Introduction to Mechanical Vibration

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Lecture - 40
Static and Dynamic Coupling

So welcome to the lecture on multi-degree of freedom system. So today we will discuss the concept of Static and dynamic coupling. So as we know that we can have the - we have defined the degree of freedom so n degree of freedom system if it is a multi-degree of freedom system, so it needs an independent coordinate to represent its configuration at any instant.

Generally, we take these coordinates from the equilibrium position, for example we take from center of mass we can select any other set of degree of freedom.

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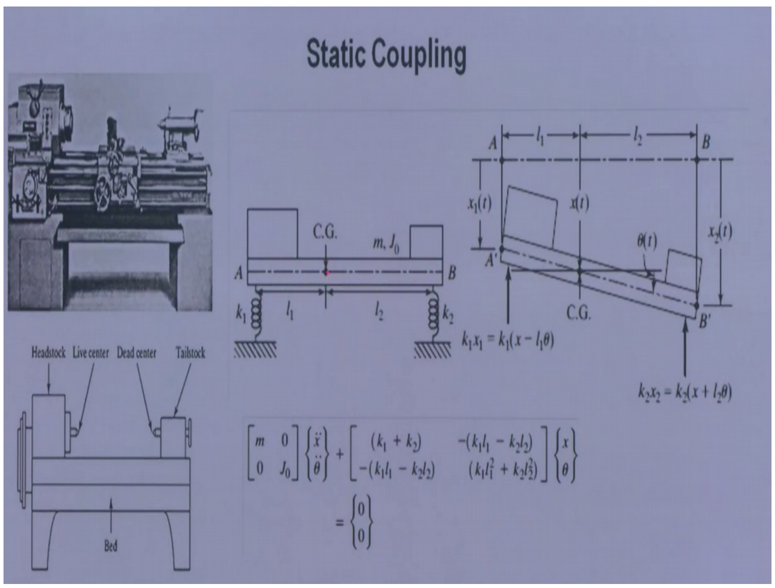

For example, if we have there is a lathe machine and there is a model of this machine, so this machine it has Headstock, Tailstock there is a Bed and this column so we can assume a model of this system and so here there is the elastic stiffness of these columns there is this - this bed okay and we have the center of mass or center of gravity of the system here, so we can because the motion of the system can be in linear as well as in angular directions.

So we can select a degree of freedom passing through center of mass so we can take one the motion of if here x t that shows the motion of the center of mass and the angle theta the rotation angular motion of this system, we can take another set of degrees of freedom that is x1 t and x2 t these are another set of the degree of freedom that we can that can represent the system's configuration at any instant.

Also it is possible that we select a degree of freedom that is not on CG but some other point and from there we can get the configuration in terms of these this point, so we can assume some point and its motion in linear and angular motion so we can take that set of coordinates. So all these - these sets they are known as generalized coordinates. Because these coordinates are it depends that they are our choice they are not some property of a system.

They are our selections we select these coordinates and we so for example if we select this co-ordinate so we have here the center of gravity and its motion x t and then theta t, we take the angle with respect to this axis so what happens.

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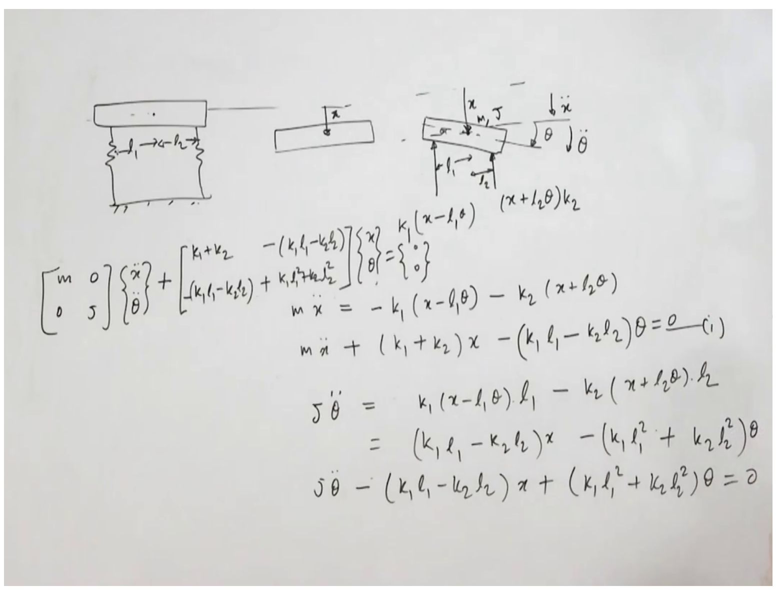

So it is a system like this so we have this system and this system is moving here, here with x t and again it is rotating here with theta t, so this is theta and this is x, now if we have this springs k1 and k2 and this distances here this is l1, l2 here so the forces acting at this point or this because the total deflection of this spring is x - l1 theta, because this is theta and here this is theta, so x - l1 theta and into k1 so that is the force that is acting on this.

Now on this - this spring so the net displacement of this spring is so it is x + this that is l2 theta so it is x + l2 theta and multiplied by k2, so that is the force on this system and the mass of this system is m and the moment of inertia is J, so we assume that the mass of this system is m and the moment of inertia J at the, so center of mass, so now we can write the equation of motion of this system so m x double dot equal to forces.

So these are the, so this is x double dot and that is - so this is opposite so it is - k1 x - 1 1 theta and this is again upon it, so - k2 x + l2 theta, so we can rewrite this equation m x double dot + k1 + k2 x - k1 l1 and this is + k2 l2 into theta, so this is equation number one, similarly we for the another degree of freedom that is the rotation or angular motion, so we take, so we have moment of inertia J and theta double dot.

So now we have we take moment about this point, so we have the this is the force and that distance l1 so this is distance l1 and this is l2, so this is giving a moment about this point, okay, and this theta dot double dot is positive as a clockwise and this is a moment in the same direction so it is k1 x - 1 1 theta into l1 and this is in the opposite so it is - k2 x + l2 theta into l2. Now we can rewrite so it is k1 l1 - k2 l2 x and - k1 l1 square, so it is - k1 l1 square theta.

And this is - k2 l2 square theta and if we take this it is like this, so J theta double dot = J theta - k1 l1 - k2 l2 x + k1 l1 square + k2 l2 square theta = 0, so this is equation number two. Now these equations one and two we can assemble in form of a matrix. So if these equations one and two we assemble so it is m 0 0 J and this is x double dot theta double dot and that is equal to, + we have another matrix so k1 + k2 and - k1 l1 + k2 l2 so it is + k1 l1 theta.

And this is -., so it is - and it is - and this is - k1 l1 - k2 l2 and this is - k1 l1 - k2 l2 and + k1 l1 square + k2 l2 square this is x and theta that is = 0 and 0. So we have arranged so we have this equation, first equation so k1 + k2 x so m x double dot and this is 0, so + k1 + k2 x - k1 l1 - k2 l2 theta = 0 = 0 and here J theta double dot - k1 l1 - k2 l2 x + k1 l1 square + k2 l2 square theta = 0.

So now we can have only the matrix form. So now we see that from this expression this matrix expression.

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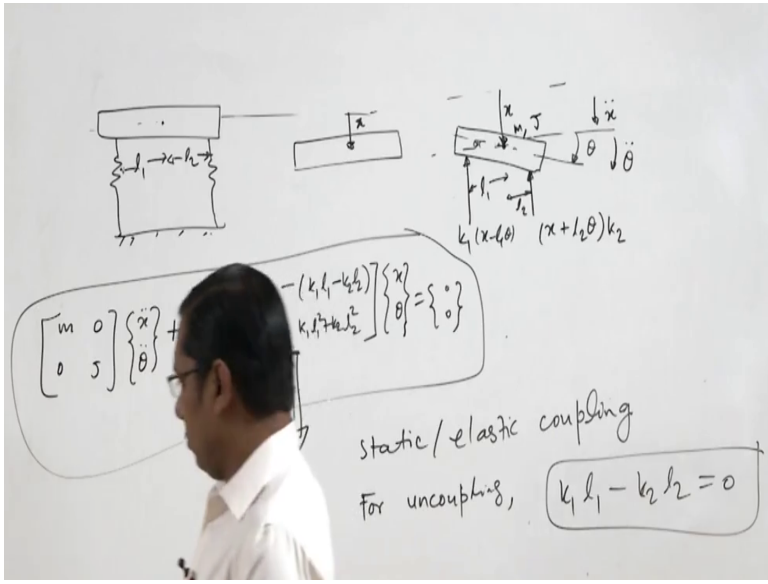

So it is - so this is the matrix expression and from this matrix expression we see that in this equation if we write when the equation in the first equation that is the linear motion degree of freedom we have also this theta term and therefore the system is coupled and similarly, in the angular motion there is the x because - k1 l1 - k2 l2 x, so this is however there is no x double dot in x double dot there is no theta double dot, okay.

But here x and theta they are, there is so when there is such a situation so we called it static or elastic coupling - elastic coupling - coupling, but it is not this system is only static coupling not the dynamic coupling there is no any coupling of the acceleration terms however if you want to uncouple this system we have to make these terms zero, because if it is zero the theta component will be zero in equation one and this is zero then x component will be zero in the theta equation.

So therefore for uncoupling k1 l1 - k2 l2 = 0 this is the condition. Now we take the another example I mean another set of degree of the coordinates and another set of degree of freedoms or coordinates - general coordinates.

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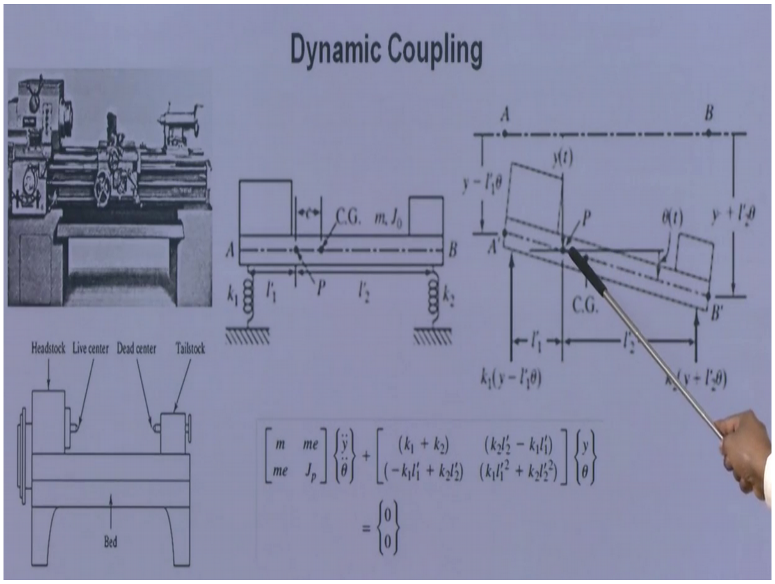

So here we can see we have now here is a point P and we are, we have selected the y t that is the linear motion of this point and angular motion theta as a generalized co-ordinate and now we will study this system.

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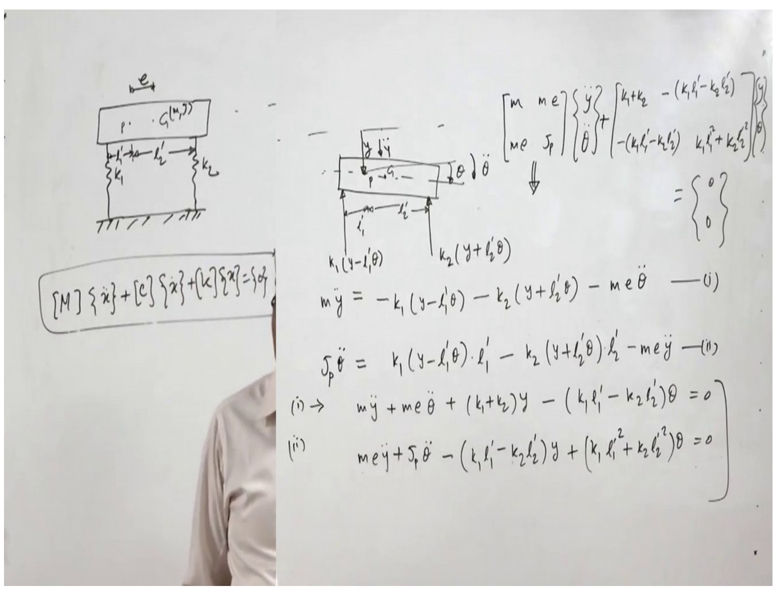

So now we have a system like this so the same system here is G and we take a point P that is at e distance here the distance is e and here we have this springs and here we have this spring that is k1 and k2. now we take this distance here this is l1 dash and this is l2 dash, so from point P to this spring okay, there is l1 dash and this is the l2 dash, now again we have to, we have displaced this system so linearly as well as angular motion.

So we have here y t no here we have P and this P so we have P this is G and here is M and J the mass M and J so mass M is center this is center of mass and J is moment of inertia about this point, now we have this y and this is theta - theta and here we have this is spring forces so now we have this points l1 dash and this is l2 dash, so we can write now we should understand that when we take it here.

Now, we write equation of motion so m y double dot =, so here it is k - k1 y - l1 dash in to theta and here it is k2 y + l2 dash into theta, so because we are here y - because it is y - and this is y + l2 dash theta, so m y double dot so exalation here is this is y double dot and this is theta double dot, so m y double dot =, so this is opposite - k1 y - l1 dash theta - k2 y + l2 dash theta. However, there is theta double dot.

So there is the inertia force that is m times e theta double dot, so it is my and inertia of force is opposite so - me theta double dot, so that is equation number one, similarly for the angular degree of freedom so JP, JP is - JP is the moment of inertia about the point P, okay, so JP in to theta =, so now about this point theta double dot is in clockwise so this is giving clockwise, so it is k1 y - 1 1 dash theta into 1 1 dash.

However, it is giving opposite so - is k2 y + l2 dash theta into l2 dash again there will be a inertia moment due to the degree of freedom y double dot, so that moment will be m y double dot the inertia force and into e, because this distance is, the distance between these two points e, so the moment is - me y double dot, because m y double dot in to e, now we can the equation two. So equation one we can write again as m y double dot + me theta double dot.

Then y terms, so we have k1 y and k2 y so it is k1 + k2 y and here k1 l1 dash theta and this is k2 - k2 l2 dash theta, so it is - k1 l1 dash - k2 l2 dash theta = 0. similarly, equation 2 can be written as me y double dot + JP theta double dot then y term, so it is - k1 l1 dash and here k2 l2 dash and that is + so it is - k2 l2 dash y and then theta. So theta term is here k1 l1 dash square so it is + k1 l1 dash square and this is k2 l2 dash square with - so + k2 l2 dash square and it is theta = 0.

So now again we can write these two equations in matrix form, so we can write them as m, me, me, JP, y double dot theta double dot = - + k1 + k2 - k1 l1 dash - k2 l2 dash - k1 l1 dash - k2 l2 dash and here k1 l1 dash square + k2 l2 dash square and this is y theta and equal to 0. So we have written the equation in form of a matrix, now we see that now for this matrix they there is no that is not diagonal.

So there is the of diagonal terms are non-zero and we can see the effect in the equation that the exalation terms are coming in the equation each equation so here in equation of y double - y motion there is theta double dot term and in theta double dot there is y double dot, so this is called coupling mass coupling or that is also dynamic coupling. And this is as we have already discussed this is the static coupling.

So here we have static and dynamic both coupling static coupling from here due to the stiffness matrix and the dynamic coupling due to, here the mass matrix. Now if you take a general system that is having the mass damping and stiffness matrix, so if we have system like Mx double dot + Cx dot + kx = 0, so this is a system. So if the stiffness matrix is not diagonal then the system has elastic or static coupling.

However, if the damping matrix is not diagonal it has a damping or velocity coupling, if the mass matrix is not diagonal the system has mass or inertia coupling. These two couplings due to damping and mass matrix that is called the dynamic coupling and this is the static coupling. So here we discussed the static and dynamic coupling. So we stop our lectures and I wish you all the best for this course and the examinations, thank you.