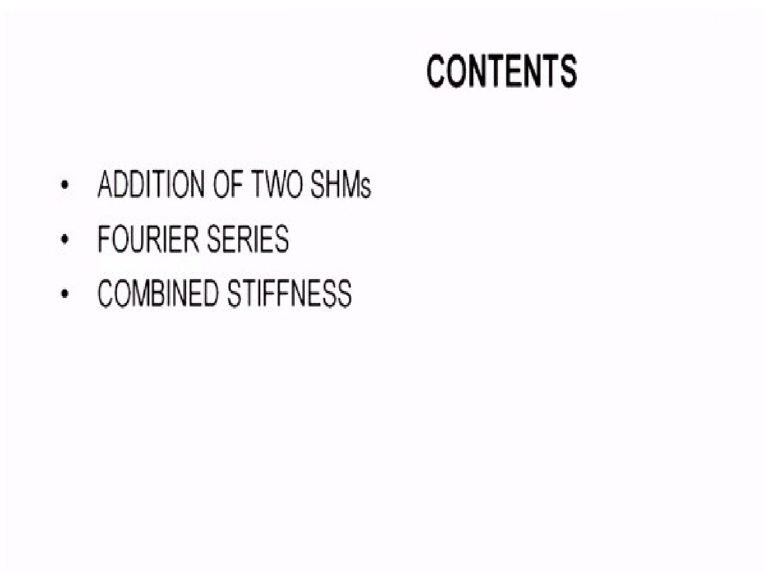
Introduction to Mechanical Vibration
Prof. Anil Kumar
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Lecture – 05
Numerical Problems

Welcome to lecture on fundamental of vibrations. So far we have discussed some basic terminology regarding vibrations. We discussed the simple harmonic motion, periodic motion, addition of two simple harmonic motions, then we discussed the Fourier series; we discussed the basic elements of a mechanical vibrating system particular the spring and we derived some formula to get the equivalent stiffness of the springs.

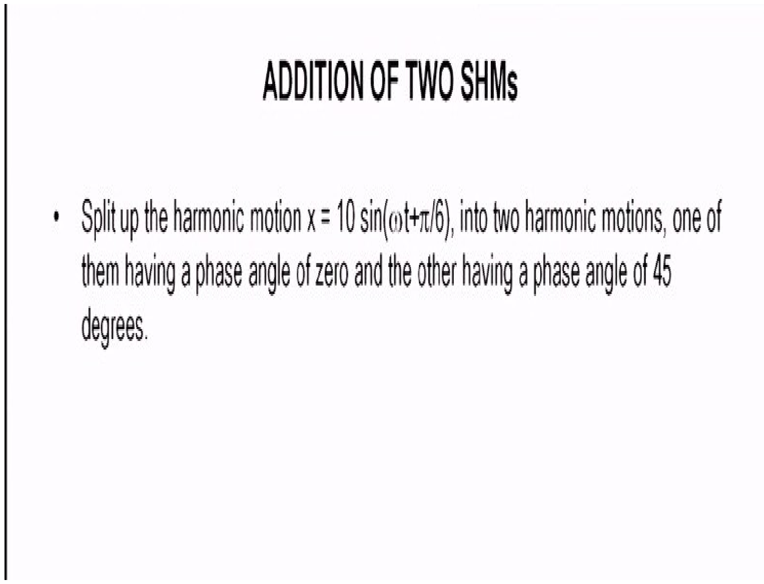
So today we will discuss some numerical problems based on that theory that we discussed in the last lectures.

(Refer Slide Time: 01:28)



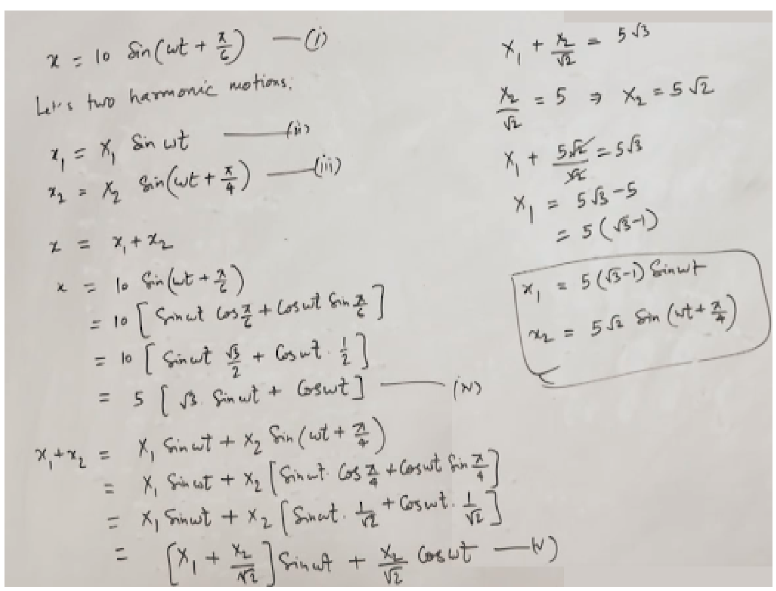
So here we have the problems related to the addition of simple harmonic motion, Fourier series and equivalent stiffness of the combined springs. So let us take the first problem.

(Refer Slide Time: 01:39)



So here it is said that we have to split the harmonic motion that is represented by x = 10 sin omega t + pi by 6 into two harmonic motions. One of them having a phase angle of zero and the other having a phase angle of 45 degrees. So if you remember we derived the formula for the addition of two harmonic motions. Here we have the reverse problem. So we are given the harmonic motion and we have to split this into two motions. So let us solve this problem.

(Refer Slide Time: 02:27)



So this is our harmonic motion and – so two harmonic motions. So let us two motions. So the first one it is saying has phase angle of 0. So because we know that if we have this omega this frequency of the main harmonic motion so the components will have the same frequency. So we will have the first one let us assume x1 = x1 sin omega t and because it has the zero phase angle so we do not need to write the zero here. So it is simple x1 sin omega t.

The second one is x2 = x2 sin omega t. Now here the phase angle is given 45 degrees. So omega t +; so it is 45 degrees and we put in radians. So it is pi/4. So now in this so this is equation one; this is equation two and this is equation three. Okay. So we have x = x1 + x2 because then only we can split this motion x * 2 motions x1 and x2 two. So here we have x = 10 sin omega t + pi by six. Let us open this so 10 sin a + b sin a + cos b + cos a sin b.

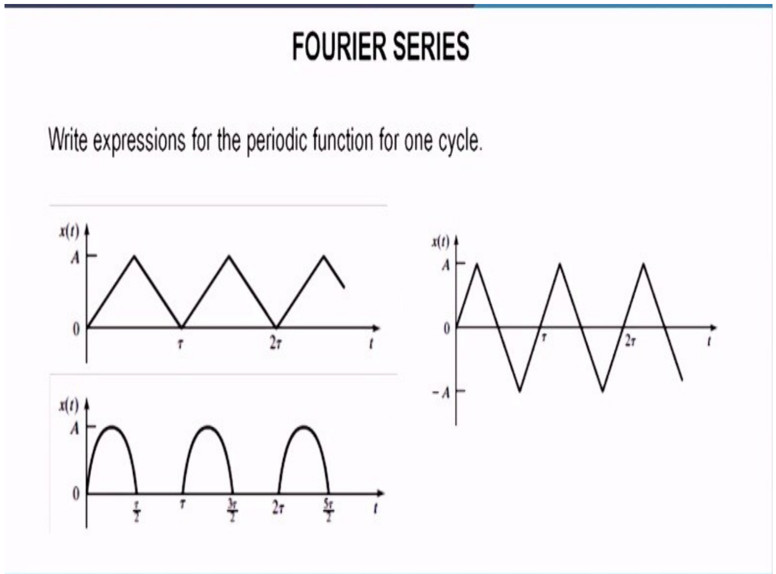
So it is sin a cos b + cos a sin b, so = 10 sin omega t cos pi/6 so it is cos 30, cos 30 is root 3/2 + cos omega t into sin 30 is 1 / 2. So we will have here 5 root 3 into sin omega t + cos omega t. Now we fine the sum of the 2. So x1 + x2 = x1 sin omega t + x2 sin omega t + pi/4 this is equal to x1 sin omega t + x2 sin omega t; so again this we can open sin a + sin a cos b + cos a sin b.

So it is equal to x1 sin omega t + x2 here sin omega t into cos pi/4 is 1 by root 2 + cos omega t to sin pi/4 is again 1 by root 2 so this is equal to x1 + x2 by root 2 sin omega t + x2 by root 2 cos omega t. So x1 + x2 by root 2 sin omega t, so this is equation let say four and this is equation five. So because we here write x = x1 + x2 so we can equate equation four and equation five means the coefficient of sin omega and cos omega t should be same.

So we can write here x1 + x2 by root 2 = 5 root 3 and x2 by root 2 = 5. So this from here we can directly get x2 = 5 root 2 and if put this x2 in this equation so we can write x1 + so x is equal to 5 root 2 by root 2 = 5 root 3 so x1 = 5 root 3 so this will be cancelled at -5; so it is 5 root 3-. So we got x1 we got x2 so we can know this from these two equations we can write x1 = to 5 root 3-1sin omega t and x2 = 5 root 2 sin omega t + pi/4.

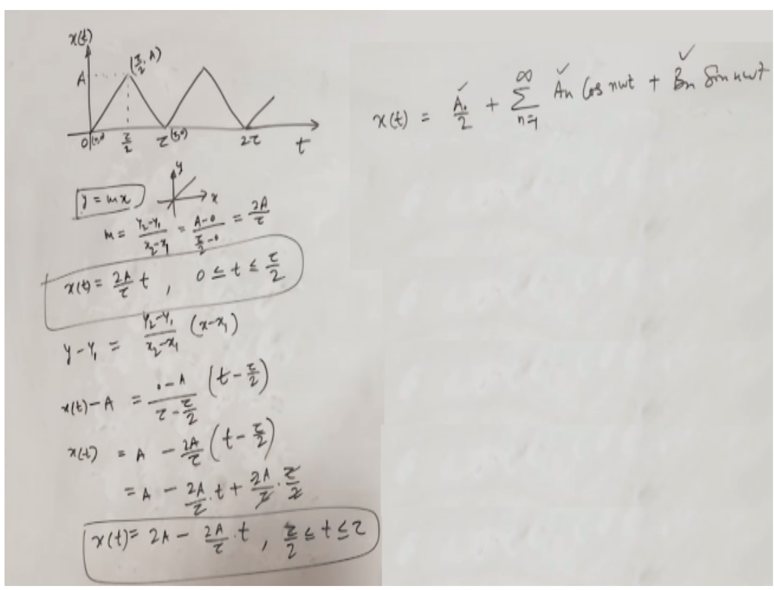
So, these are the two simple harmonic motions because they are sin and we they have been splitted. So the equation one is splitted into equation two and three. So this is the answer of this question.

(Refer Slide Time: 09:19)



So now we go to the next problem. So here we have – we have discussed the Fourier series that any periodic function can be broken or can be represented in terms of the harmonic functions. They are sin and cosine functions. So the first step is to write proper function of a periodic function. So here we have different type of periodic functions. So first have to know how to write this period functions. So we will discuss for example the problem number one.

(Refer Slide Time: 10:21)



So here we have this problem. Here x(t), t here t0 tau, 2 tau and this is A. So let us-- from the figure it is clear that this symmetric so this is tau by 2. Now here this is the one cycle because here it is a time period tau and this same cycle is repeated after equal interval. So we have to define the period function in one cycle. So we see that here there are two lines two different lines; so we have to define the equation of lines in this range.

So for this line we have y = mx the equation of line you know y = mx if we have this x axis this y axis, okay. And here we have some line passing through the origin so the equation of this line is y = mx for m is this slope and m can be obtained by y2 - y1 upon x2 – x1. So in this case y2—y2this point is tau by 2 A and this is a 0,0. So y2 is a – y1 is 2; x2 is tau by 2 – x1 is 0. So this 2A by tau. No y = mx so y = 2A by tau.

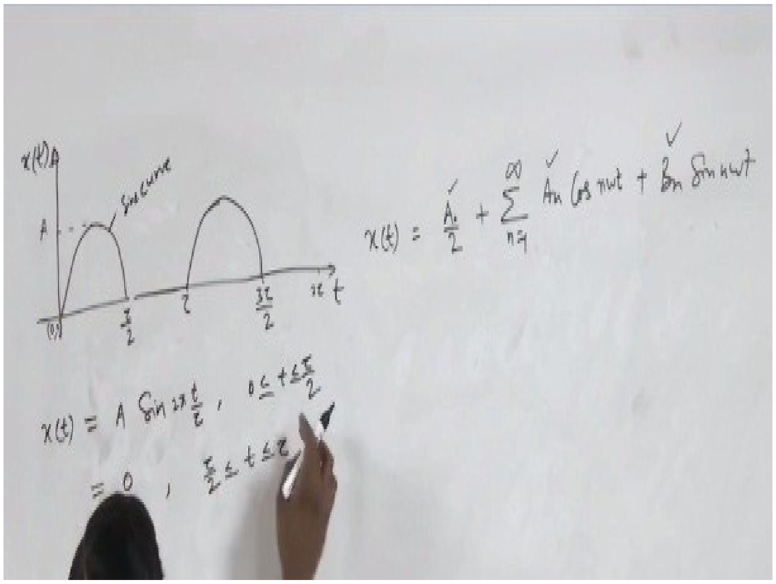
So here y is x(t) because here in --and the y axis we have the function x(t) and m is 2A by tau and x is t so this is-- so this is the equation of this line and this is valid for 0 <= t <= tau by 2. So for the half range half cycle we have defined this periodic function. Now we have to define this function so this line-- this line equation we have to write in the similar way but here we can use the because this line is not passing through, so we can use the another equation that is y – y1 = y2 – y1 upon x2 - x1 into x – x1, so this equation of line we can use.

So here y is x(t) minus; so again here we have a tau and 0. So here y1 is—x1 y1 is this x2 y2 is this. So y is x(t) – y1, y1 is A = = y2 is 0 minus y1 is A upon x2 is tau minus x1—x1 is tau by 2 and x here again x is t because we have time xt minus x1 – x1 is tau by 2. So x(t) = A, this is A and here it is minus A by tau minus tau by 2 is tau by 2 so it is 2A by tau and t minus tau by 2 so = A -2A tau into t + 2A upon tau into tau by 2, so here we have cancelled out.

Now it is equal to 2A – 2A by tau into t. So we can check whether this equation is correct. So we put these two points this would on this line. So if we put t = tau by 2 so t = tau by 2 so we will have-- it is A so 2A – 2A tau here tau by 2 so tau, tau cancelled it is a -A so 2A – A = A so we are getting A. Again for t = tau it is 2A – 2A = 0. So this is valid for 0 for tau by 2 <= t <= tau.

So here we have defined-- this is our periodic function defined for one cycle. Now if you want the Fourier to represent this into Fourier series we can write this is A0 by 2 + sigma n=1 to infinite An cos t + Bn sin and we know that how to get A0 An and Bn; we can know by using this x(t) for this cycles. Then we take another function.

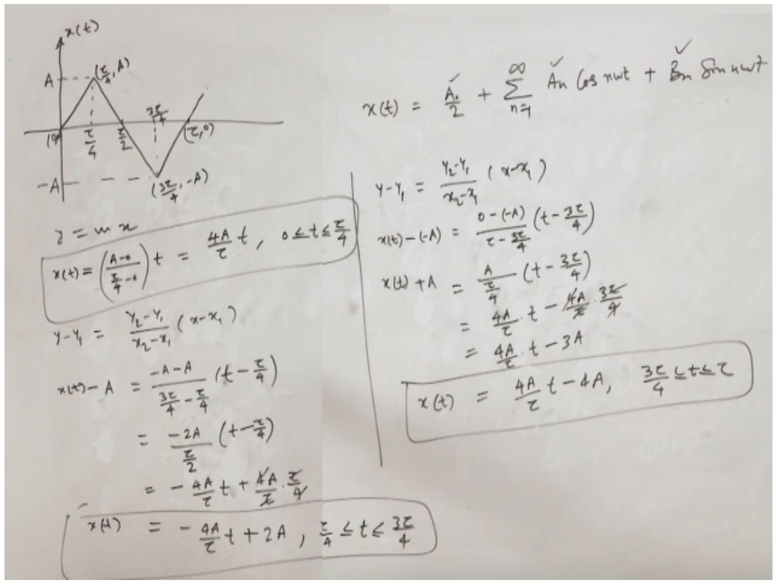
(Refer Slide Time: 16:48)



So the second function here is-- so this is the another periodic function. So we see that this function is repeating itself after this tau so we and half cycle let us it seems to be sin curve; so in the half cycle it is sinusoidal in the other half cycle it is zero. So we can write this function so let us define it x(t) = because this is purely sin curve so A sin 2pi t upon tau. So it is 2pi f t or omega t so omega is 2pi by tau.

And this is valid for 0 <= t <= tau by 2. Then it is 0 for other cycle like tau by 2 <= t <= tau. So it is sin curve in the half cycle till tau by 2 and then again it is zero for other half cycle. So this is defined and we can use this in this equation and we can find the coefficient and we can represent this periodic function into harmonic function. Now we have another periodic function.

(Refer Slide Time: 19:09)



So of course this cycle is repeating after this tau. So we have here tau by 2 and let us assume this is tau by 4 four and this is 3 tau by 4. Now we have this equation is simple y = mx so here it is x(t) so here x(t) = m so m as I said = this is tau by 4 and 8 and this is 0, 0. So here we can write y2 - y1 so A-0 upon x2 – x1 tau by 4 - minus 0 into x is t so this is equal to 4A by tau t and here it is 0 <= t; <= tau by 4.

So it for quarter cycle it is defined now we have to define this next part. So this is a straight line that is from tau 4 to from 3 tau by 4. So this straight line will represent the periodic function in this range. So from this we have to write so here we need to know this point here 3 tau by 4 and this is –A. So first we find this like y - y1 = y2 – y1 upon x2 – x1, x – x1. So here y is x(t) – y1 so y1 is here A and y2 is –A and –y1 A by x2 is 2 tau by 4 – x1 that is tau by 4 and x is t and x1 is tau by 4; x1 is tau by 4. So this is tau by 4.

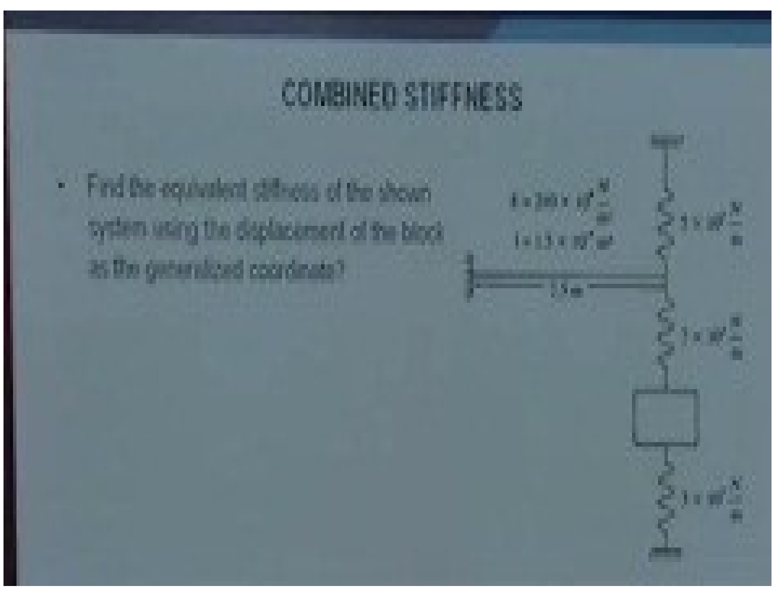
Now we can write it. So here it is –A – A that is -2A; and this is 3 tau by 4 minus tau by 4 so it is 2 tau by 4 and tau by 2 and here t minus tau by 4 so = -4A by tau t; and minus, minus +. Here we have 4A by tau because this is 4A by tau into tau by 4 so 4 will cancel out tau will get so and so. Here we have x(t) = -4A by tau t + this is A and this is 2A. so this is the equation of this lines so we can check if put tau A by 4 so t = tau by 4.

So it is A-- -A + 2A so it is A. So this is valid in the range 0 less than t-- sorry not 0 the tau by 4 <= t <= 3 tau by 4. Similarly, we can find the-- we can find this equation, equation for this line. And equation for this line is we can write because here it is tau and 0 so we can similarly we can write; so here y2 so y – y1 = y2 – y1 upon x2 x1; x – x1 so here y is x(t) – y1 is –A = y2 is –A minus y2 is 0; here 0 – y1.

So it is –A upon x2 is tau minus x1 3 tau by 4; t - x1 is 3 tau by 4. So x(t) + A = A upon-- so tau - 3 tau by 5 it is is tau by 4; t – 3 tau by 4 so = 4A by tau into t – 4A by tau into 3 tau by 4 so it is 3A; so here x(t) = 4A by tau t – 4A, so this is valid for 3 tau by 4 <= t <= tau. So we have this expression that is valid for quarter cycle then it is valid for tau by 4 to 3 tau by 4 and this is valid for 3 tau by 4 to tau.

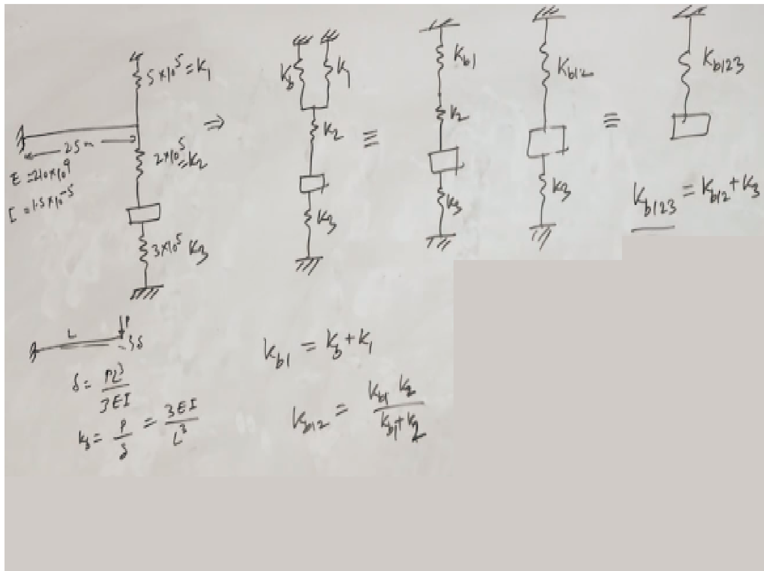
And here we can see that also this slope is positive of this line and it is cutting at negative axis to the x(t). So we can check also this. So here we understand that how to express the periodic function in one cycle because this is going to be used in Fourier series to represent this into harmonic functions. And again this function we can use here and we can find the coefficient of the harmonic series.

(Refer Slide Time: 26:47)



So the next problem is about the combined stiffness as we discussed that if there are springs in series springs in parallel then how to find the equivalent stiffness; so we will discuss this problem.

(Refer Slide Time: 27:14)



Okay. So this is our system and in this system we have beam here it is a beam fixed free beam, and at the free end here this is springs connected and the mass connected. Now we assume that the mass will move in axis so we have to find the stiffness. So for fixed free beam we know that if we have some force P here and this is deflection; this is the length EI so delta = PL cube EI upon 3EI. So he stiffness k = P by delta = 3EI by L cube.

So it means this beam is can be represented equivalent to a spring means we can if we replace this with the spring having stiffness of k = 3EI L cube. Now we can calculate this k so let us say this is Kb so we can make this system equivalent to like here we have this is Kb and let say this is K1 this is K2 this is K3 so Kb, K1 and this is K2 K3. Again we can represent this like-- so these are in parallel so the parallel stiffness. So Kb, K1 = Kb + K1.

So the equivalent stiffness of this will be one spring Kb1 then other spring that is it is K2 and this is mass m and this is K3. Now these are in series so the equivalent of these so this is K3 let us say this is Kb12 so Kb12 = Kb into K1 upon Kb + K1. No, Kb1, K2 – Kb1 + K2. So this is equivalent Stiffness Kb12 is Kb1 into Kb upon Kb1 + K2. Now again here so these are in parallel.

Because they have the fix support and other end connected to the mass so these springs are parallel. So we can again represent this so this is Kb123 so Kb123 = Kb12 + K3 okay. So now we can find finally the equivalent stiffness of this springs, okay. So, thank you for attending this lecture and see you in the next lecture.