Introduction to Mechanical Vibration
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Lecture - 04
Vibration Analysis Procedure

Welcome to the lecture on Vibration analysis procedure. So, what is Vibration analysis procedure? Here Vibration analysis procedure means we are going to do the mathematical, discuss the mathematical procedure. Here, the content is that we have a physical system

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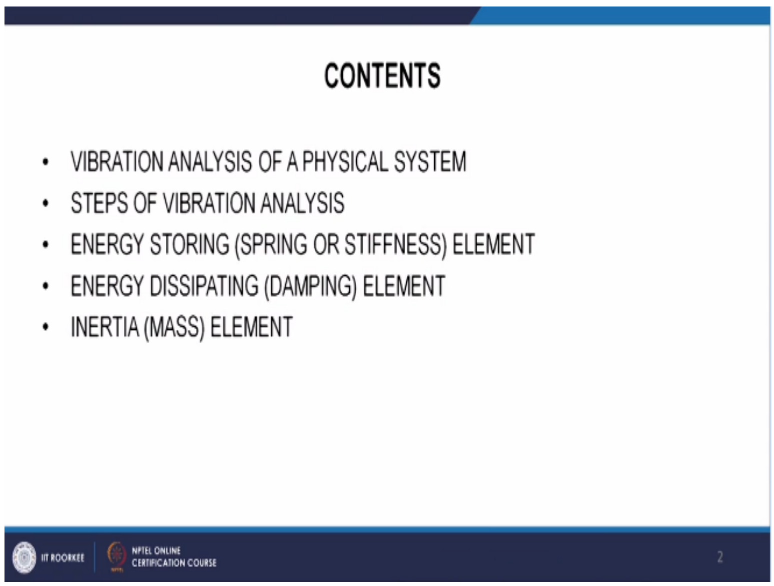

and how to do the vibration analysis of that system? What are the steps for the vibration analysis? Then some basic elements of a mechanical system like stiffness or spring, damper and mass will be discussed. So, here we have a physical system and this physical system may undergo vibration under certain external force. So, that physical system how we will analysis? So, here analysis means mathematically.

So, how we will model that system? How we will write the equations? How we will solve that mathematics and based on the solution how we will correlate with the physical system? So, here usually we have discrete system. The mechanical systems they are two types, discrete and continuous. So, here we are dealing with discrete system means we are going to divide or represent the system in terms of the three basic elements that is mass, stiffness and damping.

And we will follow certain steps that are for the vibration analysis procedure. So, here first we will make the mathematical model and if we have a physical system how we will develop a mathematical model? Of course mathematical model will be in form of some mathematical equations, some differential equations and those differential equations will need some solution. So, what are the methods to solve these equations?

And we will get the solution then. These results, how we will interpret? How we will co-relate with the physical system? What are properties that we want to know, we want to know the response of the system. So here are the four steps

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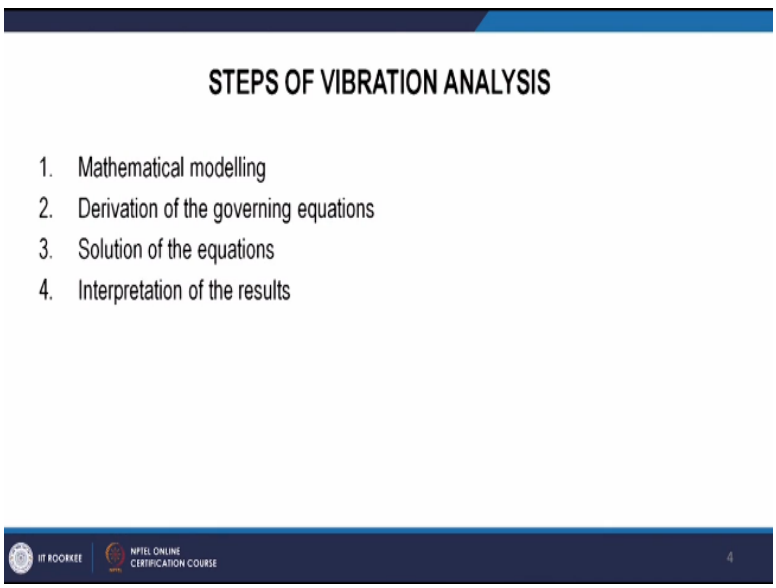

Basic steps they are mathematical modeling then derivation of the governing equations or differential equations then the solution of the equations and then interpretation of the results. So, mathematical modeling so our physical system is usually very complex and we want to represent it in terms of mathematics. It means we have to follow certain assumptions. We have to take out the features that are important.

Not the all the features but only the features that are more important based on our objective of the study of that system. What is the purpose of studying that system? What is the particular objective? What do we want? For example, if a bike is moving on the road we want to know the characteristic on normal road. What is the vibration coming? We want to know or we want to know when it is on the bump.

What is the response of the system? So means, what is the objective? So, based on these objectives we will include certain important features on our model and we will represent these important features through mathematics. So, the system could be linear or non-linear? Linear systems they are very simple and quick to solve but non-linear systems they represent the complexity and certain features that are also important in certain conditions.

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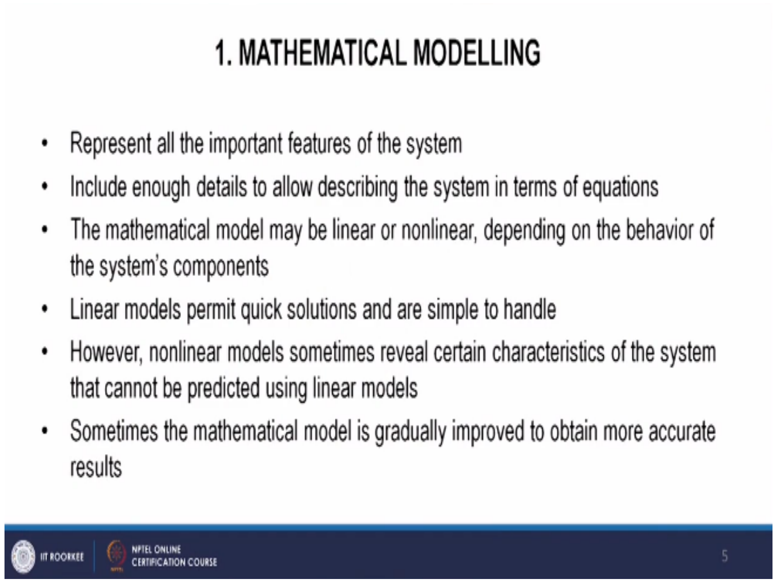

So, in order to have the mathematical modeling we cannot directly reach to a complex model. But it is better that we first develop a basic model, a simple model and then we get some ideas of the response of that system then we introduce the level of complexity. We increase the degrees of freedom we include the more specific elements. For example, here we can see.

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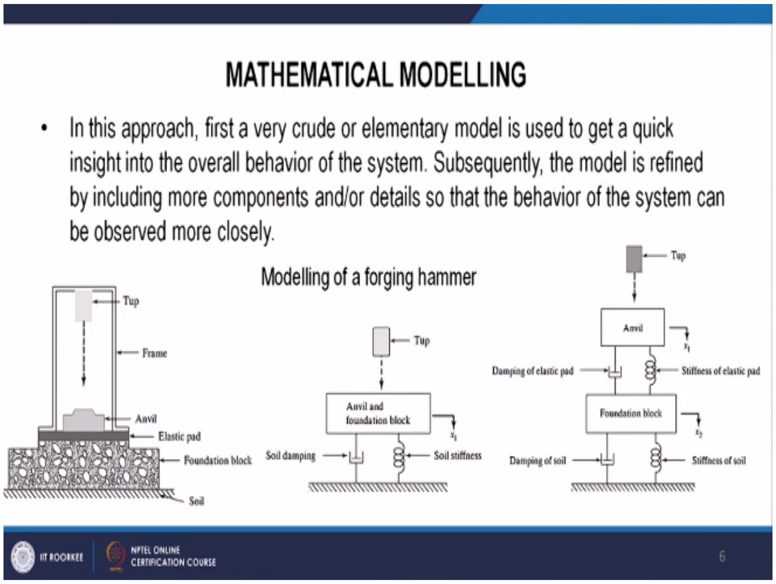

That we have one forging hammer so this is the hammer that is tup is falling on an anvil. So anvil is a steel block on which the metal is kept and the hammer is coming from the top to shape it. Moreover, to stop the vibrations or the effect of impact to go to the foundation. There are elastic pads that were guide the dissipation device, they dissipate the vibrations.

Now, we have to develop if we have to develop a model we can in a first stage we can develop a very simple model like you can see the first model very simple. It is a single degree of in a model. So, here we include one mass element. So, one mass element, it contains the mass of anvil, the frame, the foundation block as well as if there is some mass of elastic pad. Mass of elastic pad is very less in compared to others so it could be neglected or it could be included.

So all the mass is just one element and then here we take the damping of soil and the stiffness of soil. In this case, we have not considered the damping of elastic pad or other elements. So, this is our first basic model. Now, we can solve this we can find some results then we can improve we can refine this model. So, in refining process we may include the features of the certain other components of the system.

For example, we can see the second figure, second model that is the two degree of freedom system model and in this we have separated the anvil and of course this also include the mass of frame also this anvil and frame mass and the damping of elastic pad because elastic pad works as a damper. It stops the vibration to go to the foundation block so to reduce the vibrations transmitted to the foundation block.

So, we use the damping of the elastic pad as an element and then stiffness or spring constant of the elastic pad. Then here is foundation block and then damping soil and stiffness also. So, here we have refined our model this model is more complex than the earlier one. The first one is very simple single degree of freedom simple. It is easier to solve and related to the two degree of freedom system.

So, this is how we increase the complexity into the model. Then we have mathematical model we have represented our system in terms of basic mass, damper and stiffness elements. Now, the next step is derivation of governing equations. So, any discrete system can be represented as differential equations we have to represent it as a differential equation but before that we have to make the free body diagram of each component.

So, free body diagram of each component we have to isolate that particular component we have to show the external force, reaction force and inertia forces on that component and then applying certain laws like Newton laws of motion or D'Alembert's Principle or energy method we can solve that we can develop the differential equations. So, here

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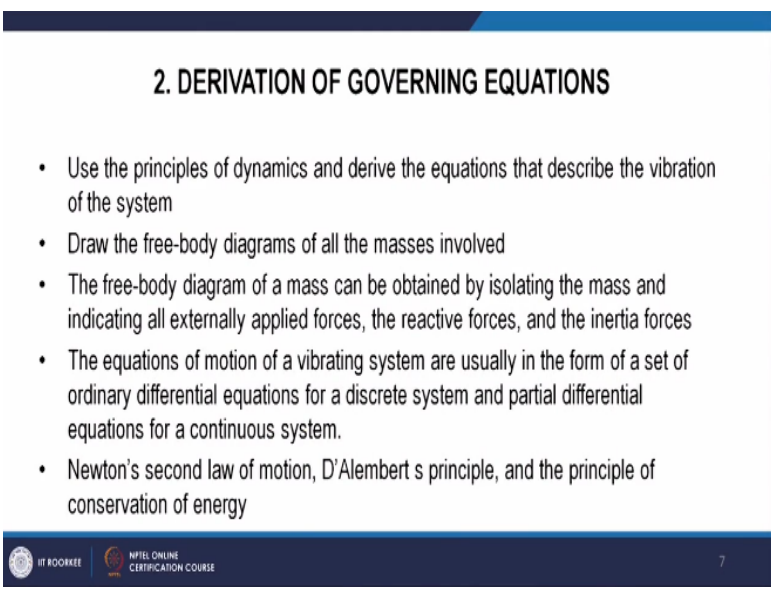

We have to make the free-body diagram from that for each mass each degree of freedom system and we can write the differential equations. For continuous system we use the partial differential equations. However, for discrete systems we write the ordinary differential equation then we go for the solution of the equations. So, the equations they are differential equations they must be solved in order to get the response of the system.

So, response means the velocity, the displacement, and acceleration varying with time so this response to solve these differential equations we have certain standard methods. We can use the theory of solving the differential equations we can use the Laplace transform we can use some Matrix method or numerical methods. If we have numerical methods, it is easier to apply the numerical methods while using the computers.

So in case of none liner systems numerical methods are more helpful because of the added complexity in the system. So using these methods, numerical methods we can solve our differential equations and then we get the results. So results as I said is in the form of the response of the system it could be displacement response, displacement time plot, velocity time plot or acceleration time plot.

From these responses we have to interpret the results means we came from physical system to a mathematical system we worked with the mathematical system we solved that mathematical system we got certain results now that results we have to go back to the, take back to the physical system. We have to correlate these things because our objective was to understand the physical system. The mathematical system was one middle step that helped us to understand this.

So what was our objective? What was that purpose? We want to know that and so we from that solutions we have to make relation with the physical system we have to know whether we want to change certain design then what is the value of that. So, we must have the clear view of the interpretation of the results. So, here there is an example, an example of mathematical model of motorcycle with a rider. So, here is a motorcycle and there is a rider on a road.

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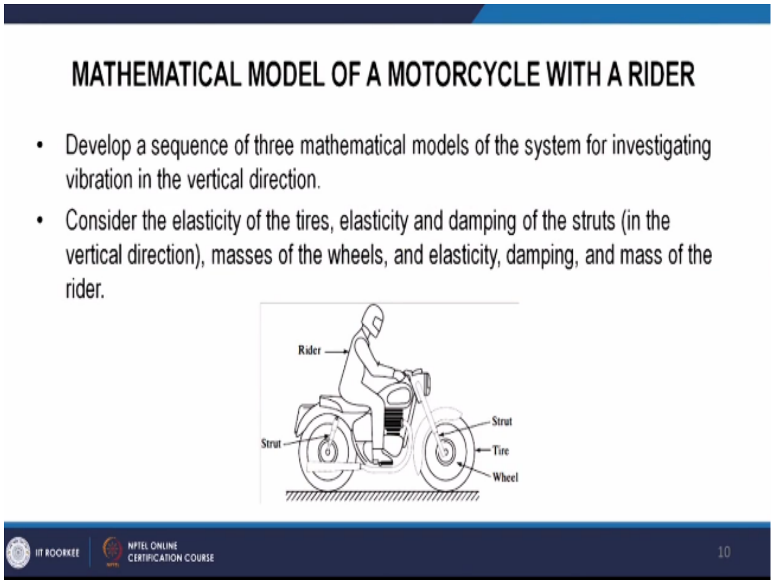

And suppose we have to develop a mathematical model. It is said of course the rider is a human person, human it has the mass, it has the stiffness because it has bones that works as a stiffness and the damping because we have this muscles they have some damping property. They can damp out the vibration. Moreover, the bike motorcycle has its own mass and it has certain damper in the strut.

So there is damper and of course some spring so there is some stiffness and damping in the strut. Moreover, wheel has its own mass and tire has its own stiffness. When we make the model so we feature out that these are the components that certain basic elements we have to add. So here we have the models. So very first model.

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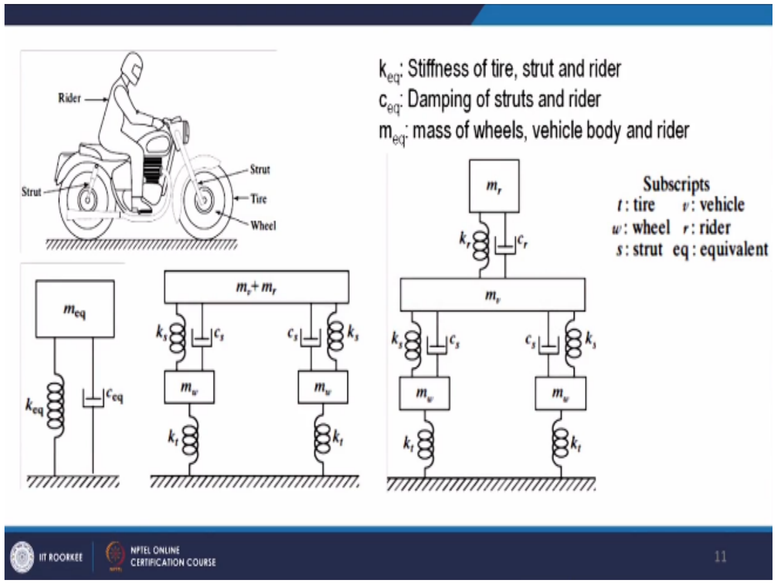

A single degree of freedom system so here we put m equivalent so equivalent mass of the system that is a rider + vehicle and wheel. So, we have the mass of these components then stiffness and damping equivalent stiffness and damping. So, equivalent stiffness could be of tire, of strut and of rider. So, it is a combination of these three then equivalent damping could include the damping of the rider and the damping of the strut.

So, the second model which is we have made taking more masses and more damping and more stiffness like we have separated the wheel mass so we have mass of wheel and we separated and the stiffness as tire. So, tire is connected with wheel so we have k t stiffness and mass and mass is then connected to the vehicle so other about rest of the body and the rider. So, here strut has the stiffness and damping so strut stiffness and damping is given, these elements from both the fields.

So, here this coming and this is the mass of the mass of the vehicle and the rider. So, this model has a of course two degree or we can say three degree of freedom. If we assume this m as symmetric and everything is symmetric then it may have only the similar characteristic both masses may have similar behavior. Now, we can even add more complexity. In adding more complexity to this model we can separate out the rider.

So, here we have tire, wheel, the strut then mass of the vehicle and because rider is sitting on the vehicle we can take rider as separate and this mass of rider, stiffness of rider and damping of rider. So, we have added more complexity to the model and the degree of freedom we have increased and of course here we will get in this case the separate responses of each component like we can get the response of the rider.

The response the vehicle, response of the wheel so more insight this model will provide. But computationally it is more time taking model. Now, we will discuss some basic elements like for our vibration modeling mass stiffness and damping they are the three basic elements they have their certain roles for example we have the stiffness or spring property. They are the energy storing elements during the vibration of the system they store the energy when it is more and as a potential energy.

And they can give back that energy for the motion of the –for the vibration of the mass. So, we assume that the stiffness element has negligible mass and damping. It is pure stiffness member it is mass less and it has no damping. So of course the mechanical system any mechanical system like beams, like bars, cables they all have the spring property. They have stiffness property so spring property means if we apply certain loads they will deform.

And if we have certain bars certain beam we apply the load there is the deflection of those elements and so they have certain stiffness. That stiffness we represent as, k = f by x or force equal to, we apply force f. So, f = k times x where k is spring constant or spring rate or stiffness. So, we can see here that if we have this

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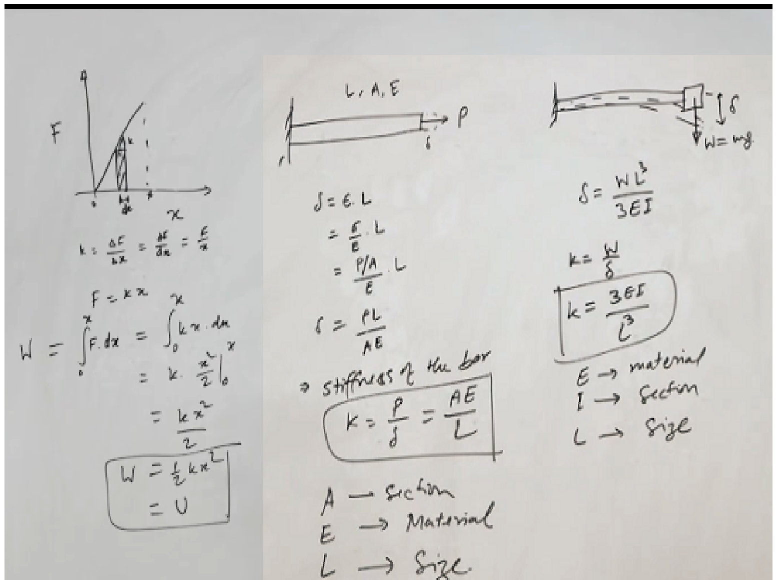

F and this X and we have this spring, linear spring so we have this k and this slope that is k so here we can have k = delta f by delta x or = d f by d x or f = F by X if it is linear. So, here f = k time x. Now, how much energy is stored in the spring element? So, we can see here if we have this x so let us say this element and this is d x so we have energy is the area so f into d x so if it is d x this is f so f into d x now we integrate this area from let us say zero to x.

So, this is the work done so f = k x so we can write k x into d x, zero two x. So, equal to k into x square by two zero to x that is k x square by 2. So equal to half k x square. So this is w that is energy. So, this is work done on the spring and same energy is stored here that is half k x square. Now, we can take certain example. For example, there is a rod.

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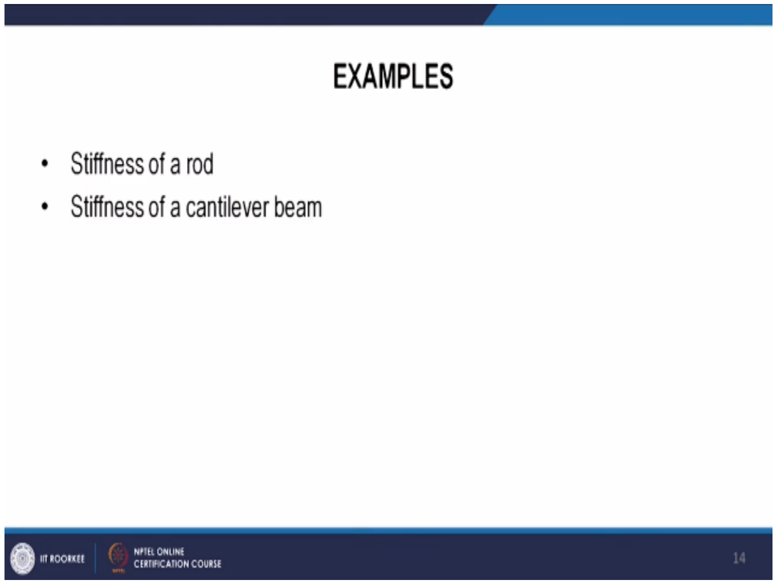

So, this is a rod that has length L area of cross section A, elasticity E and some force P is applied and there is some deformation let us say delta. So delta = epsilon into L and epsilon is sigma upon e into l. Sigma is force upon area so P upon A upon E into L so = PL by AE. Now, the stiffness of the bar K = force upon difference so we can write P upon delta so it is AE by L. So, here we can see that the stiffness k = AE upon L.

So, stiffness is property of A that is section, E that is material because it is the material properties and L that is the size. So stiffness depends on the material the section the size all these factors of this bar. Similarly, we can find out some stiffness of cantilever beam and we assume that this is a cantilever beam where at the end there is the mass concentrated mass and this beam is massless and the mass is only concentrated at the end.

So, here there is certain weight w that is m into g. we know that if there is some force applied at the end. So deflection delta = force into L cube by three E I. And so K = W by delta force upon deflection so it is three EI by L cube. Again here we see that the stiffness K is function of E I and L and so E is material, I is section, it is section properties. This is representing the size so stiffness is depending on these factors.

Now spring combinations so these spring elements can have some combinations so they can be either in series or in parallel we can develop the equivalent relation that if we have a series combination what will be the equivalent stiffness. So, let us have the series combinations so springs in series. So springs in series we have

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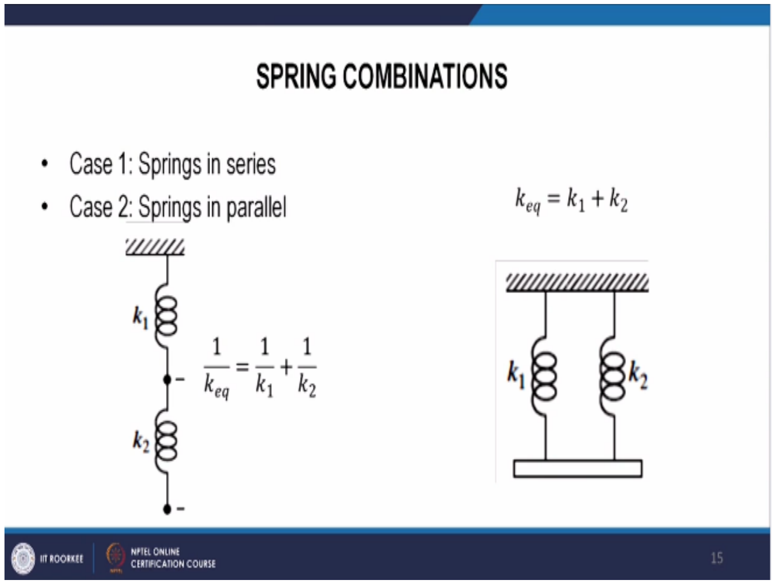

Let us say this is we have this two springs in series and they have their stiffness k y and k two.

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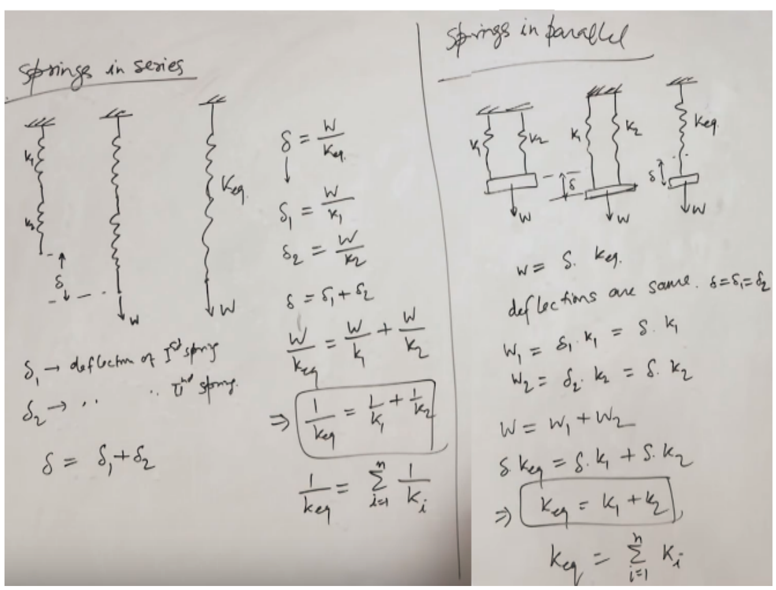

Now, we apply certain force w and so they have certain elongation. So, let us say this is delta so this delta let us say in each spring there is delta one is the deflection of first spring, delta two is deflection of the second spring. So the total delta, = delta one + delta two. Because the net deflection will be sum of two deflections.

Because the deflection in this will be total deflection that has occurred here. Now, we can write our equivalent this spring and if we apply w and this is k equivalent this much gives the same deflection so delta = w by k equivalent so this spring. It is equivalent if it gives the same deflection under the n force. It means the internal does not matter. Internal is same means if I apply the same for W, I get the same deflection from this spring.

So, the relation delta = W by K equivalent is true for this. Now delta = we can write and we can also write delta one = w by k one and delta two by w by k two because the same force w is acting because they are in series the same force W is acting on each spring. So we can write so here delta = delta one + delta two. We write delta = W by K equivalent and delta one = W by K one and delta two is W by K two.

So, we can write that one upon K equivalent = 1 by K1 + 1 by K two. So, this is the relation that you can see here if they are in series. So the reciprocal of the equivalent is equal to reciprocal of the stiffness of the spring. So here a general expression we can write = sigma one by K I for I = one to N if we have n springs. Similarly springs in parallel. So, springs in parallel we have these two springs so let us represent like this so we have this K1 and K2 and we have there is some force W and we find that they have been deflected.

So, this is K1 K2 and there is certain deflection here delta. Now, I have an equivalent spring here so if I apply this W so here is K equivalent and it is giving so again it is deflected in delta. Now, here for this equivalent we can write that W = delta into K equivalent. The delta will be same for both the individual spring because they are in parallel. So delta, the deflections are same therefore we can write here that W1 = delta 1 into K1 and W2 = delta 2 into K2.

Moreover, this W is weared by the two springs. So, W = W1 + W2 and this is equal to delta into K1 and = delta into K2. So, as delta = delta 1 = delta 2. So, W is delta into K equivalent and W1 = delta into K1 and it is delta into K2. So, this implies that K equivalent = K1 + K2. So, for n springs it is sigma K I, I = 1 to n. Okay. So, this the spring combinations. Now we can see another element

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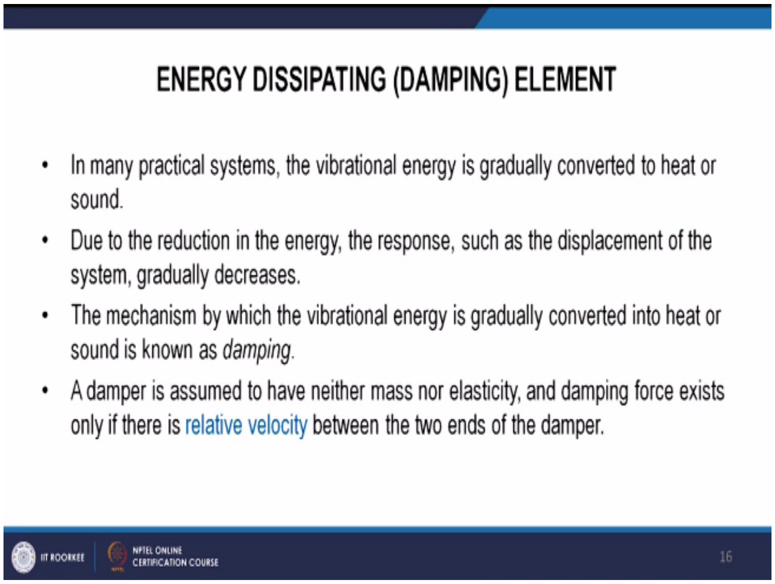

That is damping element this is energy dissipating element because in any vibrating system if there is some dissipation of the energy in terms of heat or sound and due to this the vibration of the system decreases, the amplitude decreases until unless we are giving it some external continuous force so this is known as damping element and it works based on the relative velocity between the two points of the system.

If there is relative velocity, then it works and the basic damper is viscous damping. So viscous damping it is a damping that can be due to the movement of

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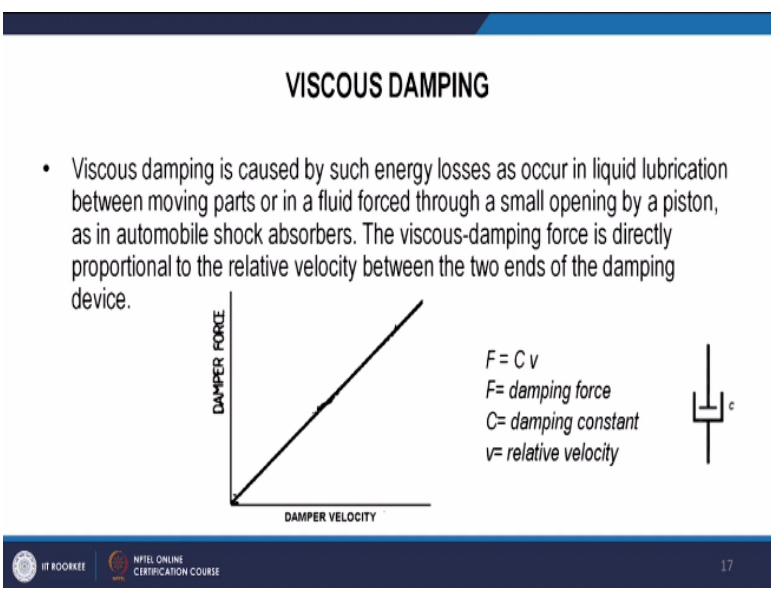

Fluid parts liquid if there is a piston that is moving in a fluid. So, this kind of damping can come into play and based on this we design the viscous damper. So, for a viscous damper the force velocity relation is linear. So, here we can see the damper force and velocity relation so that is linear. So, f = C v where F is damping force, V is relative velocity and C is the damping constant and we represent this element like this.

And the combination of dampers the similar formula applies as the springs.

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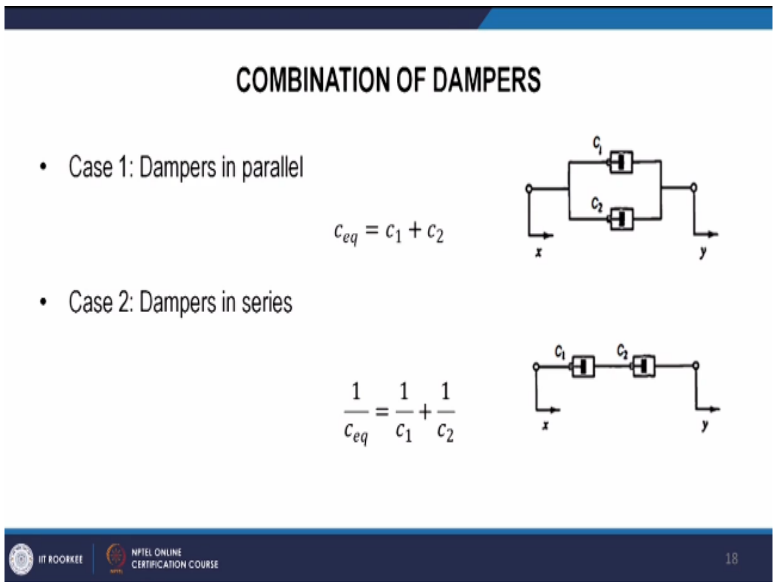

So, therefor if tampers are in parallel thus equivalent will be C1 + C2 and if dampers are in series the equivalent will be 1 by C equivalent = 1 by C1 + 1 by C2. Now, the third element and basic element is mass. Because mass is inertia element it opposes the motion of the system or it opposes to come to rest or it opposes to come into motion. So, it can gain or lose the system can gain or lose kinetic energy by this element.

And in a vibrating system there is the exchange of energy that is spring, the potential energy and the kinetic energy of the mass. So, this mass in the linear system can be represented with the moment of inertia in a rotational system. So, we will discuss now more on this application when we will discuss the single degree of freedom system to degree of freedom system and etcetera. So, we stop our lecture here.

Thank you for attending this lecture and see you in next lecture. Thank you.