Modelling and Simulation of Dynamic Systems

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Lecture - 11
Basic system models - Mechanical Systems

I welcome you all, in this lecture on basic system models mechanical system which is a sub module for the course on modeling in simulation of dynamic system. so last five lectures we have seen how can we model the systems which are there in the multi energy domains using bond graph. In next few lectures I will be discussing modeling of similar type of systems using another method what we call it as the free body diagram.

So, essentially here what we will be doing is that initially will be defining some essential building blocks in order to model these types of system and those building blocks could be used to model real physical systems. So in next few lectures we will been discussing about mechanical systems, electrical system, hydraulic, pneumatic and thermal systems.

So, this is what I intend to do in these lectures. So we have talked a lot about mathematical modeling. Now to have a quick review of why a mathematical modeling is needed.

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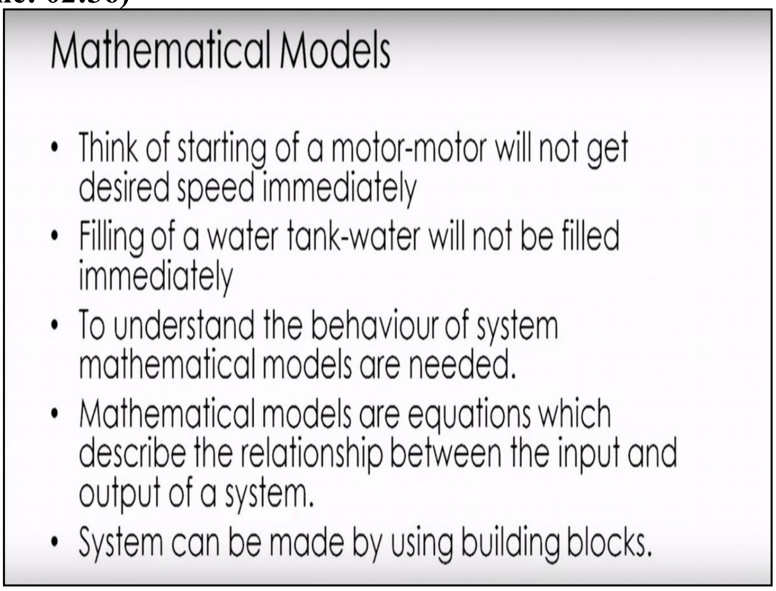

If, you think of starting of a motor, motor will not get the desired speed immediately it takes certain time to in order to get the desired speed so we may be interesting in knowing that how with time how this speed varies and then finally goes to the desired speed. Likewise suppose I want to fill of a water tank water will not be filled immediately, also there will be gradual increase in the height or change in the height of the water level with time and ultimately the water will be getting filled up. So if we want know what rate this height is changing or what rate the water is being filled up then it will help us in knowing the behavior of the system. Now as I said, to understand the behavior of systems mathematical models are needed.

These mathematical models are nothing but equation which describes the relationship between the input and output. So, what is input to the system and what is output from the system these mathematical expression basically gives you the relationship between the input and output.

As I said as we have these case of bond graph modeling there what we did that we identified the system components what all system components are made of inertia element damping element and compliance element like that here also we what will be defining is that the building blocks.

Those building blocks one building block will be exhibiting or taking of only single property and these buildings blocks combinations can be used to model the actual physical system we will be taking up certain example also.

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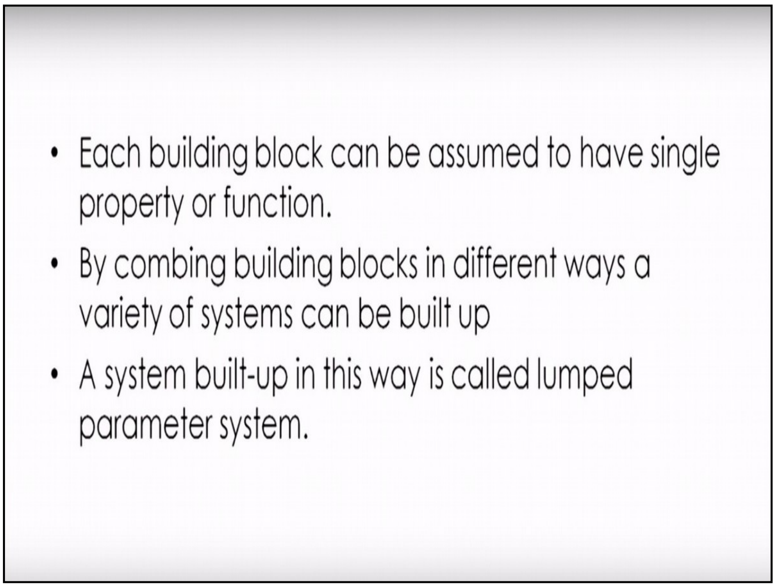

Now, as I said each building block can be assumed to have a single property or a function so one building block it could be either compliant one or it could be damper one or it could be either inertia one. So each building block can be assumed to have single property or a function and by combining building blocks in different ways a variety of systems can be built up. So this is how we are going to proceed and a system built up in this way what is a called or known as lumped parameter system. Now mechanical system building blocks.

My aim for this lecture is to discusses about the various mechanical systems building blocks and how by combining these blocks we can build a model of the mechanical system. In my next lecture will be taking up the electrical system

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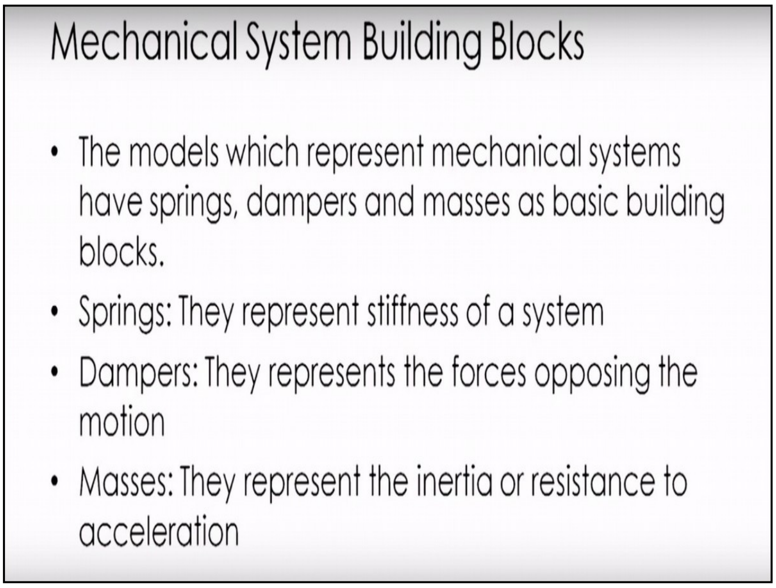

Now coming back to the mechanical system the models which represents mechanical systems have spring damper and masses as the basic building blocks. Now what does the roll of spring here is spring they represent stiffness present in the system. So if we take the example, very first example which I give you during our lecture class the first lecture in this course I took an example of the water tank.

So there the pillar of the tanks basically play the roll of the spring. So that way the spring represent the stiffness of a system then damper they represent the forces opposing the motion so in your system if you have certain force which opposes the motion then that force you can model with the help of a damper. Similarly the masses they represent the inertia or resistance to acceleration. That is in your physical system if there is some element which exhibits a behavior like this that resistance to acceleration then that element can be modeled as mass.

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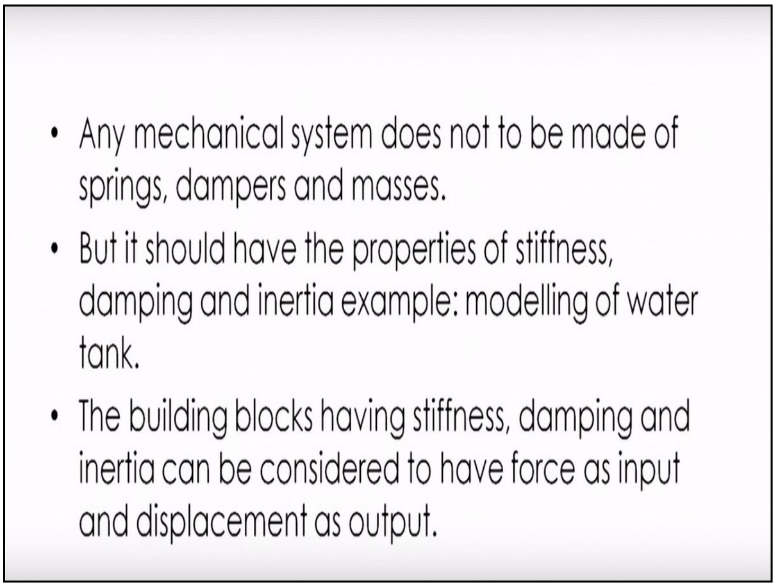

Now any mechanical system does not to be made of spring damper and masses. It could be as I said combinations or even the individual element could also be there, but it should have the properties of stiffness damping and inertia.

So, here as I said when we are talking about a physically system it is not required that, that physical system is actually made of those types of systems. That is actually made of a spring or damper or mass rather as I said they should exhibit the property of the spring damper and the masses and I give you the example of a water tank. The building blocks having stiffness damping and inertia can be considered to have the force as input and displacement as the output. So this we will be seeing here.

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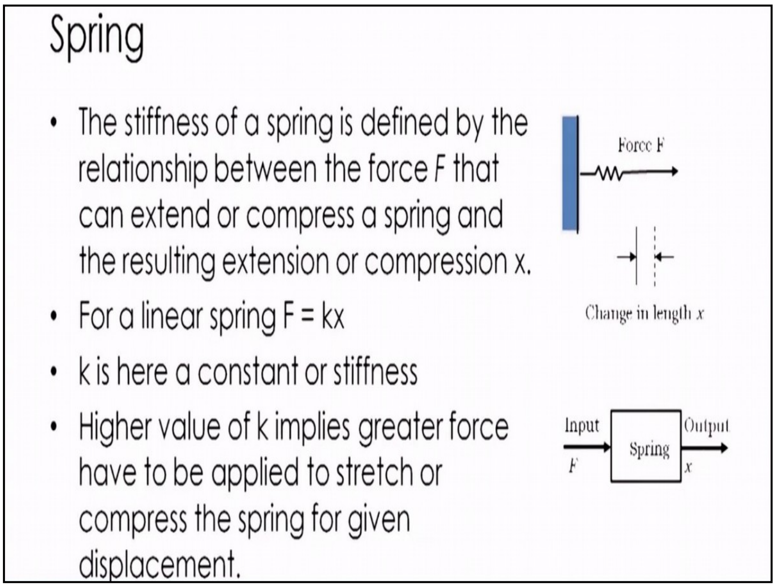

Let us begin with the first basic element that is spring now you see that we have schematic here being represented a spring is being represented and say force f is applied on this spring and because of this force there is a change in the length. Here in the block diagram mode I can just represent it like this, that there is an input f here to the spring and because of this input there is certain output x.

Or there is a change in length x of the spring. Now this spring have got a property what we call it as stiffness and this is defined by the relationship between the force f that can extend or compress a spring and the resulting extension or compression x. So this is how the stiffness of the is spring can be explain and this is how mathematically it is defined. That is f equal to kx where is k is the constant or what we call as the stiffness, what does this mean basically it is that higher value of k you will be requiring more force for the same deformation.

So, if you want the same deformation that your stiffness is increasing then naturally you will be requiring more and more force. In order to get the same deformation. Next building block is the damper. This building block represents the types of forces that are felt when one tries to push an object through fluid or move against the frictional forces.

Here you can seen the schematic is basically a cylinder piston arrangement where a force f is being applied here this side is liquid and this liquid offers resistance for the motion of the piston. So this type of situation is say action against the frictional force these can be modeled using damper. Now here you see faster the object is pushed greater is going to be the resisting force.

So the resisting here depends on the rate the object being is pushed or depends on the velocity of the damper. The damper which is used to represent damping force consist of a piston moving in a closed cylinder so here if we represent it by a block diagram you can see for an input force output here say output is x and this relationship between force and x is in terms of the derivative. That force is equal to damping coefficient into the velocity.

Now when the piston is moved the fluid on the other side tries to flow through or pass the friction and this flow produces the resistive force which I just explained to you and ideally this damping force is proportional to the velocity here as I have explained in my previous slide f is equal to cv and we c is the constant.

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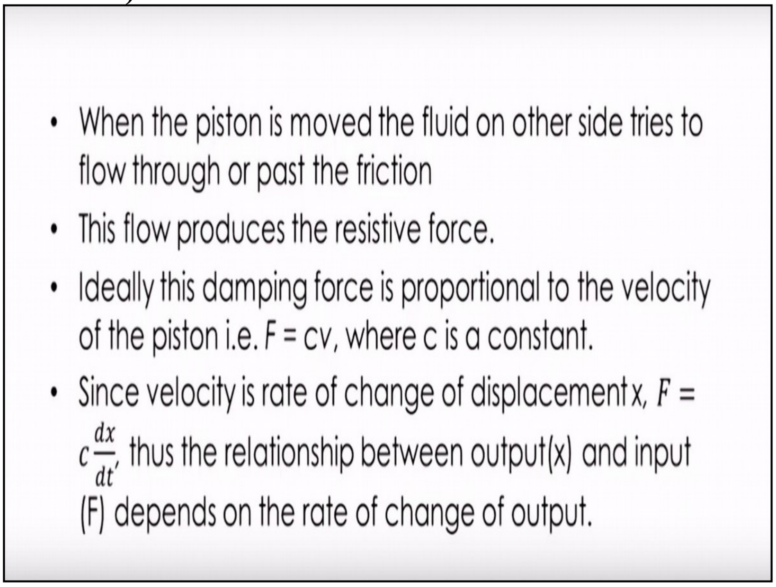

Since velocity is the rate of change of displacement x we can write f as c dx by dt so thus the relationship between output and the input depends on the rate of change of output. So this is what as I said the input is equal to constant into rate of change of output that is the rate of change of displacement position.

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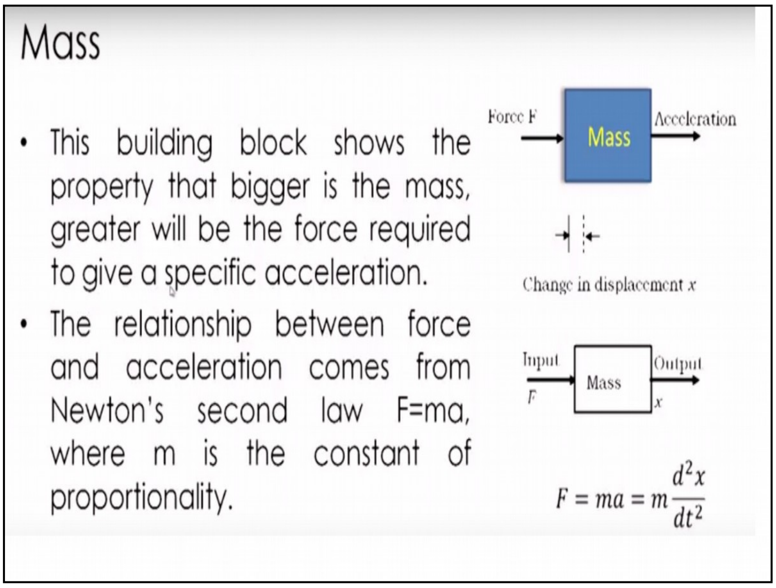

Next building block is mass, so, again this building block shows the property that bigger is the mass greater will be the force required to give a specific acceleration. Now, this relationship between the force and acceleration comes from the Newton second law which we all are aware of where f is equal to ma and here m is the proportionality constant or the mass Now let us see here in this **schematic** you have a mass a force f is applied and this force causes an acceleration of course there is a change in displacements.

Now, the input here is f and the output x but here f is related by the second derivative of the output that is mass into acceleration.

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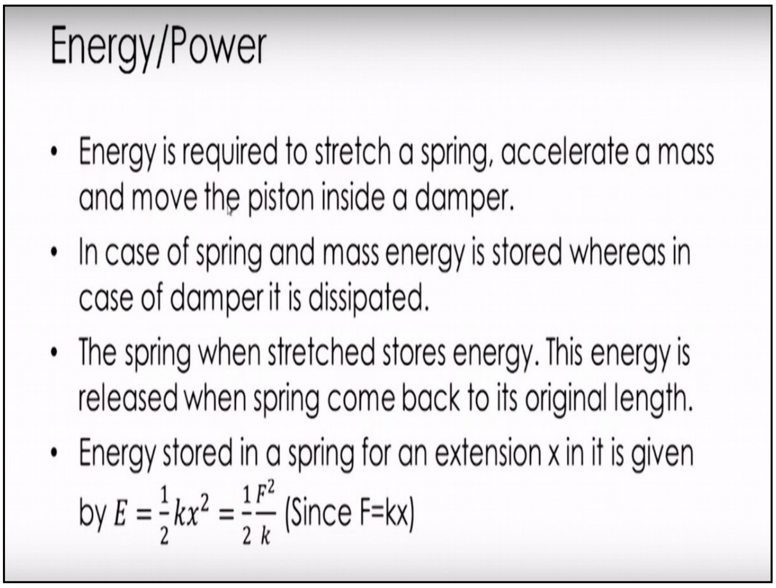

Now coming to the energy power, energy is required to stretch a spring, accelerate a mass and move the piston inside the damper. Now you see in case of spring and mass energy is stored whereas in case of damper the energy is dissipated.

We have discussed a lot about this in my lecture on bond graph. Now when the spring is stretched it stores energy and when it is released it releases the energy. Now fine and of course when the energy is released the spring comes back to its original position gets back to its original length. Now energy stored in a spring for a given extension is given by e is equal to half k x square or if I substitute for x here that is f by k it is half f is square by k.

So this is the energy, which is going to be stored in the spring. You can recall that it was based on the conservation of the power that we have drawn the bond graph. So, we try to correlate what we studied in the bond graph modeling and what we are studying here using the block diagram approach. Now which essentially will be using the free body diagram?

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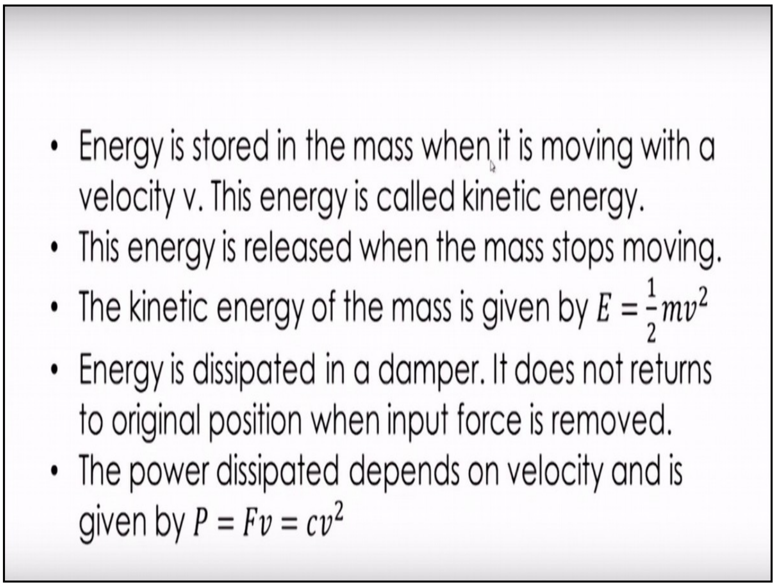

Now, energy stored in the mass when it is moving with a velocity v and this type of energy what we call it as kinetic energy and it expression all of us know that, half of the mv square where m is the mass and v is the velocity.

So the energy is released when this mass is stopped so, whenever this mass is stopped that energy it released. Now for case of damper as I said the energy is dissipated, it does not to return to its original positions when the input force f is retracted, so saying in a cylinder piston arrangement which is behaving as a damper when you retract the force when you draw back the force the piston is not going to come back to it's original position because the power has been dissipated and this dissipated power is equal to force into velocity or if I substitute for force it is cv square.

So these are building blocks for the translational system, translational system means the system which has got translatory motion or many times, we call at the which has got the linear motions also. So here essentially what we did we made the building blocks relating the applied force which is input and the displacement acts which is output. Now there is a similar counterpart in the rotational system.

For example, for the mass in the translatory system we have the moment of inertia in the rotary system or for stiffness in the spring in the translator system we have the tensional spring in the rotary system likewise for damper in the translatory system we have the rotational damper in the rotary system. So there is one to one analogy between the translatory and rotary system but let us see how these are.

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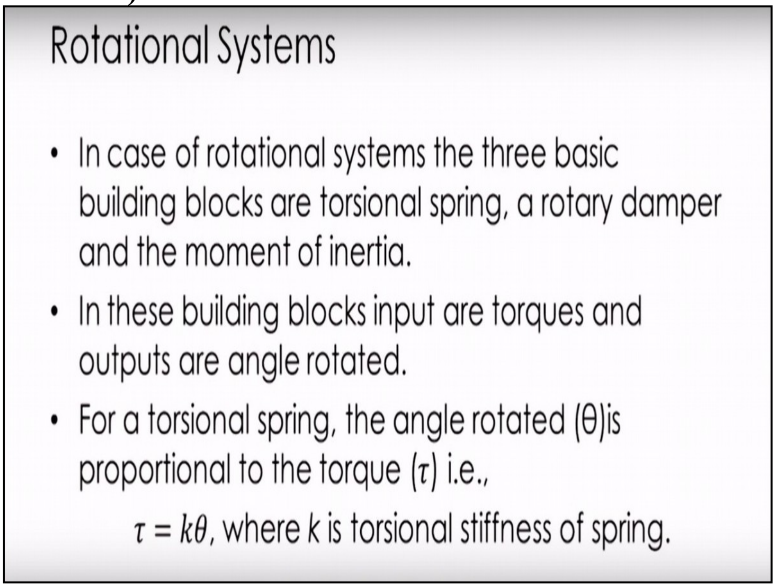

Now as I said in case of rotational system the three basic building blocks are the torsional spring, a rotary damper and the moment of inertia. So with the help of these basic building blocks we can make the model for any real physical system. Now these building blocks are subjected to input torque and the outputs are the angle rotated.

Now for a torsional spring as you know the angle rotated is proportional to the torque applied. So we have the relationship tau equal to k is the torsional stiffness of the spring as this is just similar to what we have seen f equal to k x x for the normal spring.

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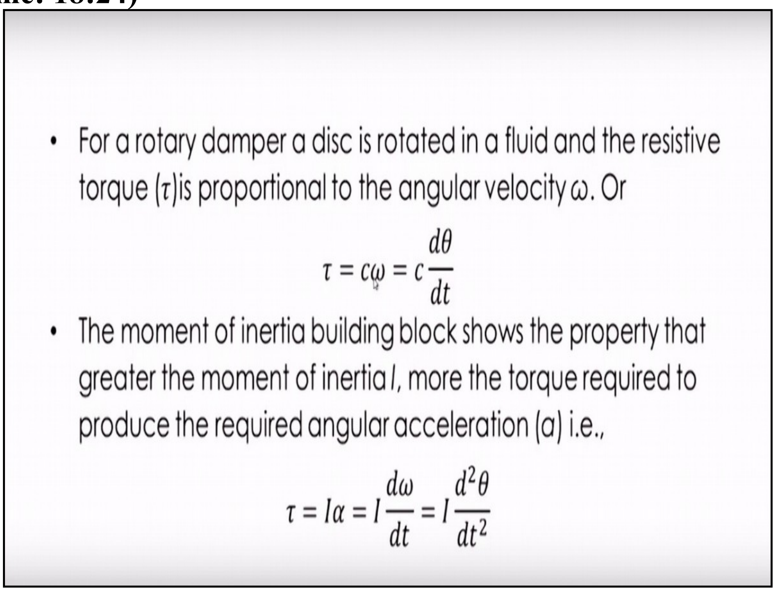

Now, for a rotary damper a disc is rotated in a fluid and the resistive torque is proportional to the angular velocity. In this case tau is proportional to omega and we can remove that proportionality constant by putting c here and here then we can write omega as d theta by dt so here basically you have the relationship between the input tau and the output rotation, but this relationship is related with the first derivative of the rotation. Likewise the moment of inertia building blocks shows the property that greater the moment of inertia more the torque is required to produce the required acceleration.

So, if I am going to compare the two system with one has lesser moment of inertia and one has more moment of inertia so naturally for a given acceleration if moment of inertia increases naturally your torque requirement is going to increase and here then I can write alfa as d omega by dt or I can write in terms of the displacement here so angular displacement so it is i d square theta upon dt squared.

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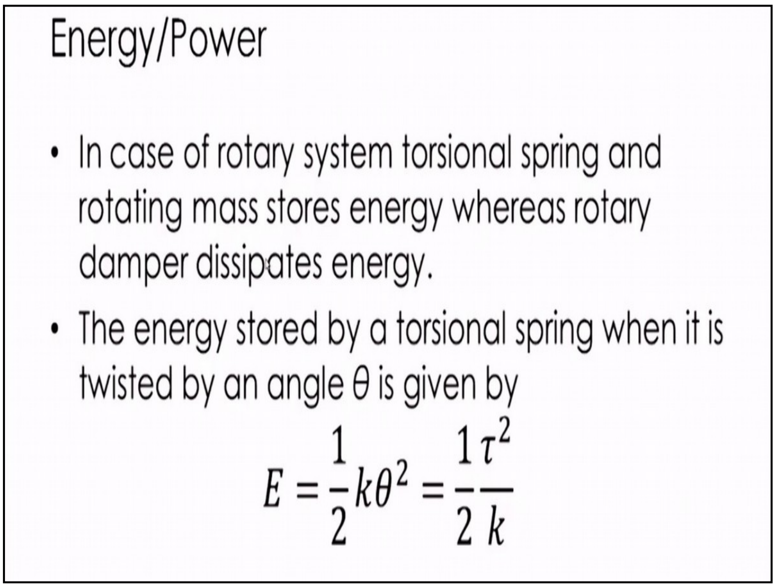

Then coming to the energy or power in case of rotary system, in case of rotary system tortional spring and rotating mass stores energy, whereas rotary damper dissipates energy the behavior is same. The energy is stored by tortional spring when it is twisted by an angle theta is given by half k theta is square or I can substitute for theta here I can get i half tou square by k. Similarly.

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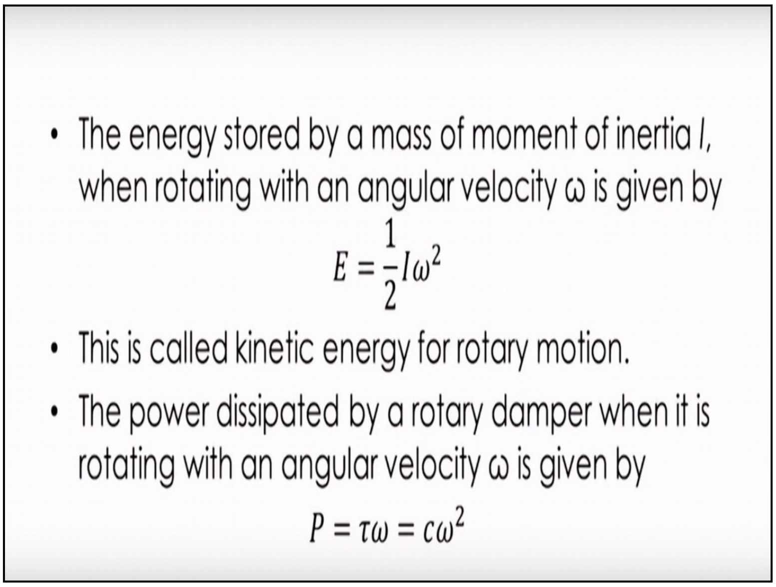

The energy is stored by a mass of moment of inertia I when rotating with angular velocity omega is given by half i omega square and this what we call it as kinetic energy rotary system or for the rotary emotion and the power dissipated by the rotary damper when it is rotating with angular velocity omega is given by tow omega or this is if by substitute for tow for this c omega is square so this is about the rotary system the three building block for the rotary system now let see an example building of a mechanical system.

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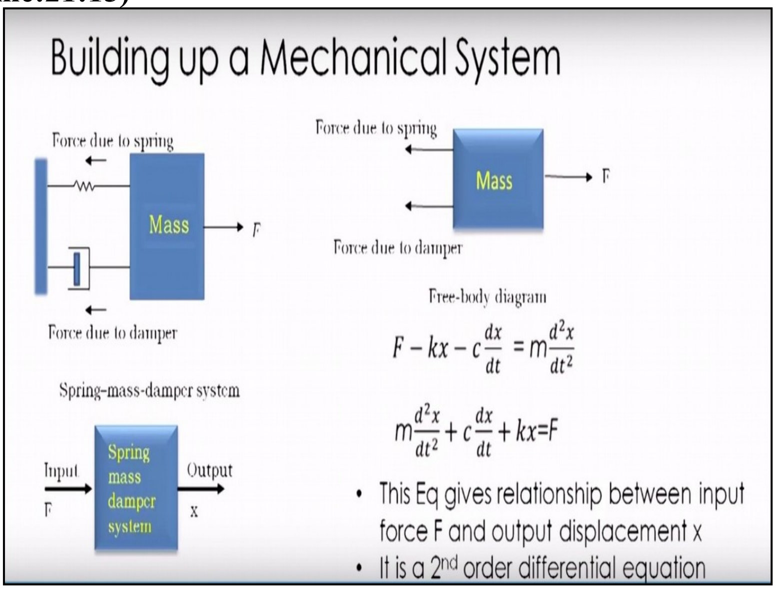

So suppose if, I have these as the mechanical system is spring mass damper system we have be spring we have mass and we have a temper system now this is mass temper system subjective force f is as input and output is a displacement x k .so here in the block diagram represent ate is spring mass a system input force and output x now i say was telling what we do is essentially t we draw for the first we draw the free body diagram of these.

So we have a mass here, the forces f exiting in direction and when the subjected to displacement basically see here we have mass say I apply force f let see my the coordinate directions x now when the force f is apply the expiring will be we can represent at this free body diagram spring force and this will be acting direction as kx and similarly the force due to damper this will be cx dot .

So this is how the free body diagram can be drawn. The free body diagram that I told the body and represent of all the forces acting on that body this is there now to write the system equations mathematically equation for this what is then is the in coordinate direction you have to find out the forces the resulted and goes the resulted forces are responsible for the acceleration of the mass.

So, this reluctance force here are f minus k minus cx dot system this going to be equal to mx dot are form here I can write this as mx double dot plus cx dot plus kx equal to f so this is the system equation for this system.

Here it is represented like this m2 square dt square plus dx by dt plus kx equal to f I here is see this is relationship between the input force f and output displacement x it is a second order differential equation of we can take example of many physical system k to model using these type of building block is

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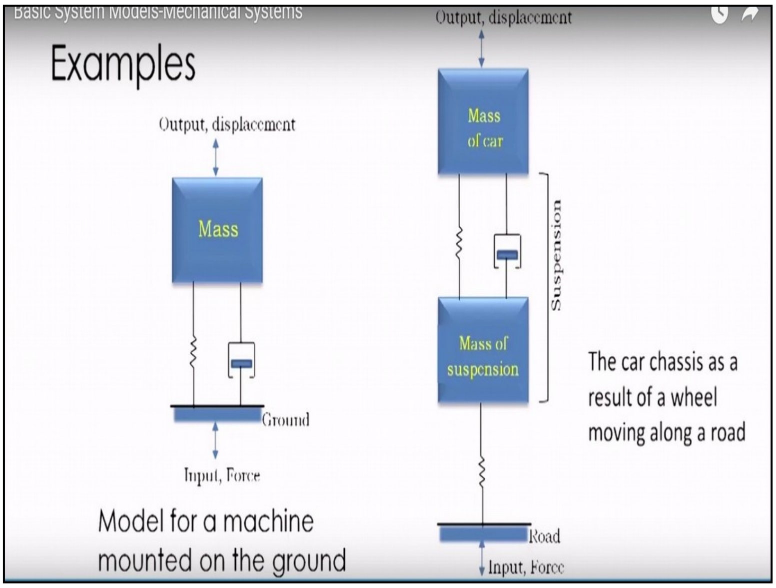

One example could be the machine mounted on the ground so you have the mass there are the ground excitations ground force which is coming from here and here is a output displacement.

So we can model this system as using the same building blocks or I can say modeling car we have seen a lot the modeling of the car born graph modeling so here say you have mass of car is there is the is a suspension system here the represented by spring and ampere then mass of the suspension there are is the there is and they ground excitation so again this system can be model using spring and temper in the mass another example the what we can see there are

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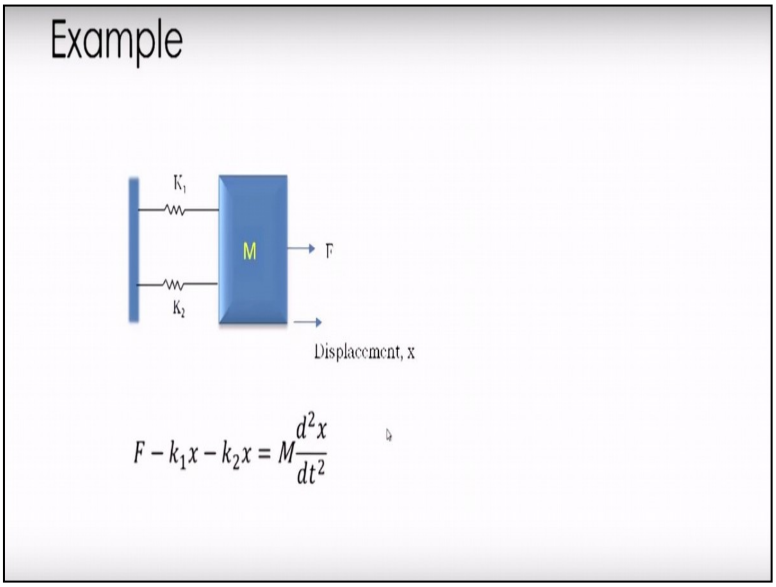

We can a take this example of if spring mass spring mass system so they are two spring which are is here which are in parallel and here for this system also we can right the system equation simply so what we do is here is force and these by say cordites dedication so here f minus k1 x minus x2 x equal this to m x double dot so this is what the equations the is going to mx double dot is going equal to f minus k1 minus x minus k2x.

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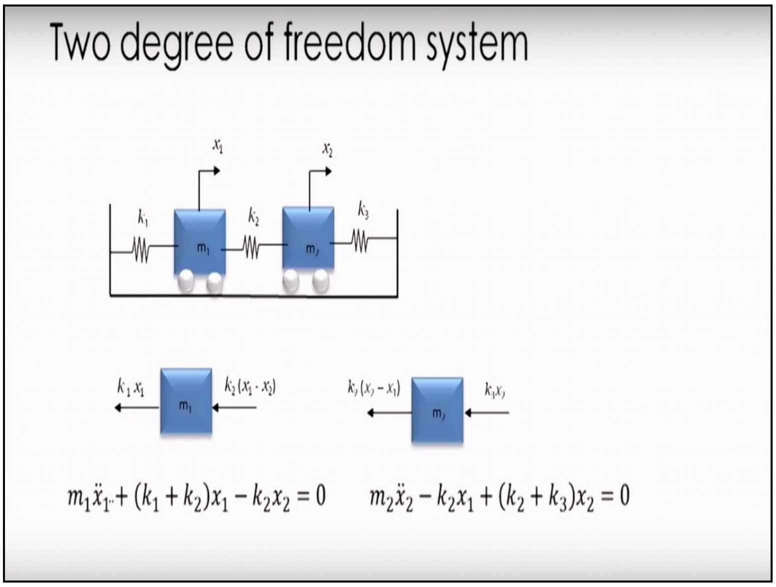

We can take by little complicated system like this here so two degree of freedom system now these two degree of freedom system there are the two mass which are the two connect by spring of the k1 k2 and k3 .so here what we k2 we take a direction a save positive direction x1 like this x2 like this.

We draw free body diagram of both the masses now for say past mass when its given small displacement x1 direction the force is by this is going to be k2 x1 minus x2 here and the forces by this spring mass is going to be k1 and x1 in this direction now what we do we find out for that case the unbalance forces those forces will be here for these case is k2 x1 minus x2 and these one these k1 x1 and this my x1 since positive since.

These one the forces in this direction is minus k1x k1x1 minus k2minus x2 and using minus sign because direction up going to that or this one and this going to be equal to m1 x1 double dot you can seen get here all write likewise.

I can write for the second mass here m2 so other end of the expiring here is fix at will be having 0 velocity so if these my x2 direction Then the force by this is spring is going k3 x2 on the force by other spring going to be here k2 x2 minus x1 so this is going to be and then I can written the forces induction minus k3x2 minus k2x2 minus x1 and this equation to m2 x2 double dot and when simplifier is I get this equation.

So, this way again I can draw the equation here we can again a ia can draw the equation here we can introduce the damper equation also and we can represent damping sour's and cooingly the equation would be mode fine

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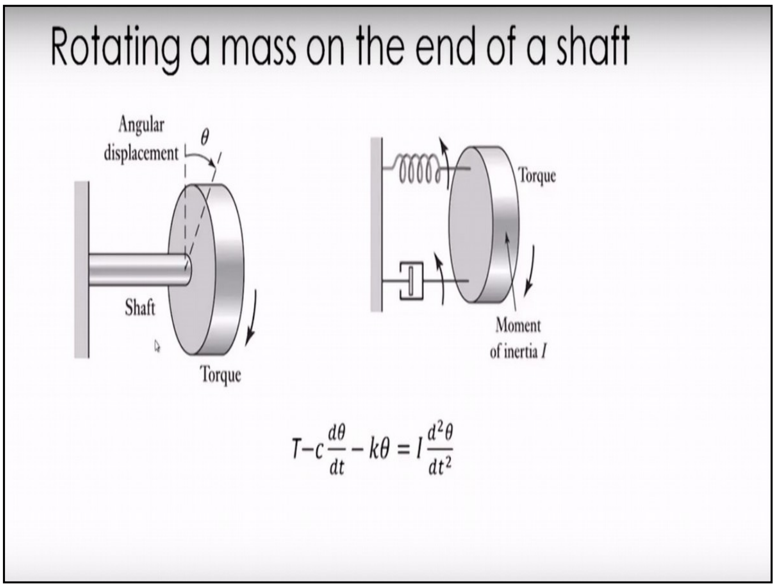

The last example I am going to take its rotating a mass at the end of shaft suppose here in the shaft here and this shaft at the shaft at the of there is the mass here and I want to represented are mathematically model is system .

So say the dark apply in the direction this my positive sense for the machining of the theta then what we do basically is that this shaft behavior we can represented by a faunal is spring and a rotary damper and of course I can have got this mass with the moment of inertia I then order to Wright the expression for this system here what we do is that that is say I have this system I have a factional spring and there is say tensional damper find this is I remember.

Positive theta and a play a dark here now this is tensional spring going to apply the of resistor top of minute say k theta and the rational is going to apply dark of minute c theta dot so what are my unbalance here unbalance stork t minus k theta minus c theta dot and this .I can equal to as I theta I double dot the basically what we have here are that is the t minus t minus c d theta by dt minus k theta equal to I theta double dot is I d2 theta by dt2 plus c d theta by dt t k theta equal to torque.

So this way we can model this rotation mass at the end of shaft so with I rouses lecture on modeling of mechanical system the next sum model looking at the modeling of electrical system. Thank you.