Introduction to Mechanical Vibration
Prof. Anil Kumar
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

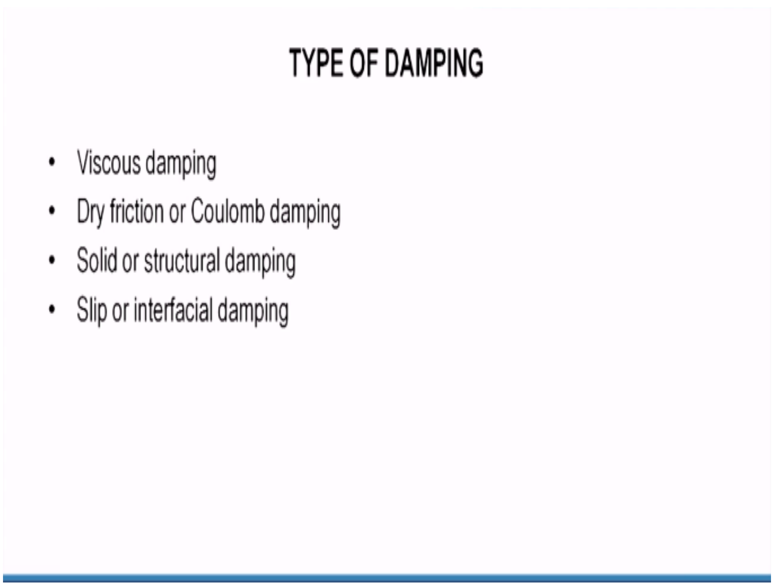
Lecture - 08
Damped Free Vibration

Welcome to the lecture on Damped Free Vibration. So in the previous lecture we discussed the free vibration of single degree of freedom system having mass and stiffness. There was not a damping element so there was not a dissipation of energy in the system. So the vibration of the system was maintained forever. There was no reduction in the amplitude of the vibration.

Now if we introduce a damping element in the system means we have M we have K now we have the C the damping element in the system then what will happen like in the case of the free undamped vibration we had the vibration of the system with the natural frequency omega N now will the damped system will vibrate with the same frequency as omega N or different, will the amplitude of the vibration remain constant or it will change with the time, will it decay with the time, will it increase with the time.

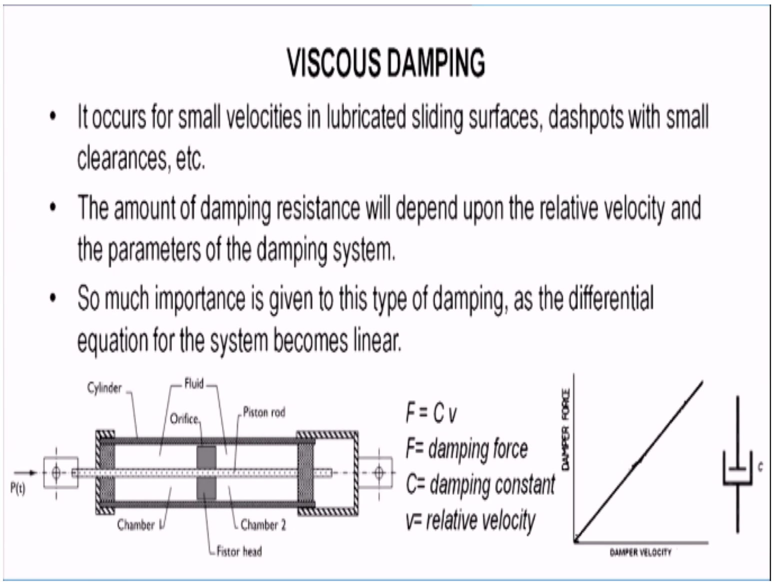
So all these questions that we will discuss in this lecture.

(Refer Slide Time: 01:55)



So we will discuss type of damping because when we want to discuss the damped free vibrations there are several types of damping the most common is the viscous damping because this is very important and simple so it is used vastly then we have the dry fiction or Coulomb damping then solid or structural damping then slip or interfacial damping. So here viscous damping, viscous damping occurs when a piston in certain fluid that has some viscosity. The piston moves in the fluid and the fluid is allowed to pass through some orifice so there is the small velocity of the movement of the piston.

(Refer Slide Time: 03:04)



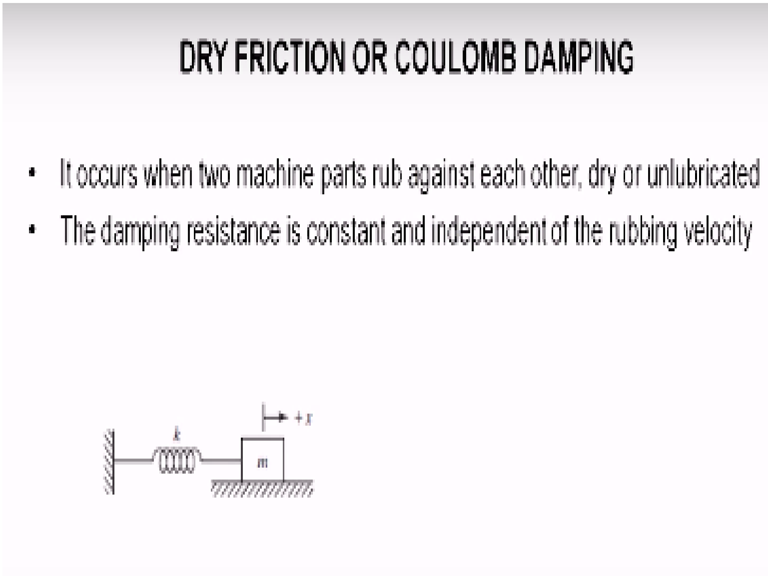
So this kind of damping we can see that the force and velocity relationship that is linear. Here is the force F = C into V where F is the damping force C is the damping constant and V is the relative velocity. Here we can see one damper and if some force is applied here is the piston and it is moving and here is the orifice through which the fluid is passing so this kind of damper is called the viscous damper because they are based on the principle of viscous damping.

Why we give this so important to this kind of damping because here the force viscous this damping force is directly proportional to the velocity so X dot. So if we use this put this force in the equations differential equation of motion of the system that makes the equations differential equation linear so that is why we this damping is more important. Several times we use the equivalent of the other type of damping are equivalent viscous damping for the other type of damping just for this reason.

We represent this damping C like this symbol then another type of damping is friction dry friction damping or Coulomb damping. So whenever a mass is sliding on some surface there is friction between the two surfaces and due to the friction there is some energy dissipation because the basic role of the damping is the energy dissipation so like in the viscous damping it was the energy dissipation in the fluid and energy dissipation is usually results into heat and sound.

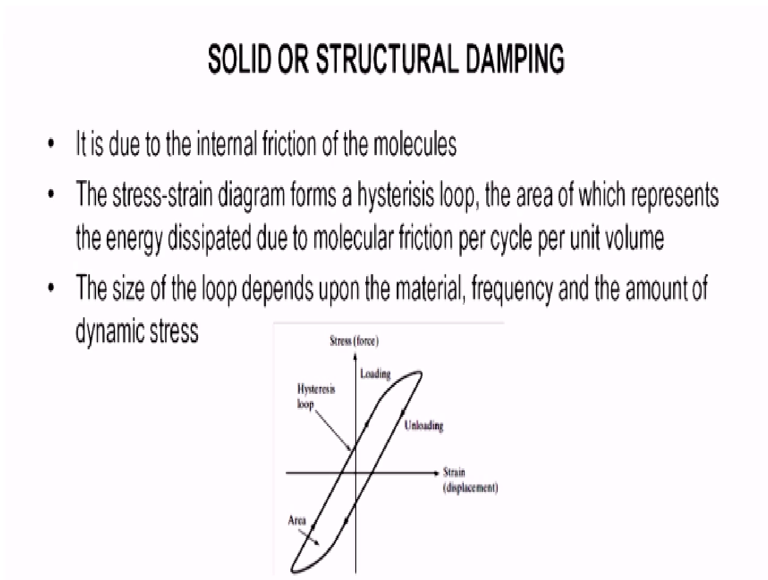
So here when a mass is sliding on a surface due to the friction there is energy dissipation into heat. So when there is dry friction here the damping resistance is constant because it depends on mu and therefore it is independent of the rubbing velocity.

(Refer Slide Time: 06:19)



So here we can see we have mass that may slide on the surface and due to the friction between the two surface there could be the energy dissipation and this kind of mechanism is called Coulomb Damping.

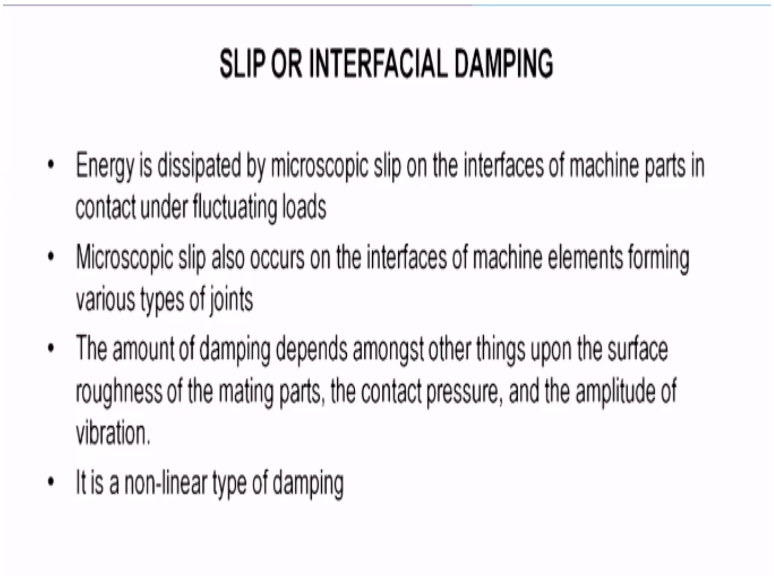
(Refer Slide Time: 06:31)



Now we have another type of damping that is called Solid or Structural Damping. It is also called a Material Damping or Hysteresis Damping. So there is various name to this damping. So we can see the stress-strain diagram and stress strain diagram when there is a material that has this kind of damping. The stress strain diagram may form a hysteresis loop this loop that is the area there is some area between stress and strain and the area shows the energy dissipated in one cycle of this loop.

So this energy shows the energy that is due to the friction in the molecules so there are the molecular deformation in the material. So therefore this damping depends on material, the frequency and amount of the dynamic stress.

(Refer Slide Time: 07:43)

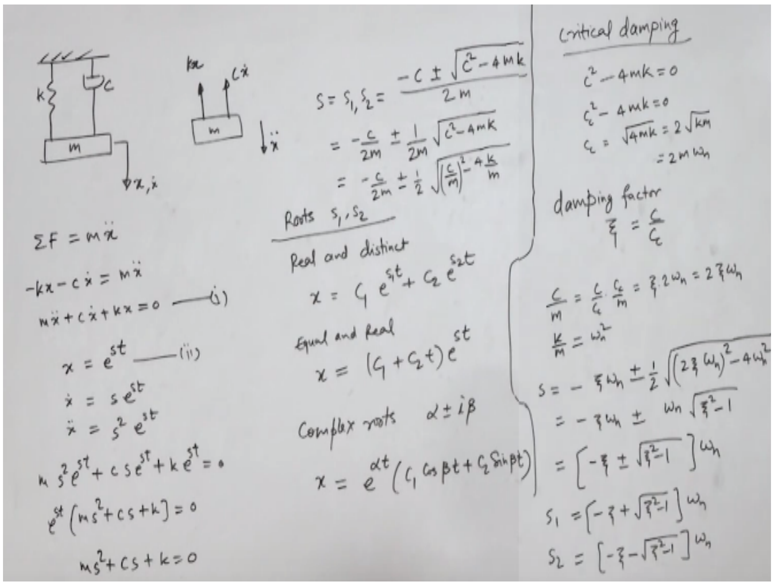


There is a one more type of damping that is Interfacial Damping or Slip Damping so we have several mechanical parts and there is roughness between these parts and when they come into contact they meet. There is some micro level of interfaces and rubbing and that makes the energy loss, energy dissipation. For example, there is some joints like bolted joints and some machine parts so this damping depends on the roughness of the surfaces the contact pressure and the amplitude of the vibration.

Now in this lecture we will discuss, we will take the viscous damping as we said that in the taking the viscous damping there is certain advantage that it makes the differential equation linear and so it is easier to solve these kinds of equations. Therefore, we start with a single degree of freedom system having viscous damping. So we can see here we have a spring damper and mass system and again it is a free vibration case.

So due to the free vibration we will give some initial disturbance to the system and then we will isolate this mass to make a free-body diagram means we have to show the forces that is acting on the body and then we have to apply the Newton's law of motion to write the differential equation.

(Refer Slide Time: 09:42)



So let us do it so we take the single degree of freedom system so this is the system having spring damper and this is mas. So we disturb the system so we put here X now we make a free-body diagram so this is the mas M and because we have pulled this system X and it has the velocity X dot so the forces. So let us assume this is the direction of X double dot and because we have pulled the system with X we have spring force KX and this is X dot.

So we will have here CX dot because when we pull it the damper or spring will apply force opposite to this pull. So this is our system. Now we have to write the equations so here we have sigma F = MX double dot. So sigma F we have two forces spring force and damping force so here it is - KX because their direction is opposite to the direction of X double dot that is why we are giving negative so - KX and - CX dot = MX double dot.

So we can write MX double dot + CX dot + KX = 0 and this is the equation of motion of free vibration of this system. Now you see if we put C = 0 means if we remove the damping our equation results MX double dot + KX = 0. So it is the same as undamped free vibration case. Now we have to solve this equation and to solve this kind of differential equation we assume a solution X = E power ST.

So we have assumed one solution X = E power ST and we put this into this equation1 so put this X = E power ST. So X dot = S into E power ST and X double dot = S square into E power ST. So we put these values in this equation so M into S square E power ST + C into S E power ST + K E power ST = 0. So here we can take E power ST outside so E power ST MS square + CS + K = 0.

So because this is exponential so this cannot be zero. So E power ST cannot be zero that is why this term should be zero. So MX square + CS + K = 0. Now this is a quadratic equation in S so we can write the roots it will have two roots. So let us write the roots so S = so it is S1 and S2 = so - B + - under root B square - 4 AC upon 2 A so 2 M. So - B, B is C so it is - C + - under root B square so it is C square - 4 A, A is M and C, C is K upon 2A.

So 2 and A is M now here we can write so - C by2 M + - half by 2 M under root C square - 4 MK. Now the solution of this differential equation if we have roots so roots these roots S1 and S2 they can be real and distinct. So we will have solution X = C1 E power S1 T + C2 E power S2 T. If they are equal and real so we will have solution X = C1 + C2 T E power ST.

So because S is S1 or S2 anyone if they are complex roots so we have if complex roots so that is let us say alpha + - I beta. So this is if these kind of complex roots we have S1= alpha + I beta S2 = alpha - I beta. So we have X = E power alpha these C1cos beta + C2 sin beta T. So these are the solutions depending on the different conditions of the roots so roots could be either real and distinct or real and equal or they could be a complex pair.

So in these three conditions the solution will be given by these equations. So now let us so this is our equation now we define certain terms so let us see. Now we define critical damping so critical damping comes when we have the equal and real roots so equal and real roots means this term is zero. So C square - 4 MK = 0. So here we have and this Cis CC so we call it CC square - 4MK = 0.

So CC = under root 4 MK = 2 root KM or we can write it 2 into M into omega N because omega N is root K by M and M will cancel out. So the critical damping is the damping that we obtain when our roots are equal and real it means this part is zero so this under root part is zero so we get CC = 2 root KM or 2M omega N. Another thing we define that is damping factor.

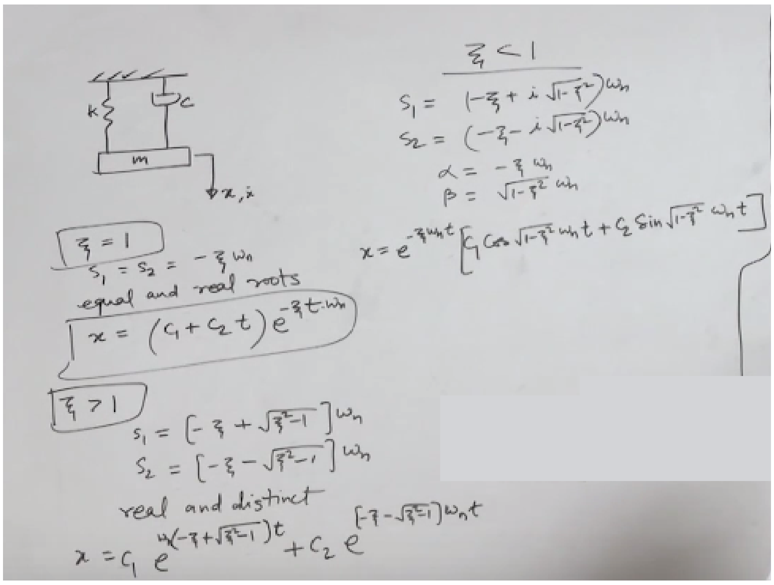
So damping factor zeta = C by CC. So C is the actual damping value and CC is the critical damping value. So the ratio of this is C by CC. Now we write this equation of S in this term so S equal to so first we write C by M so C by M = C by CC into CC by M. So zeta into CC by M CC by M is 2 omega N so = 2 zeta omega N. Now we write here we can write here - C by 2M + - 1 by 2 root C by M whole square - 4 K by M because here M will be M square when it will go inside.

So 4K by M K by M is 4 K by M and K by M = nothing, but omega N square because omega N is root K by M. So now we can write these values in this equation so S equal to - C by M is 2 zeta omega N and it is by 2 so it is - zeta omega N + - 1 by 2 under root C by M square. So C by M square is 2 zeta omega N square - 4 into omega N square.

So here we will get - zeta omega N + - 1 by 2 and this is 4 and this is 4 so 2 will come out and it will cancel out and omega N square omega N will cancel out so this will be cancelled out omega N root zeta square - 1. So here we can write = - zeta + - under root zeta square - one omega N. So these are our roots so our S1 = - zeta + root zeta square - 1 omega N and S2 = - zeta - root zeta square - 1 okay.

So these are the two roots S1 and S2 that we have obtained now depending on conditions we have to fit these in this three conditions. We have to see in which condition these roots are coming okay. So now I remove this part.

(Refer Slide Time: 23:22)



So now here if we have zeta =, zeta = 1. So if we have zeta = 1, so this is 0, this is 0 we will have S1 = S2 = - zeta. So it means we have equal and real roots so we will have X = C1 + C2 T E power S is - zeta into T, okay, for zeta = 1. So if zeta is greater than 1 so zeta is greater than 1 we have S1 = - zeta + root zeta square - 1 omega N and S2 = - zeta - root zeta square - 1 omega N. So here we see that because zeta is greater than 1 so this quantity is greater than 0 this is greater than 0.

So these S1 and S2 are real and distinct. So we will have this equation so we will have X = C1 E power S1T. So S1 is - zeta + root zeta square - 1 T + C2 E power - zeta - root zeta square - 1 T. So it is - zeta into omega N so here it will be into omega N. Here into omega N and here is also into omega N so we have - zeta + root zeta square - 1 omega N into T + C2 E power - zeta - 2 zeta square - 1 omega N into T.

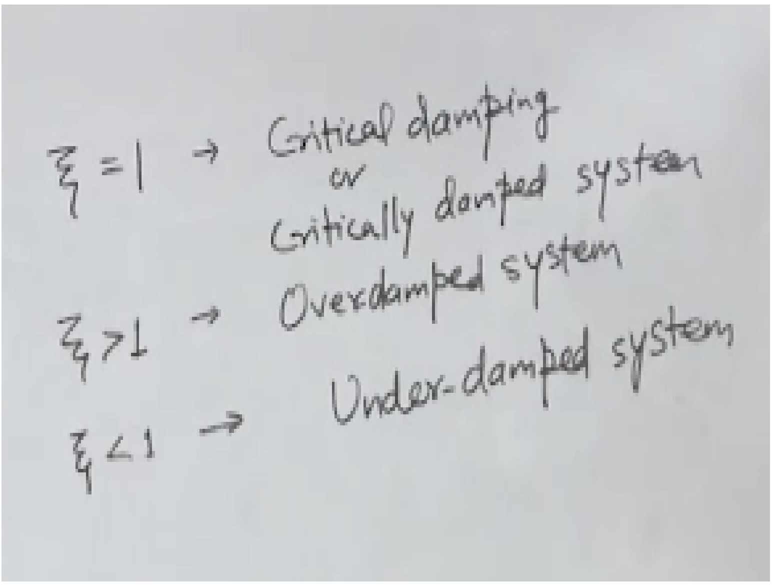
So here we have the condition when zeta is greater than 1. Now another case we will have so let us have the other case. So when zeta is less than 1. So we take a case when zeta is less than 1. So when zeta is less than 1 we will have S1 = - zeta + I root 1 - zeta square because we can write here is - 1 - zeta square. So - root so - can be 1 into omega N and S2 = - zeta - 1 root 1 - zeta square omega N.

Now here we see that we are getting a complex pair root. So here if we compare from this alpha + I beta we have alpha = - zeta omega N and beta = root 1 - zeta square omega N. So we can write the solution X = E power - zeta omega N T C1 cos root 1 - zeta square omega N T + C2 sin root 1 - zeta square omega N into T.

So here we have reached to three conditions based on the damping factor zeta. So zeta = 1 so zeta = 1 we get equal and real roots and the solution, solution means the response of the system that is X = C1 + C2 T into E power - zeta omega NT. When zeta is greater than 1 we have got the two real and distinct roots and the response of the system X = C1 E power omega N - zeta + root zeta square - 1T + C2E - zeta - root zeta square - 1 Omega NT.

However, for the case when the damping factor zeta is less than 1 we got the complex pair roots and for complex pair roots we have - zeta + - I root 1 - zeta square omega N and we have alpha, beta and we write X = E - zeta omega NT and C1 cos root 1 - zeta square omega NT + C2 sin root 1 - zeta square omega NT. So these three conditions lead to us the three different types of damp systems.

(Refer Slide Time: 30:48)



So these three different type of damp system like zeta = 1 that is called critical damping or critically damped system. If zeta is greater than 1, we call it over damped system or overdamping of the system. When zeta is less than 1, we call it under damped system or underdamping of the system. So we see that in these three different conditions of damping that is critical damping over damping and underdamping the response of the system is different.

So because from the expressions of the response X we can see that their response is governed by different equations and therefore we have if you want to study a system we have to decide that what the damping value we want I mean we want our system to keep in the under damped region, over damped region or critical damp region because all these different regions has their own characteristics and that we will discuss in detail one by one in the next lectures. So thank you for your attention and see you in the next lecture. Thank you.