**Introduction to Mechanical Vibration**
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**Lecture – 06**
**Undamped free Vibration**

Welcome to the lecture on Free Vibration of a Single Degree of Freedom Systems. Today we will discuss the Undamped free vibration.

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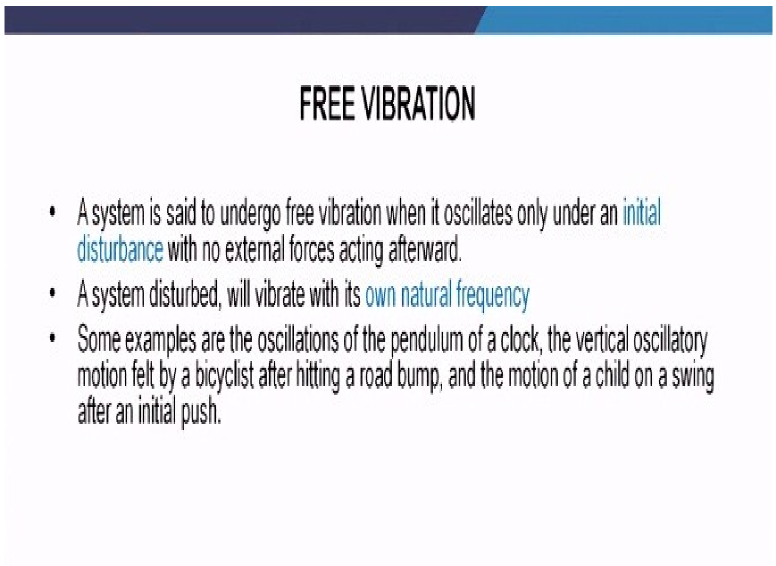


So what is Free Vibration? As we have already defined, so any system that vibrates due to its own elastic properties. And how it happens when we disturb system; we get some initial disturbance and we leave that system, then the system starts vibrating and that is called Free Vibration because there is no any other external force applied on the system. Because, in that condition the system vibrates with its own elastic property.

So it vibrates with its natural frequency. So the frequency of free vibration is defined as the natural frequency. So here are several examples, for example if there is some pendulum we disturb it so we just give a small displacement and we release then it will start vibrating. And that vibration will be free vibration and the frequency of vibration will be the natural frequency. Similarly, when cyclist is going through road.

And there is some bump and suddenly the bump is coming and due to bump there is some disturbance to a cycle and it starts vibrating. So this is an example of free vibration.

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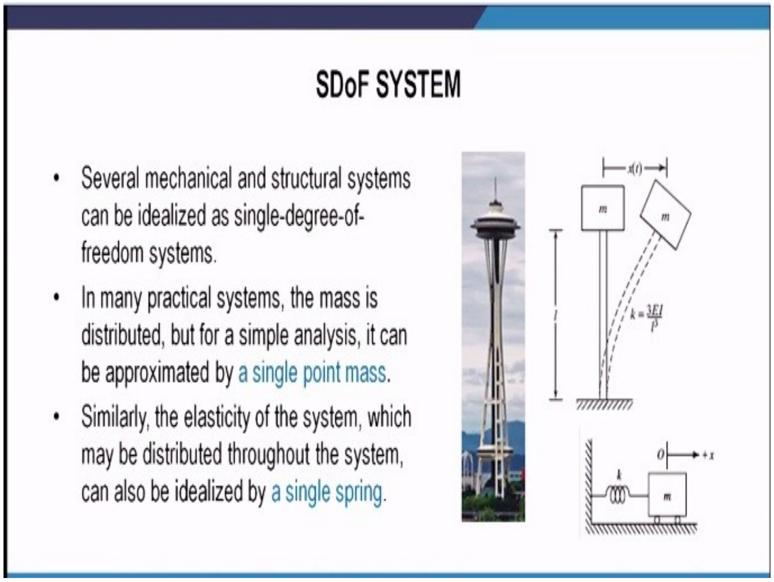


So, because we will discuss the Single Degree of Freedom System the free vibration of single degree of freedom system and we have defined the single – the degree of freedom. So the Single Degree of Freedom System needs only one coordinate to define its motion at any instance of time. And single degree of freedom system consisting of spring and mass is one of the simplest system.

And when this system is disturbed from its initial equilibrium position, okay it will vibrate as a free vibration and with its own natural frequency. A system consisting of only spring and mass okay. It does not consist of any damping element any damper. So the vibration that starts after disturbance that will remain the amplitude will remain constant because there is no any dispassion in system here and so the amplitude does not decay with time.

Several mechanical and the structural systems can be assumed and converted to a single degree of freedom system with certain assumptions. And while we convert to single degree of freedom system because it is easier to solve equations of single degree of freedom system, they are simple system we can study quickly and we can get some quick insight into the system by doing so. For example, here we can see there is some tower and here we can also the-- this is a continuous system but the major mass is consolidated at the peak of the tower.

So we can neglect the mass of this other part and main mass we can concentrate the mass here and the peak and the rest of the column is mainly working as a stiffness element and so we can provide stiffness. We can assume as a spring or stiffness element. And we can convert this system into single mass stiffness system. So here we can see because here at the support it is fixed and we can assume it has a fixed free beam or a cantilever beam.

So here is the mass, and when it is disturbed it will vibrate as a cantilever and derive stiffness of cantilever S 3Ei by L cube. And this is the equivalent representation because again this k we can represent as a spring and mass like this and we can assume it has a single spring mass system. Therefore, why this single degree of freedom system study is quite important.

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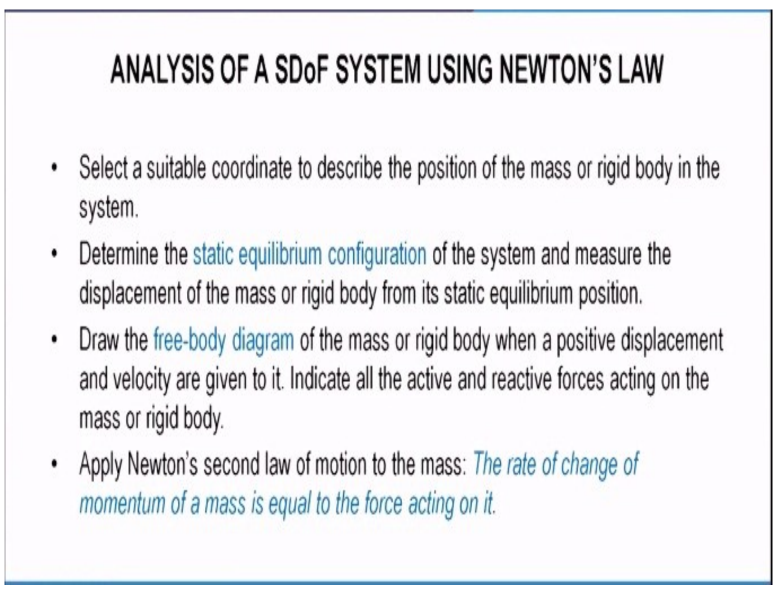


Now there is when we want to study the free vibration of single degree of freedom system there is certain procedure. How we will analyze the system? So the first thing is that we have to define our systems coordinate like this is a single degree of freedom system what is the coordinate of its vibration; whether it is translational; whether it is rotational or angular and so. So we have do define the coordinate.

We have to-- then we have to disturb the system, so it means that we have from equilibrium position we have to displace it by certain displacement giving certain displacement x and in that instant we have to isolate the system. When we isolate the system and we have to show make the free body diagram. It means we have to show all the action, reactions, external forces on the system they are internal forces, external forces and reaction forces.

So you have to show all the forces on the system as well as, so in the, the coordinates direction of the coordinate like x what is its direction of motion that we assume positive. Then on this free-body diagram we will apply the Newton’s law here. There are another methods that also we can apply like D'Alembert's principle and Energy method but here first we will discuss the Newton’s law that say that the rate exchange of momentum of a mass is equal to the applied force. So it means d by dt mv here we can write d by dt.

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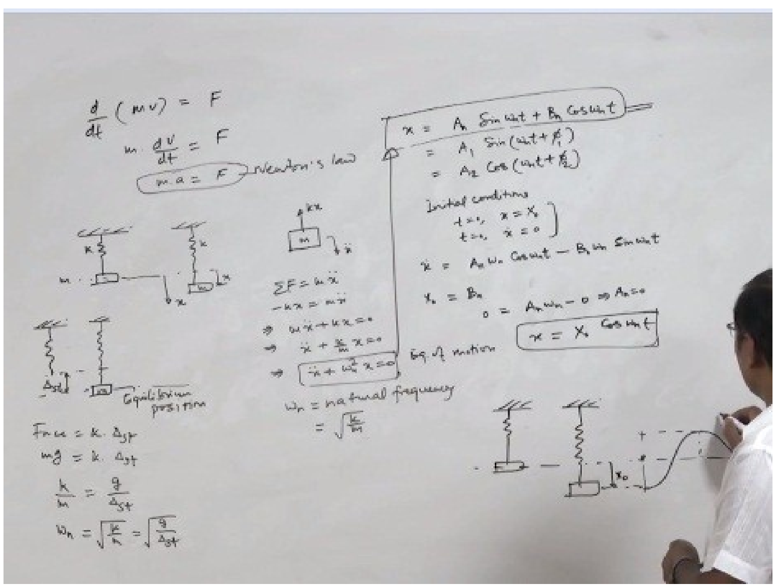


So this is momentum = the applied force. So here we can—if m is constant we have dv by dt = F and dv by dt is acceleration so m into a = F, so this is Newton’s law so that we will use. So let us take one this simple system, so let us say we have this simple system. So this is mass k. Now one thing we must remember that spring-- this is a spring. This is a unstretched spring when the mass is attached to this spring. So this spring is displaced.

Let say this is delta st static due to the mass it is. Now this is the equilibrium position. Because the—now the system will vibrate about this point so it will vibrate. And here we can write, so because force equal to stiffness into displacement, so delta st. Here the force is nothing but the mass so mg = k into delta xt; so here we can find that k by m = g by delta st. And here we know that natural frequency of the system is a root k by m so it is also root g by delta st.

So here what I say-- if I say the equilibrium position means this the equilibrium position about which it will vibrate. So now this is our system and we assume that this is direction x coordinate that the system is going to be disturbed and so it will vibrate. So let say at some instance of time it of its vibration it is here okay, so this is k this is m this is equilibrium so this is x here this is x. Now we have to show the free-body diagram.

So free-body diagram for this mass so this is our mass. So what are the forces that our acting on this. So when we have pull this mass is here so x is -- spring is stretched by x so the spring force k will act like this. Okay. There are no any other forces acting on this when this is direction of the acceleration, so here we will apply sigma F = mx double dot. So here sigma F that is opposite of the x double dot so it will be - kx = mx double dot.

So this means that mx double dot + kx = 0 or x double dot + k by x = 0, this simply as x double dot + omega n square = 0. Here omega n equal to the natural frequency that is equal to root k by n. So here we can see that when we started the free vibration of simple spring mass system and we followed the Newton's law. We reach to this equation of motion.

So this is the equation of motion or differential equation of this motion and from here this-- we convert into this forms so this form that is multiplied that is the coefficient of x that is the factor of x. That tells you the information about the natural frequency. So we are getting natural frequency of the system through the free vibration. Now the solution of this system this kind of differential equation is x = An.

And sin omega nt + Bn cos omega nt or as you know the two are the harmonic motions so they can be represented in terms one harmonic motion. So we can also write like A1 sin omega nt +

phi 1 = A2 cos omega + phi 2, okay. So these are three ways of writing the same equations, so this is a solution of this differential equation. Now An, Bn they are the constants that depends on the initial condition of the system, so what was my initial condition?

So let us assume some initial conditions, so here is initial conditions so let us say that at t = 0, x = x0 and at t = 0, x dot = 0. So here we assume that at t = 0 we displace the system through some distance so we assume that x 0, okay that displacement so initial displacement that we gave. And of course we left the system from the rest so at that time the velocity was 0 so that is the very general initial condition that we have taken.

Now we put the-- this initial condition into the equation this equation. So of course we have also one initial condition for the velocity so we have to get velocity by differentiating this equation so x dot = An into-- so sin omega nt the sin omega nt if we differentiate we will got cos omega nt into omega so omega n omega n into cos omega nt. And if we differentiate cos omega nt we will get - sin nt into omega n so it is - Bn into omega n into sin omega nt - Bn omega n into sin omega nt.

Now we put this values so t = 0 x = x0 so here x0 =-- so if we put t = 0 and this will be 0 and this will be one so it is equal to Bn. So we get x0 = Bn. Now we put in the second equation the second initial condition. So x dot = 0 = – here omega n it is An omega n - this is 0 because t = 0 this is 0 this means that we are An = 0. So we are getting An = 0 and Bn = x0, so we put this equation.

So x = An = 0 so this term will be 0 and we will get Bn so Bn is x0 into cos omega nt. So here we see that in case of free vibration our system is vibrating in a harmonic manner with the cos. So here we have our system and we displace this system. So here we displace as x0 so this is x0. And here you see this is the at zero and this is cosine so it is at cos is here is maximum at zero and this is a cos omega nt; this is a x0.

So the system is vibrating with harmonic manner, so here we can see from this system. And the frequency of vibration is omega n. Now we will take some other examples because here we get that if we want to know the natural frequency of the system we have to induce the free vibration and we provide the equation of motion and bring into the form of this x double dot means escalation plus some coefficient into the displacement equal to zero.

And this term will be equal to omega n square frequency square.

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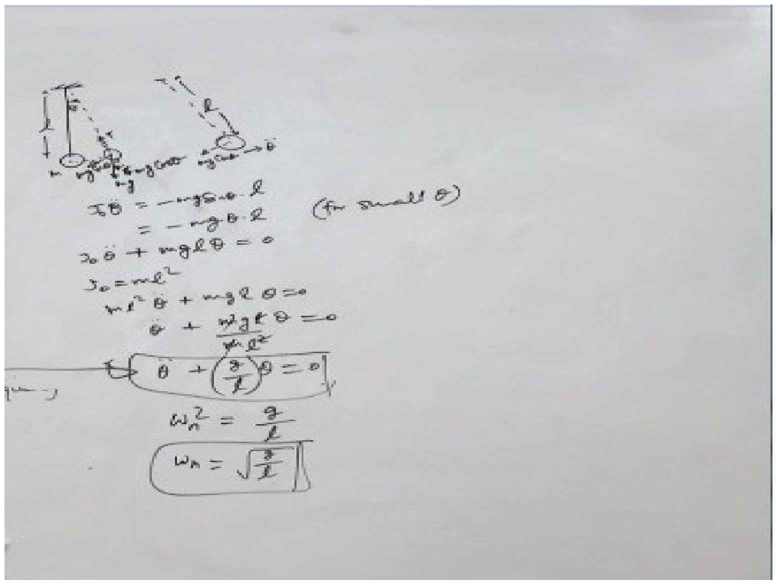


Now we take some other example, so here we have a rod that is mass there is no mass and here is a mass concentrated here with mass m and this is L length. Now we want to know the natural frequency of this system. So we have to disturb the system. So let us disturb the system with angle theta. And this situation we balance the forces, so here is the mg working downward. And this is angle theta so this mg cos theta and this is mg sin theta.

And here this is some force that lets T, so this is going to balance this. But this force is giving here the torque. So for this system we have-- this mg sin theta and this I L this length L. And let us say this is the theta so because theta is this so theta double dot. So j-- because here this is rod and mass so j0 theta double dot = - mg sin theta so this is the force into the torque so it is the L into L. And for this the small theta we can assume sin theta = theta.

So here we can write - mg theta into L for small theta. So here we can write j0 theta double dot + mg L into theta = 0. Now j0 = ml square so we write ml square theta double dot + mg L theta = 0 so theta double dot + mg L theta ml square = 0; so m cancels out L cancel out. So we have theta double dot g by L theta = 0. Now if we compare these two equations.

So we see that this term is omega n square so omega n square by = g by L so omega n = root g by l, so is this the natural frequency of this system that depends on the escalation and the length of this rod.

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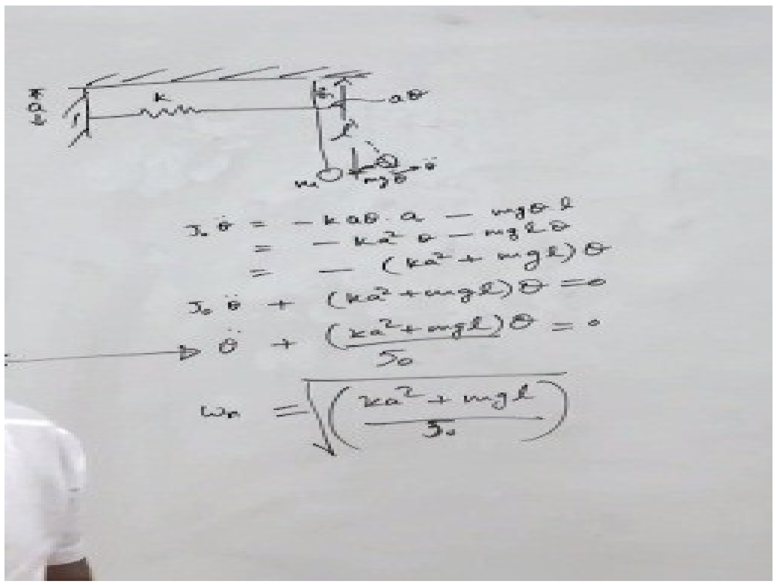


Now we can take another example. So now we have this system. So here is the spring and this is a this is length L again this is a rod and this system that system is here. Now we want to know the natural frequency of this system. So we have to displace this system in this direction. So we-- will have in that situation we will have after we displace it this is the situation. And because here we displace with theta so this is a, and this is theta so it is a theta.

And here we have theta double dot again the similarly the forces that are working here mg into sin theta so here mg theta that we assumed. So, here mg for a small theta mg theta. So now we can apply here so the forces so j into theta double dot = - k so this is A theta and this k so ka theta will be the force acting here so - ka theta into a because we have to take the moment about this.

Again here we have - mg theta into L mg theta into L. So this is the force acting so opposite to this so that is why there is the - sign. Now here we can write - ka square theta - mgl theta. So = -

ka square + mgl theta. So here we can write j0 theta double dot + ka square + mgl theta = 0. So here we can theta double dot + ka square + mgl theta upon j0 = 0.

Now again if we compare this equation here we will have omega n square = ka square + mgl upon j0. And we can get omega n by taking under root like this. So we can we got this natural frequency for this system. Now we can discuss about the vibration of the torsional system. So torsional systems here we have the torsional system we can discuss. So let us take an example of torsional system, single degree of torsional system.

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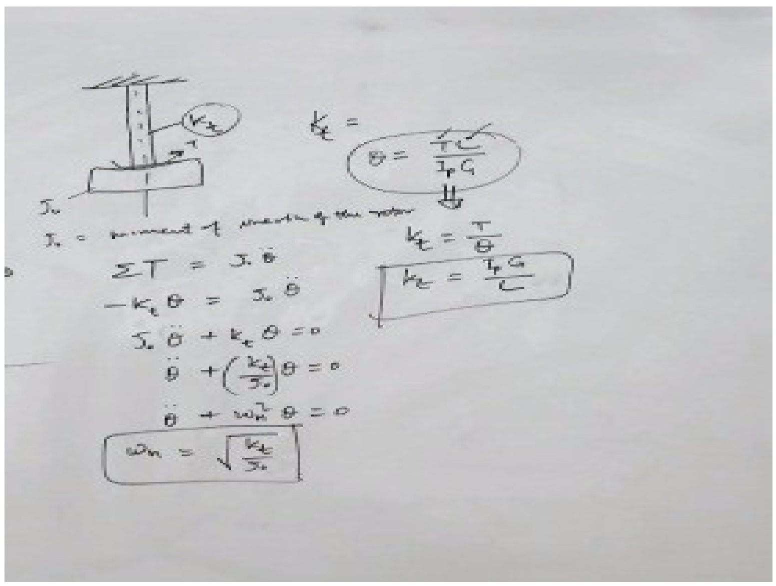

So similar to the spring-mass system that is the simplest system having the stiffness and mass, we can have a torsional system having let us say this is some shift and this is a rotor. So if we are giving some torque here T initial disturbance torque and we give some initial disturbance in terms of torque and we release it the system will start vibrating in the-- about its axis. And in this condition we have to write the equation of motion.

So let us say this is the j0 that is j0 is the-- this is moment of inertia of the rotor, okay. So this Newton's law in case of the-- this system will be sigma torque = j0 into the angular. So let us say we give some initial theta displacement so sigma torque = j0 theta double dot. So this system working as a stiffness so it has the torsional stiffness kt. So similar to the linear stiffness.

We have the torsional stiffness kt and this is the resistance of the system about rotation about some axis so kt. So if we are giving some initial rotation, okay. So here - kt into theta force will be acting opposite to this rotation. And here we can write j0 theta double dot + kt into theta = 0. And we can write theta double dot + kt by say zero theta = 0. Now again if we compare with this expression so theta double dot + omega n square theta = 0. So we get omega n that is the natural frequency.

So omega n = kt by j0. So we can get the natural frequency of the torsional system. Now what is the kt the stiffness, so we know that theta = TL by-- so here we know for soft member the angle of rotation theta = torque into the length of the soft into the polar moment of area into the rigidity. So from here we can write that kt = torque upon theta; similar to the force upon displacement.

Here the displacement is angular displacement and forces is torque so we can write torque upon theta so this IP into g upon l. So here gain this-- this is the formula for the torsional stiffness of the-- torsional stiffness of the member torsional member. So here we will stop and thank you for attending this lecture, and see you in the next lecture.