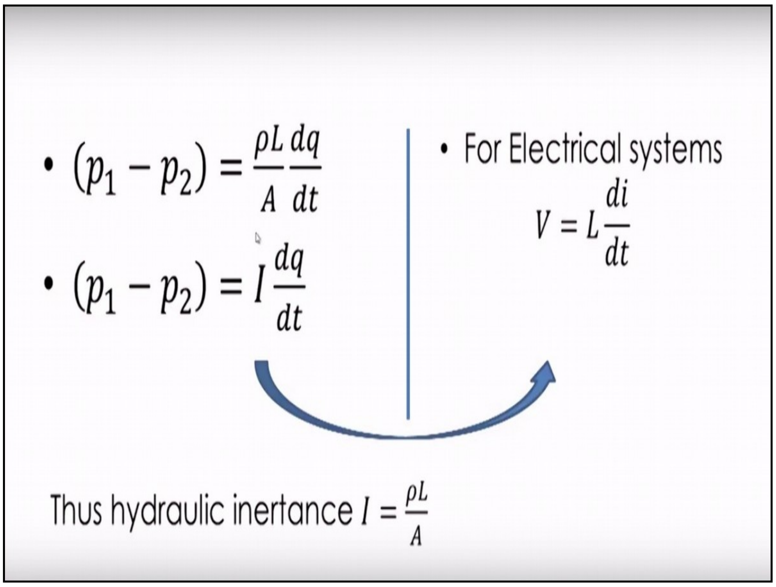

So, this is what I ultimately have p1 minus p2 equal to rho l by a so if we look at this expression. This is what we get yes this a a get cancelled here but this a comes to the denominator and the right hand side.

So i have p1 minus p2 equal to ro l by a into dq by dt and if i right this rho l by a as the hydraulic inheritance i in can replace the in can here so my p1 - p2 is equal to i dq by dt fine and of course we know for the electrical system for an inductor building block our relationship is v equal to l i by dt and if you compare again what we have been doing the comparison of electrical and hydraulic systems the voltage corresponds you can see that pressure difference here i corresponds to the volume of rate of flow and this inductance the electrical inductance corresponds to the hydraulic inheritance here.

So, this way we can define the hydraulic inheritance the hydraulic resistance and the hydraulic capacitance. So this there so to summarize the hydraulic resistance is there if there is change of cross section. The conduit it which the liquid is flowing hydraulic capacitance this there if the water is stored.

In some container or tank and energy is stored in the form of potential energy and the hydraulic inheritance is there whenever the liquid is been accelerated. So, this is how it was there now we can use these three basic building blocks to model. What we have called any hydraulic systems.

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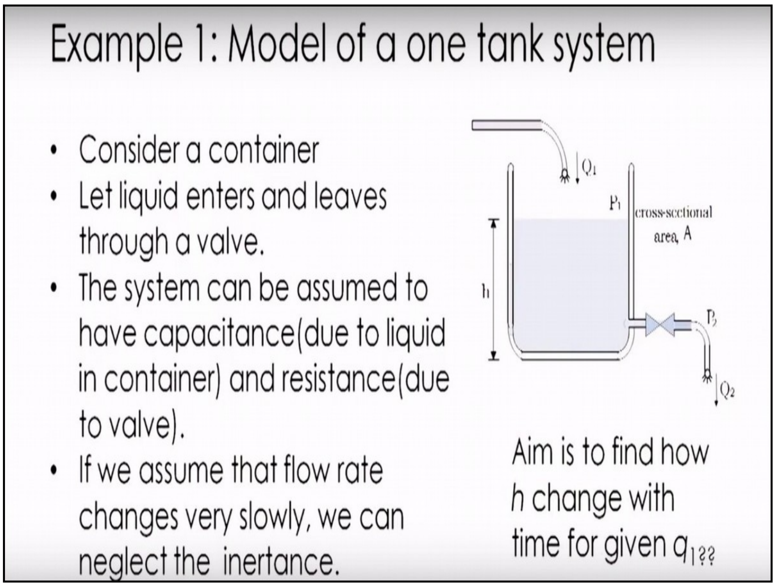

So, I will take an example model of a 1 tank system. Now, in this 1 tank system I have tank or container of cross sectional a and I have a liquid of height hl and volume flow rate is q1 and q2 volume is going out through this pipe fine. Now here basically you can see that whatever i have defined so here we can see that I assume that my liquid is not accelerating or the changes of velocity are very slow.

Then in that gas in can neglect the hydraulic inheritance, and I can model the 2 things here the first thing is hydraulic capacitance because the water is stored here for a height h and i can model for the hydraulic resistance because the water is flowing throw this valve. So, here basically we will be requiring only 2 building blocks in order to model these hydraulic systems.

So, yes what ultimately we aim for we aim for to find how h changes how height changes height basically here represents the pressure. so how the pressure changes how the output changes for the given input flow that is for the given input that is q1 how the height of the liquid pressure of the liquid in the tank changes. So this is our ultimate aim in the modeling of this hydraulic system.

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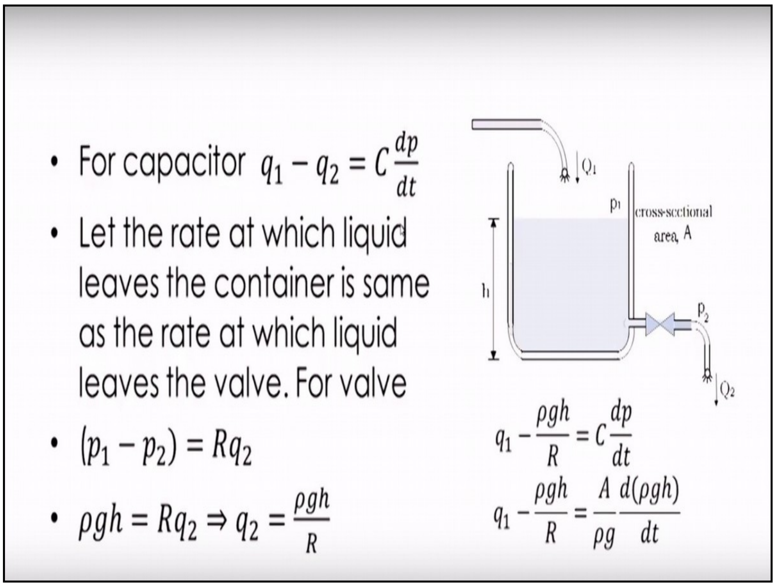

So, let see how we can do that so for a capacitor hydraulic capacitor you know that q1 - q2 equal to cdp by dt this relationship we have just worked out were q1 is flow in and q2 is the flow out. of course i am sorry here these has to be the small q s so q1 - q2 is c dp by dt .now if we assume that the rate at the liquid which leaves the container is same as the rate at liquid leaves resistance of that valve here.

That is the flow here is same as here then i can write this expressions that is general expressions the pressure drop which comes because of the flow of the liquid through the valve. so that is p1 - p2 that i can equate to the resistance of this valve in to the liquid which flowing into q2. Now, here you see the p2 we can assume to be zero because the liquid is going to the atmosphere so this p1 I can write as ro g h and this is r q 2 r q 2 I can write ro g h upon r from this one.

Now I can substitute this value of q2 which we got from the definition of the hydraulic resistance into this definition of the hydraulic capacitance. so if I substitute here what I get is q1 - ro gh upon r equal to c dp by dt or i can write this q1 - ro gh upon r see now a i m writing a by ro g and p i m writing at ro gh. So here since ba liquid density is constant so rho will be getting cancelled and this is what we get.

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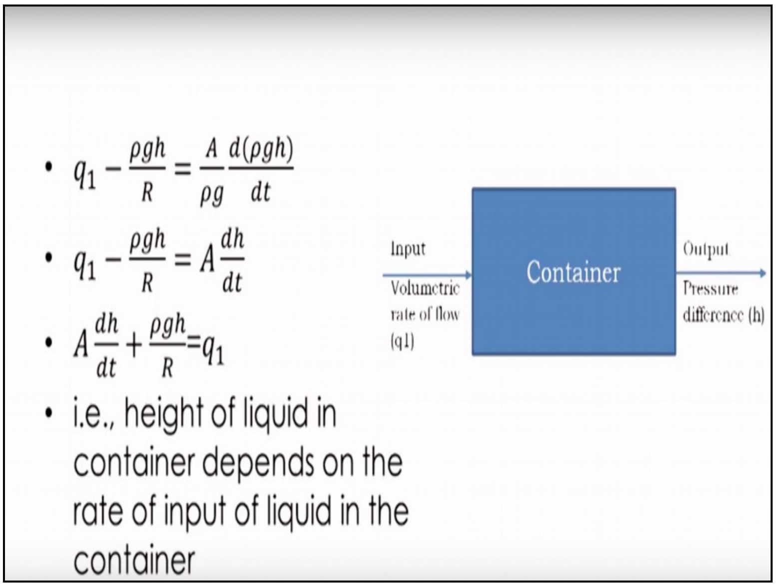

So, q1 - rho gh. Upon r equal to a by rho g this was the previous equation and this is what we get after detection of a so this terms are same so here at this rho g ro g gets cancelled so what we have is dh by dt. or i can write it like this in terms the input 1 side and the output terms other side a dh by dt plus rho g h equal to q1.

So, you can see here the right hand side got the input q1 and the left hand side tells basically the variations of h .so this equation is again the first order of differential equation and which essentially tells us that how the height of the liquid h is going to change for the given q1 and this what in terms of input and output we write we have container as i said the input is volume rate of flow and the output is pressure difference h.

This way we can model this hydraulic tank or the container. Now let make things little complicated.

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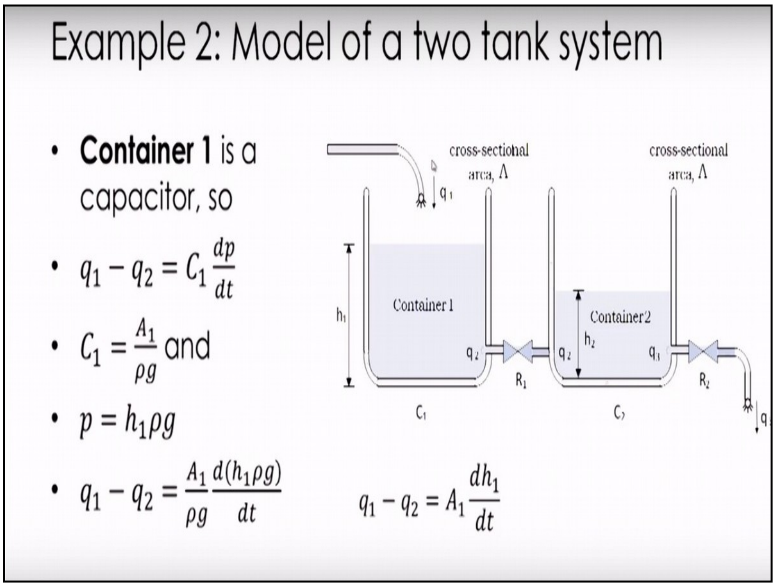

So, in this example we see that the model of the 2 tank systems. So here we have 1 tank or container and here we have the another tank or second container. Now you can see that there is the valve r1 here connecting the first and second tank and there is the valve r2 here from which is this liquid discharge to the atmosphere.

So this tank system again we can assume what assumptions we did in the previous example that we assume that changes in velocity of the liquid is very slow we neglect the hydraulic inheritance. and we consider only the 2 factors the hydraulic capacitance and the hydraulic resistance. So we will be using these 2 basic building blocks to model the first tank and the second tank. So that way we can do it.

So let us consider the first container now since the first container is the capacitor .so i can write this expression as q1 - q2 is equal to c1 dp by dt .where the c1 is a1 by ro g and the p of course h1 ro g because the height of the liquid here is h1. Alright so i substitute for c1 and p in this equation. This is what i get q1 - q2 is a1 by ro g d h1 ro g by dt or q1 - q2 equal to a1 d h1 by dt. so this basically this expressions what we have got is for the capacitor hydraulic capacitance expressions for the first tank. so this q1 - q2 is a1 d h1 by dt.

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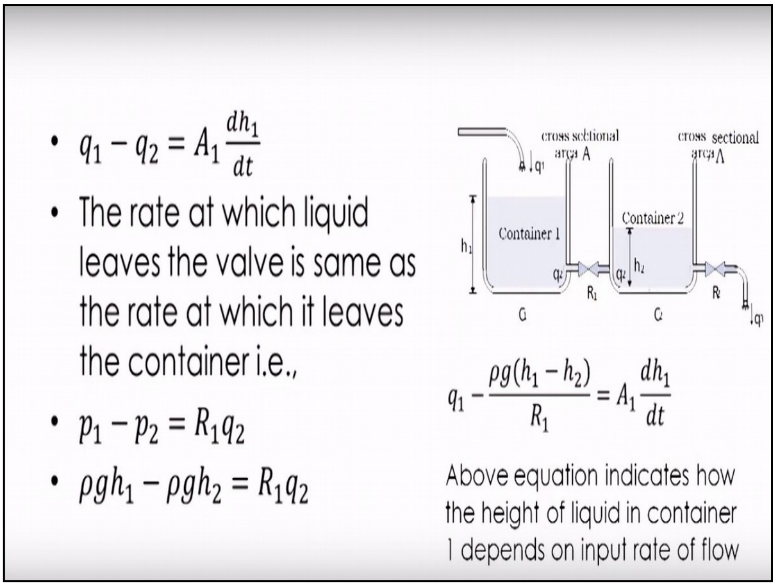

Now if I consider this valve. The valve r1 now in this valve r1 the rate at the liquid leaves the valve is same as that rate at which leaves the container. So with that assumptions I can write the pressure this side the p1 and this side it is p2 that is the pressure after the valve it is p2 so this p1 - p2 is equal to r1 into q2 where q2 is the discharge forms here.

Now I can write this p1 as ro g h1 basically the pressure because of height of this liquid column h1 and this side pressure will be basically pressure because of this height of liquid column of h2. So this p1 - p2 is ro g h1 - ro g h2 and this is equal to r1 q2.

Now, form here I can substitute for q2. So if I substitute fo q2 q1 of course we are interested in because it is input parameter alright I just substitute q2 here so this is rho g h1 - upon r1 equal to a1 d h1 upon dt so this is the expression for my first container. So here you can see that what we are interested in knowing how the height or the pressure in the first tank changes with the time for a given q1.

Alright so this is what this equation indicates now we can go for the container 2 so first we write the hydraulic capacitance equation for the container 2.

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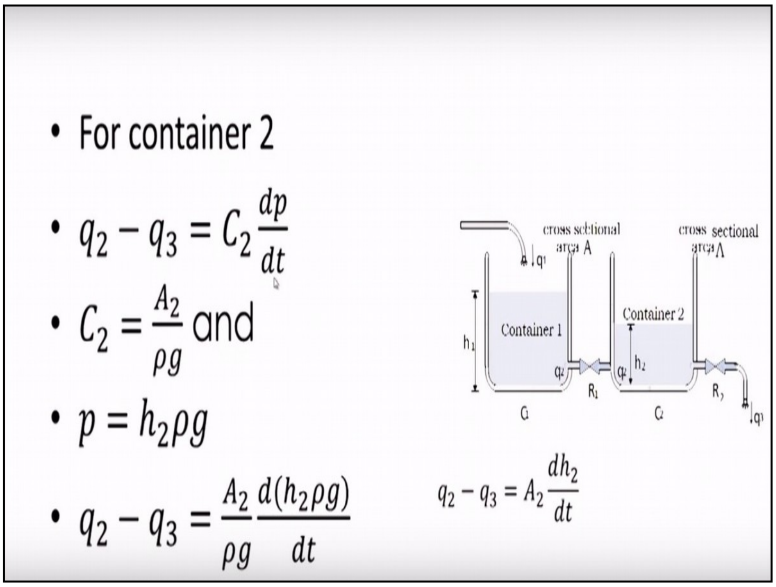

Here you can see q2 minus q3 q2 is in and q3 in out so q2 minus q3 is c2 dp by dt and this c2 i define as a2 by rho g and p i define at h2 ro g .why h2 because here the this side the pressure is zero fine. So, when I do that I substitute for q2 - q3 so this my a2 by rho g d of h2 rho g here by dot or these expressions reduces to this form. So q2 - q3 is a2 d h2 by dot so in this expression basically for this is expression basically for the hydraulic capacitance for the second container. Then let see so in this expression.

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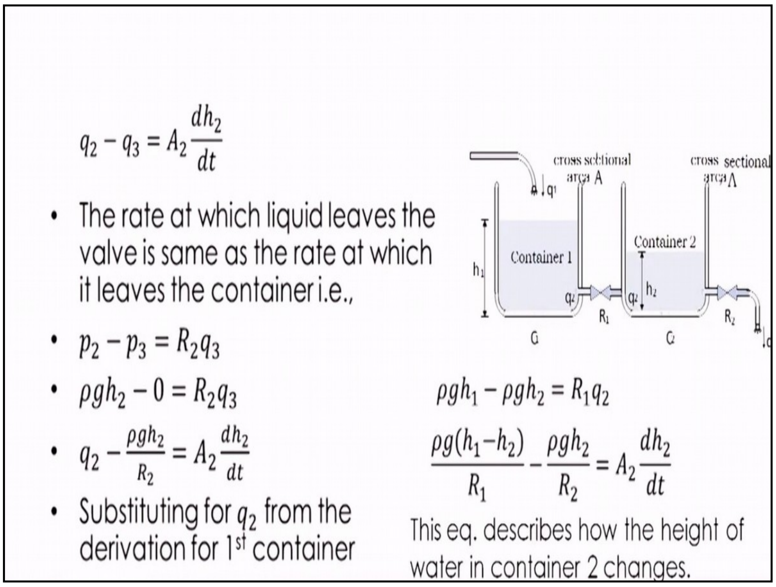

Now, again I assume that the rate at the liquid enter here to a container 2 is the same at which leaves this 1. The valve then I can write p2 - p3 is equal to ro2 sorry r2 into q3. Now, in this p3Ii am taking it at zero so this rho g h2 equal r2 q three and so this expression becomes q2 - I substitute for q3 here so it is q3 here is ro g h2 upon r2 so this is being substituted here and i get a2 g h2 by dt now further what I can do that as i said we want expression in terms of the input terms.

So I can substitute for this q2. and how ah i get q2 I write expression for the first valve this we have already d1 that is r1 q2 equal to ro g h1 - ro g h2 so form here I get the q2 term so that is ro g h1 - h2 upon r1 this I am substituting here for this term - rho g h2 upon r2 is equal 2 a2 d h2 by dt. So, this equation describes how the height of water in container to changes and as I as you can see these expression is a coupled expressions.

That is here in this h 2 expression you can see you get the terms of the first tank also that is the h 1.

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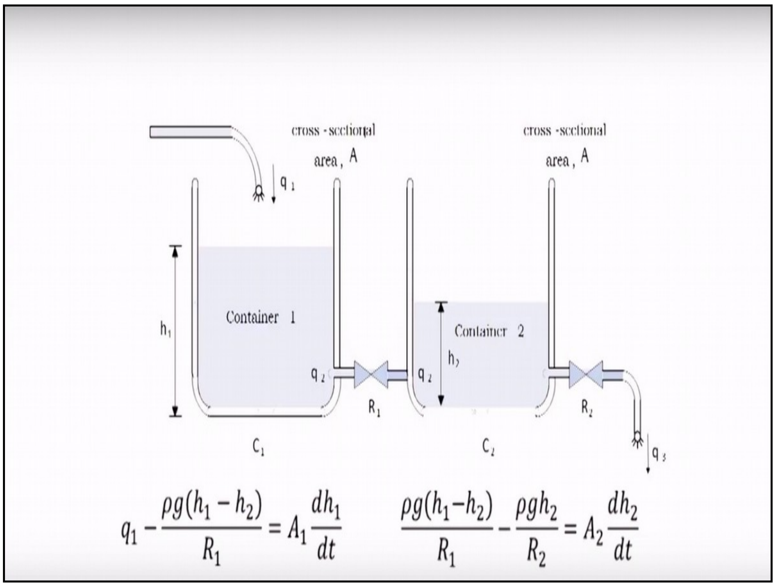

So, this is what the summary is for the 2 time systems you have the first tank and you have the second tank here. So for the first tank this is by a 1 d h 1 by dt I am interested in h 1 here and this is how q 1 is the input here and this is the second term where I am interested in knowing the h 2 and of course we are require the information of h 1 and h 2 also.

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Summary

Building block	Block equation	Energy stored/power dissipated
Inertance	$q = \frac{1}{I}\int (p_1 - p_2)dt$	$E = \frac{1}{2}Iq^2$
Capacitance	$q = C\frac{d(p_1 - p_2)}{dt}$	$E = \frac{1}{2}C(p_1 - p_2)^2$
Resistance	$q = \frac{p_1 - p_2}{R}$	$P = \frac{1}{R}(p_1 - p_2)^2$

So, I think now before I conclude I just give you I just summarize what by building blocks the three building blocks which we have used inheritance capacitance resistance these are the they are the respective expressions for the inheritance capacitance and resistance and these are the additional expressions for the energy stored or power dissipated which you can work it out by just comparing with the electrical system.

So, what is the power is stored inductor and capacitor or what is energy is stored in inductor and capacitor and how much power is dissipated by the resistor. So if you compare those 2 expressions those 2 systems you can write the energy expression for here. For example for inheritance in electrical systems it is half l i squared.

So, I have half i q squared, for capacitance it is half c v squared so here it is half c pressure difference squared likewise here for resistor it is v squared by r so in this case it is p 1 - p 2 square by r so this way we can write expression for the energy and the or the power. So, with this I conclude this lecture. Thank you