Introduction to Mechanical Vibration
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Lecture - 07
Energy Method

Welcome to the lecture on free vibration of Single Degree of Freedom Systems. Today we will discuss the energy methods. We already discussed in the previous lecture the case of free vibration using the Newtons Law. We wrote the equations of motions we found the natural frequency. Here there is another method that is energy method with the help of this method we can also get the natural frequency of the system.

So here we know that if we have a system consisting of only the stiffness and mass okay and we disturb the system it will vibrate with its own natural frequency and this is the case of free vibration. Here the energy is neither dissipated nor supplied from the external force. So the total energy of the system remains conserved that is why these systems are called the conservative systems.

Mass has a kinetic energy due to its own velocity and the spring stores the potential energy due to when it is stretched. So the sum of these two energy is constant for a conservative system.

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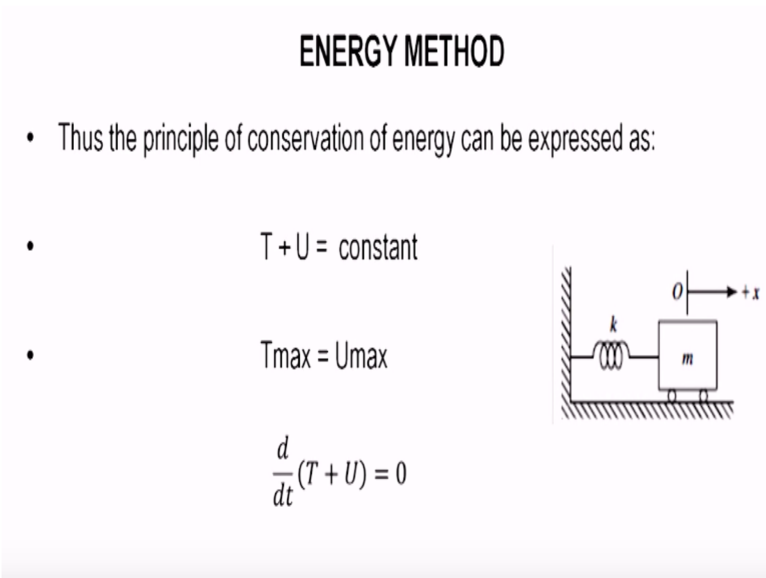

So the principle of conservation of energy we can write as T + U = constant.

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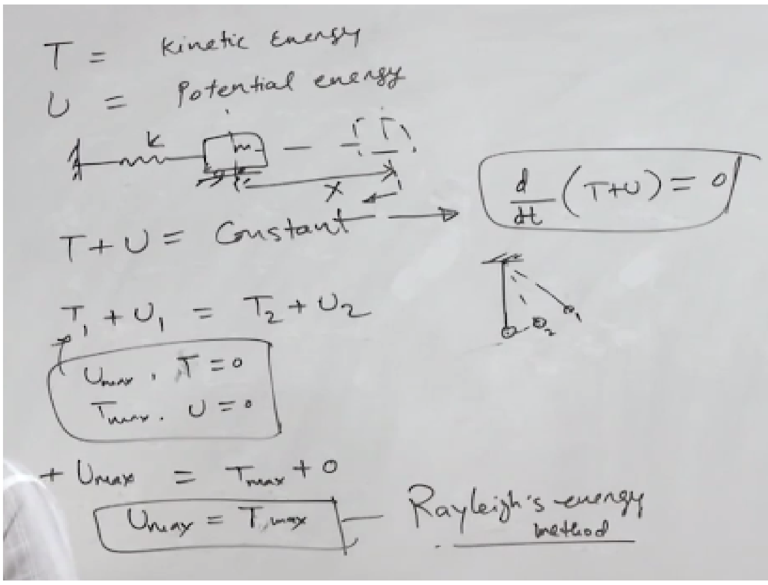

So here we have T is the kinetic energy and U is the potential energy. So this is our system let say this is so this total T + U = constant. Now if we take a case of case such that let us have so this T + U is constant at any instant of time T. Now let us consider two instants of time so one and two. So T1 + U1 = T2 + U2. So we have let us have this is our system. So this is a pendulum and this pendulum is vibrating.

So let us say at any instant of time here T1 here T2 so we have total and this is let say this is one-two. So T1 + U1 = T2 + U2. Now we take this example we see that at this point the potential energy is minimum and kinetic energy is maximum. Similarly, when it is at maximum of its displacement here the kinetic energy is zero because velocity comes zero and potential energy is maximum.

Similarly, in this case when it go to maximum this is equilibrium position it is going to its maximum here X. It stops and then it come back. So here it has only potential energy okay. So we are seeing that when potential energy is maximum kinetic energy is T is zero and when T is maximum its potential energy is zero. So we put these two conditions in this equation so let us take this first case when U max and T equals to zero.

So zero + U max = the second case this when we have T max and + U is zero. So from here we can write that U max = T max. So this is Rayleigh's Energy Method this is called Rayleigh Energy Method. Now in order to get the equation of motion we for any system we find its total at any general instant of time. We find the total kinetic energy then total potential energy.

And we write this equal to constant then we write D by DT we differentiate it T + U = 0 because it is constant term so its derivative with respect to T will be zero. So now we will take one example to find the natural frequency of the system using the energy method. If we take the simple example of spring mass system so let us take the example of spring mass system.

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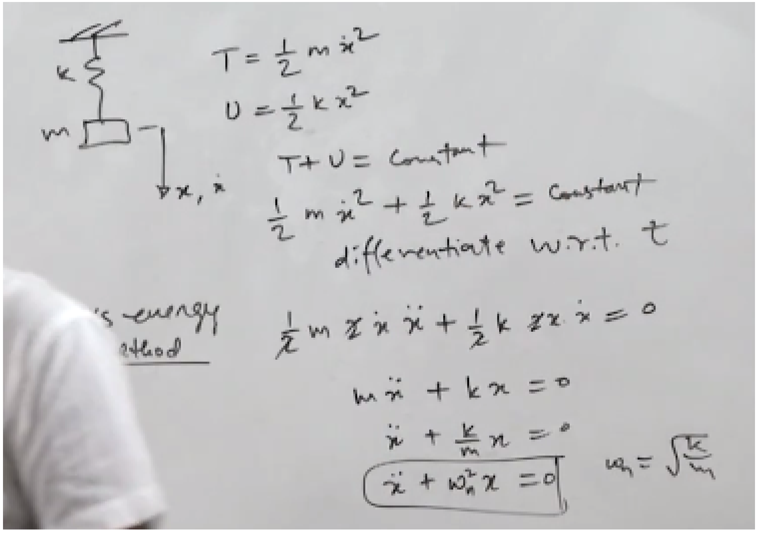

So we have this is M this is K. So we displace here it is X dot. So at this instant the kinetic energy T = half M this is M V square so X dot square and potential energy U = half K X square. So here we can see that this is the potential energy half KX square in the spring stored in the spring at this moment of time and this is the kinetic energy. Now T + U = constant. So T is half MX dot square and U is half KX square this is equal to constant. So here now differentiate with respect to T the time.

So it is half M this is two X dot into X double dot. So this is two X dot and then differentiate X dot so it will be X double dot + half K. Now X square so it is two X into X dot and this is constant so it is zero. So this will be cancelled out so we will get and X dot here so we can leave this term. So will have MX double dot + KX = zero or X double dot + K by M X = zero or X double dot + omega N square X = zero.

So here we get omega N = root K by M. So by using this energy method we reached to the same equation of motion of this spring mass system that we obtained using the Newton's Law of motion.

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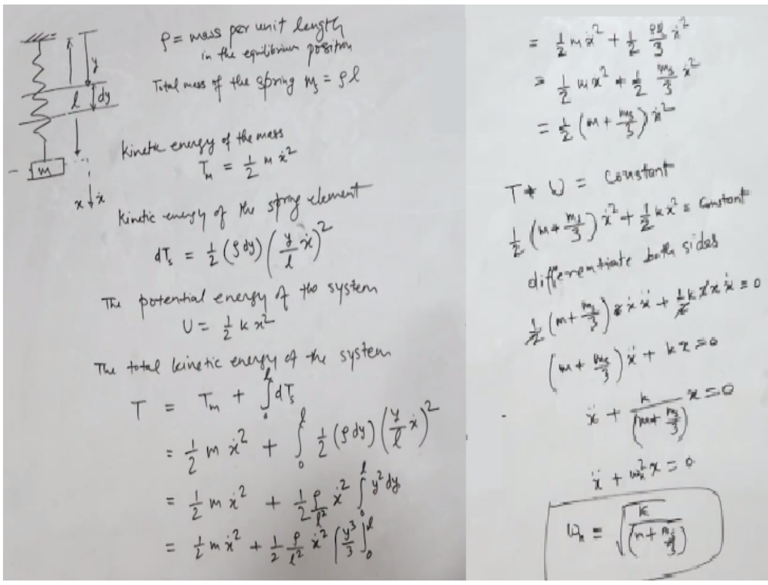

So now we will take one more system so let us take another system. So here we take a spring that is not massless. This spring has mass density that is mass per unit length so let say it has mass per unit length so it has rho = mass per unit length and this is the length L of the spring when it is in the equilibrium position. So this is in the equilibrium or un stretched position equilibrium position.

This is M so the total mass of the spring MS = rho into. L. So it is rho into L. Now we disturb this system so to vibrate so let us say here and here we take a distance Y here element DY so this is element DY. So the potential energy and kinetic energy we have to calculate. So here the kinetic energy of the mass that is we say TM = half m into X dot square. So we assume that here it is X dot X this velocity.

Now we have to obtain the kinetic energy of the spring element so we have taken a spring element so the kinetic energy of the spring element. So DTS = so half M so M of this element is rho into DY. This is M of this element into X the velocity so velocity because here it is zero velocity and at this end it is X dot. So the velocity at this Y from the support will be Y by L into X dot okay. So here this will be the velocity at this point.

So it is Y by L into X dot because here the velocity is X dot here it is zero, so here it is X dot. This is the kinetic energy of the spring element so the potential energy of the system, the potential energy of the system because this spring stretched with X total. So we will have potential energy U = half KX square. Now we have to find the total kinetic energy because the total kinetic energy comprised the kinetic energy of the mass and the kinetic energy of the spring.

So here the total kinetic energy of the system T = kinetic energy of the mass TM + kinetic energy of the spring and we can integrate this from 0 to L because this kinetic energy is only for defined for the elementary length DY and if we want the kinetic energy of the spring we have to differentiate from 0 to L. So we assume here Y = 0 and Y = L. So TM = half MX dot square and here 0 to L DTS.

So DTS we can write here half rho DY Y by LX dot X square. Half MX dot square + half K rho by L and X dot square that can be out because only we have the derivative with respect to Y and 0 to LY square DY. We have written this Y square and here L square. So this is L square, L square will be out half rho by L square into X dot square. Now we will do this integral so half MX dot square + half rho by L square X dot square and this is Y cube by 3 and we have limit 0 to L.

So we can find T = half MX dot square + half rho by L square X dot square and this is Y cube by 3 so if we put the limit so L cube by 3 - 0 Y = 0 so it is L cube by 3.So we can solve it so it is L square cancel out so here it will be only L so we will have half MX dot square + half here rho L by 3 into X dot square. Now rho into L is MS so the total mass of the spring.

So we can write half MX dot square + half into rho l is MS by 3 X dot square. So we can write half M + MS by 3 X dot square. So this is the total kinetic energy of the system that consists of the kinetic energy of the mass and the kinetic energy of the spring. Now we will apply the energy method. So energy method says that T + U = constant for any system. So T + U = constant.

So here T is half M + MS by 3 X dot square + U is half KX square = constant. Now we differentiate both sides with respect to time. So here we will have half M + MS by 3 into here 2X dot into X double dot 2 X dot into X double dot + half K to X into X dot 2 X into X dot = 0. So here 2 cancel out. So here M + MS by 3 X double dot because here is X dot X dot that will be gone + KX = 0.

Now X double dot + K by M + MS by 3 into X = 0. So now we have got the, we can compare it like X double dot + omega N square X = 0. So we can find the natural frequency omega N = root KY M + MS by 3. So this is the final expression for the natural frequency of a spring mass system with spring having a mass with mass density rho. So what we see if we have massless spring so MX = 0.

So that is why we get the natural frequency omega N = root K by M, but if we have some spring having mass MS so the one third of the mass will be accounted to calculate the natural frequency of the system. Now we will take another example of the system to use this energy method to find the natural frequency.

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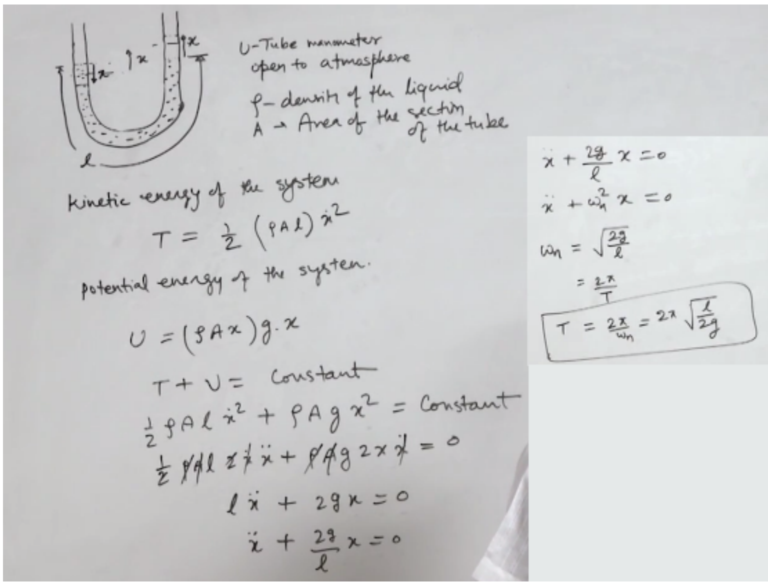

So let us take another example. So here we have U-Tube manometer that is open to atmosphere and there is a liquid column of length L here. Now if we want to disturb this column by pressing it with certain this liquid if we press it with X here. So we are pressing here and then we want to see the free vibration of this liquid column and let us assume that at certain instant of time the liquid column is here X.

And because of the law of this conservation of the mass the mas the same mass will be raised here. If it is X here it will be X here. So this will raise here X. So now we have to apply the energy method to find the time period or natural frequency of oscillations of this liquid column. So here we will have first let us have the kinetic energy so the kinetic energy of the system.

And let say the density of the fluid density of the liquid and A is the area of the section of the tube. So kinetic energy of the system is half into so each point of this column will have the same velocity X dot at that moment. So the total mass half into rho into A into L into X dot square. Half rho A so rho is the density of the liquid and so multiply with the volume that is area into the L so the total mass into X dot square so this is the kinetic energy.

Now the potential energy of the system. So first let us find this potential energy so it is like we have this mass kept here so we will have if this is the CG of this mass so X by 2 and here it is raised so it is raised by X this mass. So that mass is rho times A into X. So rho X this mass is rest against gravity so MG and into X so this is the potential energy. Now we have this kinetic energy and potential energy so this is T. So T + U = constant.

So T is half rho L X dot square and this is rho A G X square = constant. Now we have to differentiate so let us differentiate this with respect to time so half rho AL so this is 2 X double dot + rho AG 2 X dot = 0. So we have so rho cancel out, A cancel out. We have X dot so it is cancelled out. So this is 2 it is cancelled out so X double dot + LX double dot so here is LX double dot + 2 GX = 0.

So here we can write X double dot + 2 G by L X = 0. Now we compare it this X double dot + 2 G by LX = 0. We can compare it with omega N square X = 0. So we find that omega N = under root 2 G by L or omega N = 2 pi by time period T. So T = 2 pi upon omega N so that is 2 pi root L by 2 G. So this is the time period of the oscillation.

So here we see that by using the energy method we have to compute first the kinetic energy of the system. So kinetic energy of the system means because kinetic energy depends on the mass so if there is mass in the system so all that system we have to consider and system means the kinetic energy the sum of the components of this system. So for example in the previous case we had the spring and we had the mas both were having mass.

So we consider the kinetic energy of the both system. In this case we have only the liquid column that is having mass so we got only one component of the kinetic energy. So we have to find the kinetic energy of the system and then the potential energy of the system we do T + U = constant.

We differentiate it and after differentiating we get the equations that lead to us to the equation of motion and from that equation we compare it like X double dot + omega N square X = 0 and we find the natural frequency of the system. Thank you for your attention and see you in the next lecture.