Modelling and simulation of Dynamic Systems
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Lecture - 12
Basic System models - Electrical systems

I welcome you all, in this lecture on modeling and simulation of dynamic systems in this lecture will see the basic system models and under basic system models we will be looking at the modeling of electrical systems. As I told you in my earlier lectures we are currently looking after the systems which are there in the different energy domains. So in the last lecture we have seen in the systems which are there in the mechanical energy domains.

Today we look at the systems which are there In the electrical energy domains and then of course hydraulic pneumatic and the thermal systems in these energy domains we are going to look at it. Now, as for the mechanical system for electrical system also initially we will be see the three basic building blocks. And the expressions corresponding to those three basic building blocks will be looking at and then will use these basic building blocks to model a complicated electrical circuit or complicated electrical system. So, this is how we plan to do it today.

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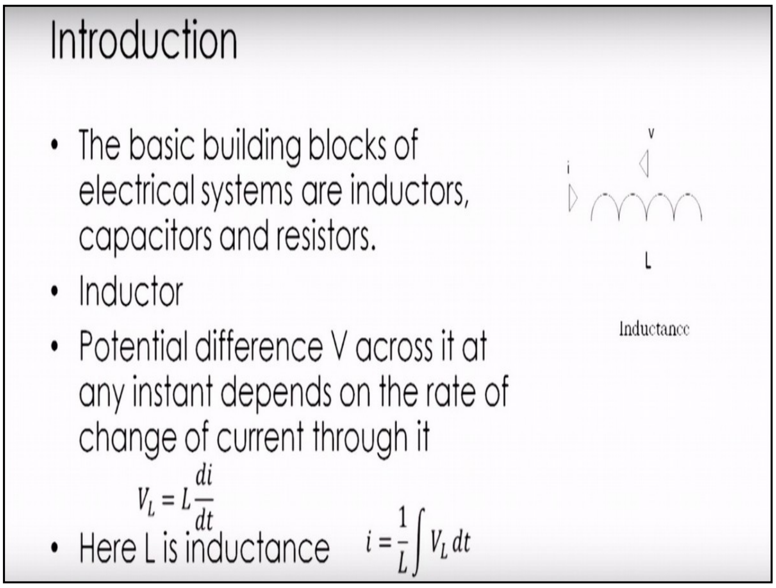

Now, the basic building block in electrical systems are inductors, capacitors and resistors and these inductors capacitors and resistors using these 3 basic building blocks we can model any system.

So let us first of all look at the inductor in this figure you can see the basic systematic of the inductor and in an inductor actually are the potential difference across it at any stand depends on the rate of change of current through it in fact, this vl is schematically proportional to di is divided by dt and when we replace this proportionality sign constant or proportionality sign we take on stand this what we put the l for that and here this l is what we call it as inductance.

So here you can see that if a current i passes through the inductor and if rate of change of current. We can represent here di by divided by dt then that into l is going to be voltage giving as the voltage across it. Here the sign which you have seeing that is basic ally for the ck emf that is the one which is in the reversed direction of the applied voltage and firm this expression.

We can find out the expressions for the current the current which is going through it at any instant and that is 1 by l integral of vl dt so this how are how we can defined the inductor or inductor characteristics.

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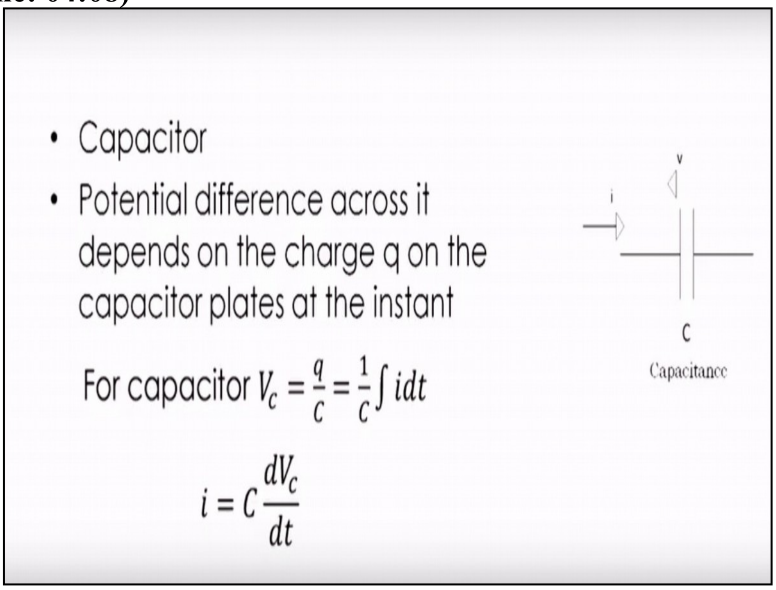

Next basic building block is the capacitor. You can see here the systematic of capacitor this is the the direction of current i and this is the game on the direction of for that back emf. We know the capacitor can be defined as the potential difference across it depends on the charge q on the capacitor plates at any instance. So at the any instant whatever charge is there across plate of the sed on that we can define the capacitor.

So, you can see the vc is q by c and here c is the capacitance what we call it as the capacitance of the capacitor and i can replace q here by integral of idt that is integration of current will give us the total charge accumulated across the capacitor plates and from here. I can find out what my current is going to be that is c dvc by dt that is if I take the differentiation of it Ii get the i is equal to c dvc is divided by dt. So this is how we define the inductor characteristic.

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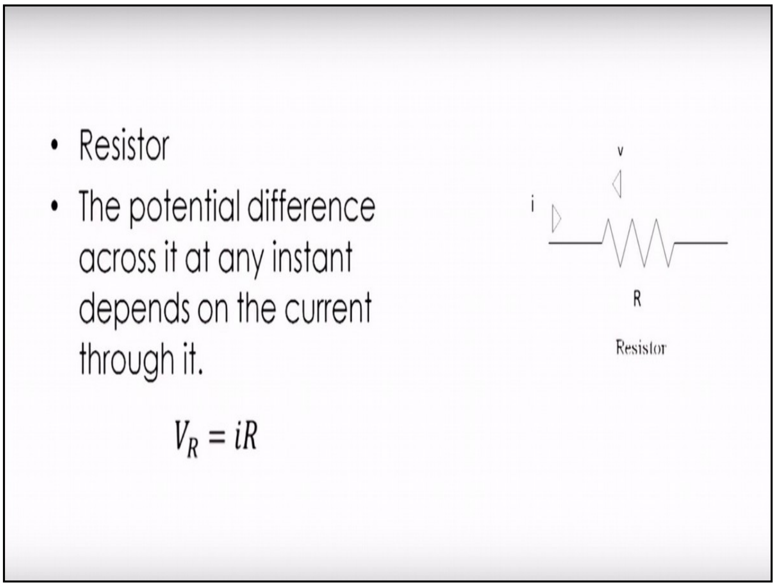

Next building block is resistor and you can this the systematic of resistor and the current resistor is i and again here the direction of back emf is strong and the applied voltage direction will be reverse of that and this the potential difference across it at any instant depends on the current through it of course this relationship all of us know very popular Ohm's law. So the that is the voltage across the resistor is going to be i into r that is and its going to be proportional to current rate and here r is the resistance of the resistor.

So these are the 3 basic building blocks which are used in writing the system equations for any electrical system so here I summarized the electrical system building blocks this is resistor the basic equation vr that is voltage drop across resistor is i into r where r is the resistance i is the current.

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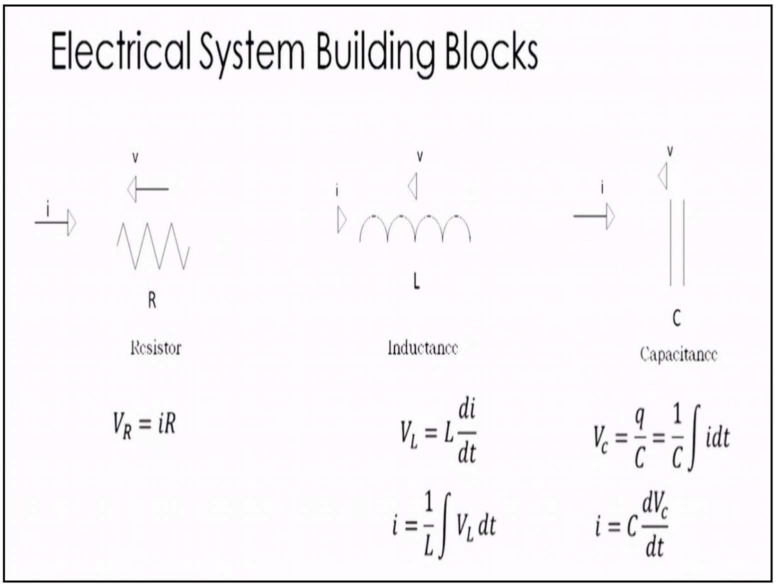

This is for the inductor here the voltage across inductor terminal is ldi by dt and from here I can write the expression for i as 1 by 1 integral vl dt and this is the voltage across the capacitor plate which is q by c is written again in terms of the current and from here I can write the current expressions. So these are the 3 basic system building blocks.

Now, let us see the energy and power associated with these elements as we have seen I have talked about these elements in our lecture on the bond graph modeling also the inductor and capacitor they are the energy storage elements both store energy whereas the resistor dissipates energy.

So energy is stored by an inductor as you can see here when there is a current i through it is given by half li squared. So this I am not working on the derivation of it you might have studied it is just review for you that, this is how we write the expressions for the energy is stored in an inductor when there is an current i through it.

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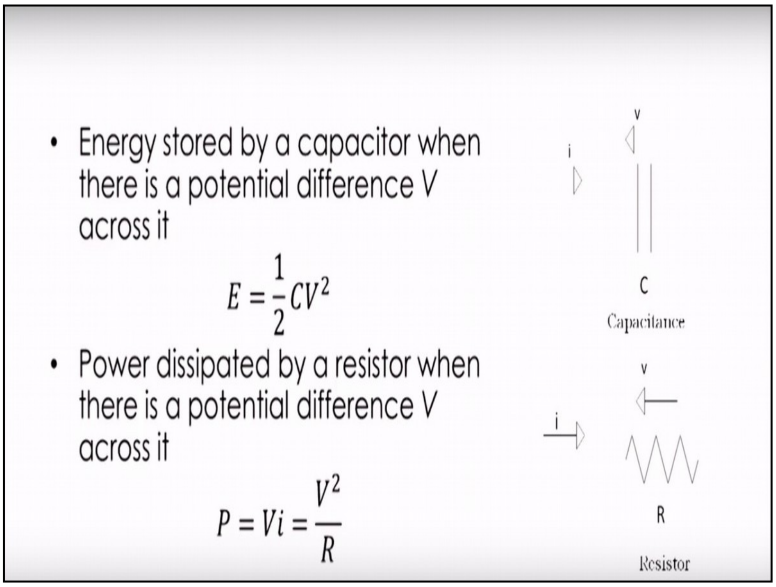

Similarly, if we talk about a capacitor the energy is stored by a capacitor when there is a potential difference v across its capacitor plates is equal to half c v squared where c is the capacitance of the capacitor and as I said resistor will be the dissipated power and the power dissipated by resistor when there is a potential difference v across the two hands of it is given by vi or I can write in terms of voltage and resistance as v squared by r. So this is how we defined the energy stored in the inductor and capacitor and the power dissipated by the resistor.

Now if we want to build up a model of an electrical system we can do so by using that two laws of the Kirchhoff's. These laws of the Kirchhoff's are what we call it the current law and the voltage law also people calls it and the first law basically tells about the current in to a junction and current out from a junction. So, if we look at this simple electrical circuit and were you have applied for voltage source of v there is resistor of resistance r 1 and there are two resistor again which are parallel to each other r 2 and r 3.

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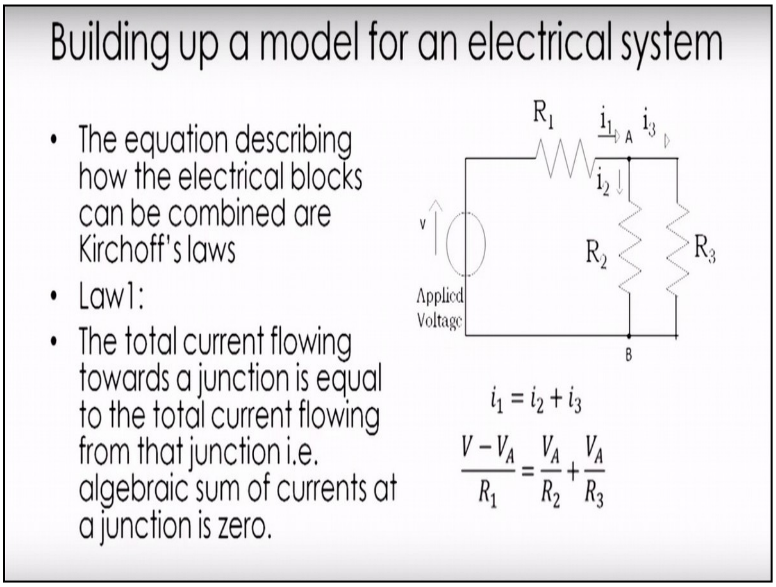

Now if we look at this junction a you can see that there is current coming into the junction which is i 1 and the current going out from the junction are i 2 and i 3 so this Kirchoff's first law it is that the total current flowing towards the junction is equal to the total current flowing from that junction or that the total current into a junction is equal to total current out from the junction or we can in other words as algebraic sum of current at junction is equal to zero.

So this is the Kirchhoff's first law and I will be talking about Kirchhoff's second law also and with the help of these two laws we can use them separately or use them in combination also we can derive the system equation for any electrical system. So for this case you can see the current i 1 is equal to i 2 + I 3 so current in at junction a is i 1 and out from junction a is i 2 or I 3.

So i 1 is I 2 + I 3 now here if I take b as my reference voltage which is brown alright then I can write this expression in terms of voltage so the voltage across r 1 is v - a i - v - va upon r 1 and now I get this expression from the resistance building block and i 2 will be the voltage here va - zero that is v a divided by r 2 and + i 3 will be v a - zero divided by r 3. So, this way i can write the expression for this system.

And this is how we can implement the Kirchoff's first law in order to get the system equation for this circuit.

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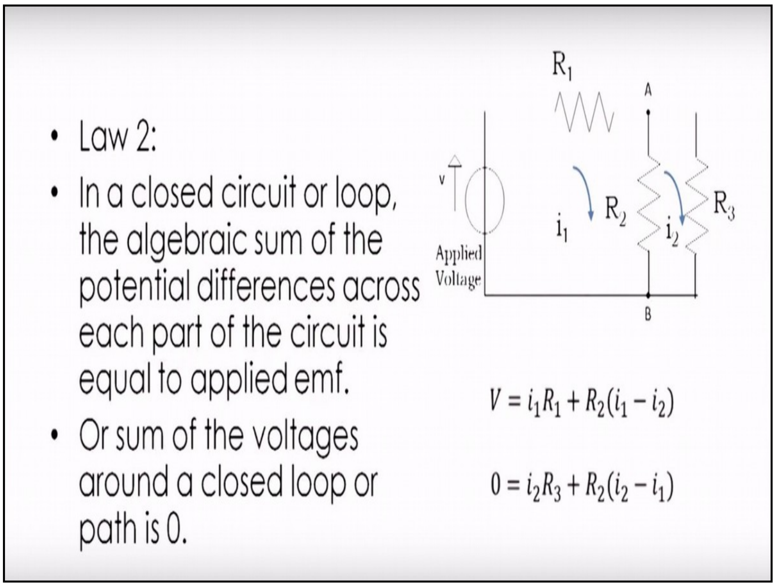

Next, let’s see the law two again I take the same circuit the one which we have see the previous one that is I have got the voltage source applied voltage here then we have resistor and there are two other resistor r 2 and r 3. Now this Kirchoff’s second law s that in a close circuit or loop the algebraic sum of the potential difference across each part of the circuit is equal to the applied emf or that is in a closed loop or path is zero this is what principle.

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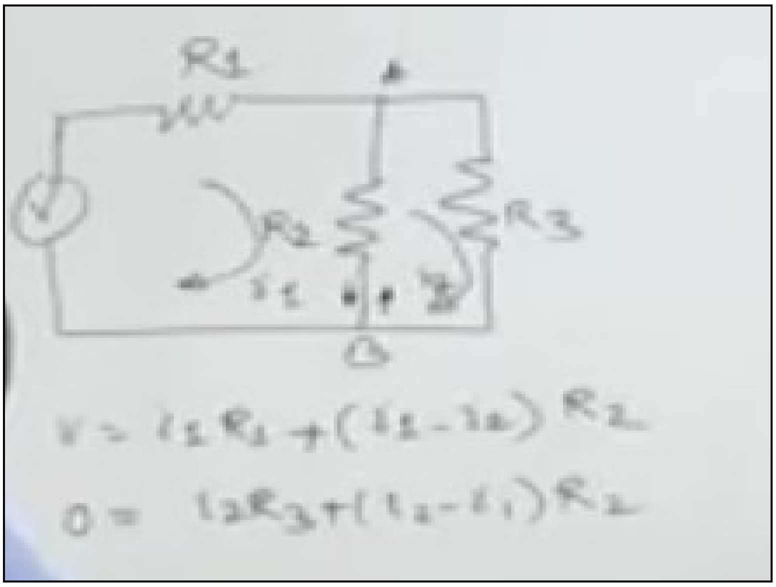

So here basically, I can just explain how can we write the expression for this circuit which we have considered here this is point and here we have two resistor and this my b and my a this resistor is r 1 and r 2 and this is r 3 now here what we do is that we assume a direction of current through the loop so suppose through this loop i assume a direction of current as i 1 and through this loop i assume a direction of current as i 2.

So, in the circuit you can see that we have got that two loops first loop is constituted by voltage source resistor r 1 and resistor r 2 and the second loop is constituted by resistor r 2 and resistor r 3. So if I write expression for the first loop it will be v is equal to same i 1 r 1 because here we are considering i 1 current to be flowing through all the elements here. So, this is going to be the i 1 e r 1 plus here basically you see that we are moving in this sense and the current through resistor this i 1 this is what we are taking as positive and this current is i 2.

So here basically this will be the direction for the i 2 and this will be the direction for i 1. So we have to find out the net current and so this is i 1 - i 2 into r 2 so this is the expression for the first expression that is expression for the first loop.

Similarly, I can write expression for the second loop now in the second loop we don't have any source alright. So Ii can just write the source voltage as zero and the current through the element is i 2 so it will be i 2 or 3 and here again you see that this sense we are taking to be positive so the net current in this direction will be what i 2 - i 1 into r 2 so this what I way we can write the expression for the second loop so this way we can write the equation for this system using the Kirchoff ' s second law or which is also called as the voltage law.

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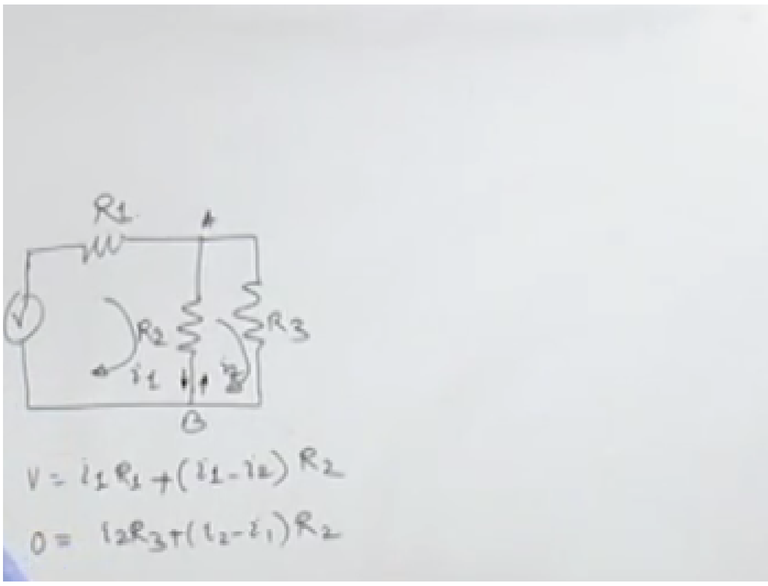

Now, next I will be explaining various examples where will be taking the three building blocks that is the inductor resistor and capacitor and will be using either the Kirchhoff's current law or the Kirchhoff's voltage law in order to build the systemic equation for the given circuit.

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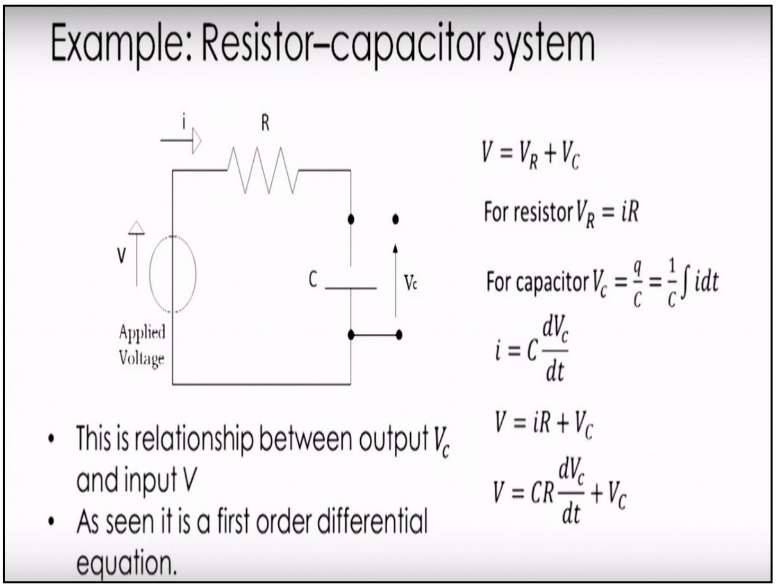

So let us take the first one alright this is an example of resistor capacitor system so here we have resistor, and we have a capacitor fine now there is a voltage source v here let the current through the circuit is high alright so what is input here in this circuit the input is the voltage the source voltage and I am interested in finding out the voltage across the capacitor so my output is my vc voltage across capacitor.

Now, I intend to find the relationship between the input voltage and output voltage. So I want to find the relationship between input voltage v and the output voltage v c so by seeing the circuit as a loop here we can apply the Kirchhoff's voltage law or Kirchhoff's second law. So here the v that is the source voltage is equal to voltage across resistor plus voltage across capacitor. So v is equal to v r + v c using the Kirchhoff's voltage law.

Now, they are that is resistor voltage across resistor we all known from the Ohm's law that this is going to be i into r where i is the current through resistor and r is the resistor fine then for the capacitor I can write vc is equal to q by c r this is 1 by c and q I can write integral idt and from here i can write I as if I take differentiation here so i is equal to c dvc by dt my this expression vri substitute ir and this vc I already have and now i replace for this r replace for this i as c dvc by dt and i get this expression.

Now, you see this expression involves the input voltage v and the output voltage vc which is of interest towards and of course the parameter of circuit that is c capacitance of the capacitor and the resistor of the resistor right so you can see that this equation is a first order differential equation.

So, this is what is lets see another example Resistor inductor system. We have a resistor and we have a inductor both are connected in series and there is a voltage source here this is again connected in series both of this element and my interest is to find out the voltage across the inductor that is the vl so again we can apply the Kirchoff's voltage law and here v will be vr + vl vr we know that it is i into r and for will of course is the output here.

So, what I actually need is I need to replace this i in terms of the vl so i go for the expression for the inductor that is i used the expression for the inductor building block. So, this vl is equal to ldi is divided by dt and form here i get i is equal to 1 by l integral vl dt and then i replace this i here and i get the expression involving the input voltage v and the output voltage vl resistor so this way we can get the expression for this circuit that is resistor inductor system.

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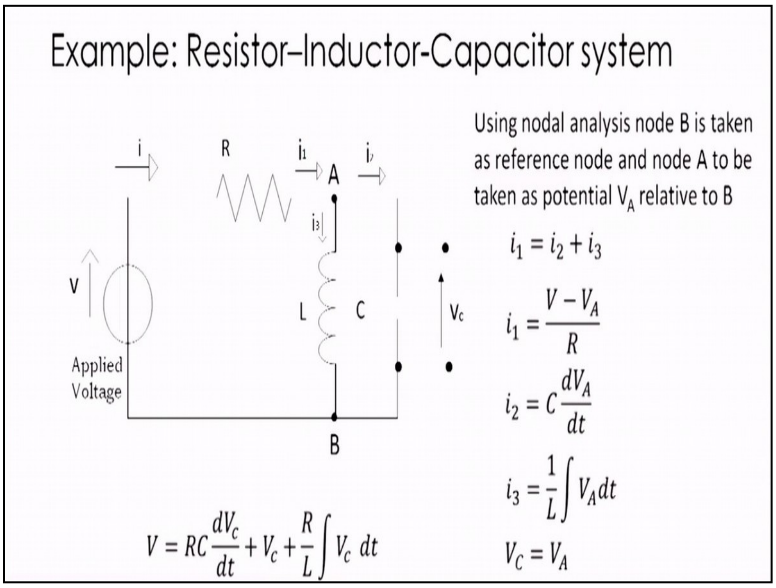

Next, let us see a resistor inductor capacitor system the ric circuit we have see the same figure same model in order to give you the first impression of the bond graph model. So let's see how we deal with here how we are able to develop the system equation using these three building blocks. So again here we have a voltage source a resistor an inductor and a capacitor all in serious and. My interest is to find out the voltage across the capacitor so the input here is the voltage source and the output here is the voltage across the capacitor so this is there.

So we can again use the Kirchhoff's voltage law and the here this v is vr + vl + vc that is the voltage drop across the resistor inductor and the capacitor and this is going to be equal to the input voltage fine. Now for the resistor building block vr is equal to ir where i is the current through resistor and r is the resistance of resistor now for inductor likewise i can write vl is ldi by dt so if i do that what i have is v is equal to ir for resistor + ldi by dt for inductor.

I am not touching vc because I want my expression in terms of vc because vc is ma output voltage so i want this expression in terms of vc only now the thing is that here you see i have got i's and i want to replace this i's in terms of vc. So i can the expression for current which is passing through the capacitor which is use to chance the capacitor as i is equal to cdvc is divided by dt. So i can used to substitute this v is equal to this vc + cr dvc is divided by dt + lc d2vc is divided by dt square.

So you can see here this equation is in terms of the input voltage v and the output voltage vc and moreover this equation is the second order differential equation fine. Let's see next another resistor inductor capacitor system and in this example rather than using the Kirchhoff's voltage law i would like to use the Kirchhoff's current law.

So let's see that my circuit is like this we have the voltage source a resistor and here inductor and capacitor they are in parallel and of course both together or in series with resistor and the voltage source so here let as assume that there is a junction a and the current here through this junction is i 1 is the in and the i 2 and i 3 are the out at this junction a and v is my reference voltage or which i can take as b.

Now my interest is to find out the voltage across the capacitor that is vc so if i go by the nodal analysis and i said by b is taken to be the reference node and node a to be taken at potential va relative to b then i can write using Kirchhoff's current r that i 1 is equal to i 2 + i 3 then i can replace for i 1 i 2 and i 3 is in terms of voltages.

So i 1 will be here v - va by r this i can get the voltage difference across the two terminals here so v - va by r using the definition for the resistors building blocks likewise i 2 i can write here i 2 is the current which use to charge this capacitor. So, i 2 is equal to c dva is divided by dt and i 3 I can write her current through the inductor as 1 by l integral va dot and once I substitute this here I substitute this i 1 i 2 i 3 in this expression this is what I get.

So this is winning equation for this circuit let us see the analogy between the mechanical system which we have seen in previous sub model and the electrical system there is a always and an analogy between the spring mass damper system and the rice circuit we have seen this analogy in our lecture on the bond graph falls you see that here I got the spring mass damper system and here I have got the series rice circuit fine.

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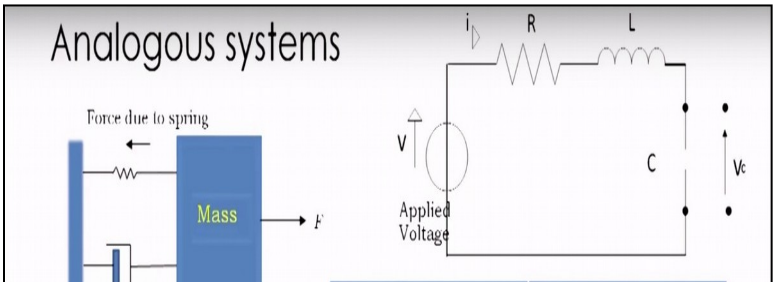

Electrical	Mechanical
Current (i)	Force (F)
Potential diff (V)	Velocity (v)
1/R	Damper constant c
Inductance	Spring
Capacitance	Mass

So if I want to write the expression for the resistance in electrical circuit this is I is equal to v by r and If, I want to write the expression for the damper by in a mechanical system then this is f is equal to c into v where f is force and c is the damper coefficient and v is the velocity.

Now if we compare these two expressions you can see that the force is an analogous to the current voltage is analogous to the velocity and the damping coefficient is analogous to 1 by r so this way I can build an analogous between the electrical system and the mechanical system.

So the current in electrical system is an analogous to the force in the mechanical system and this analogous is also known as the current force analogous and the potential difference in the electrical system is analogous to velocity in the mechanical system 1 by r here in electrical system is analogous to damping constant in the mechanical system and in similarly.

We can define that inductance is analogous to spring and the capacitance is analogous to mass in the mechanical system. So this way we can defined the analogy as have seen in told you in my initial lecture in many times we use this analogy to model the big mechanical system because it is not always possible to see variations are simulate big mechanical system. So what is d1 is that rather than that mechanical system we can model and analogous electrical system and we can do the parametric variation there in the analogous electrical system and.

We can see the effect of the various components of various variations I leave exercise for you if you can recall this was a circuit which we have seen during bond graph modeling of electrical system.

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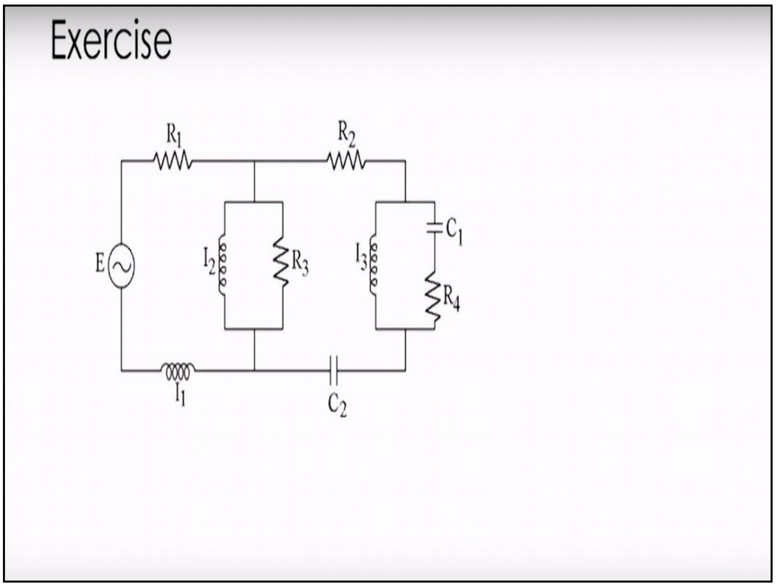

And I leave this exercise for you to derive the system equation for this system using the Kirchhoff's first and second law and yes you can work it out how you did in case of bond graph modeling was that we assume sub models here we created various sub models and we draw the bond graph of the system and then those sub models which were covered and they were gradually uncovered and we could easily draw the bond graph model for that system.

So if you do this one then you will appreciate the ease at which we could draw the bond graph model for the complicated electrical circuit like this. Thank you.