**Introduction to Mechanical Vibration**
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**Lecture - 39**
**Stiffness Influence Coefficients**

So welcome to the lecture on Multi degree of freedom system, so in this lecture we will discuss the stiffness influence coefficient. So we discussed in the previous lecture the flexibility influence coefficient and we saw that if we know the flexibility influence coefficient we can prepare the flexibility matrix and by using that matrix we can write the equation of motion. Now we have similarly, we have stiffness influence coefficients.

Because in equation of motion of a multi degree of freedom system we have m x double dot + k x = 0 for and multi degree freedom system undamped system.

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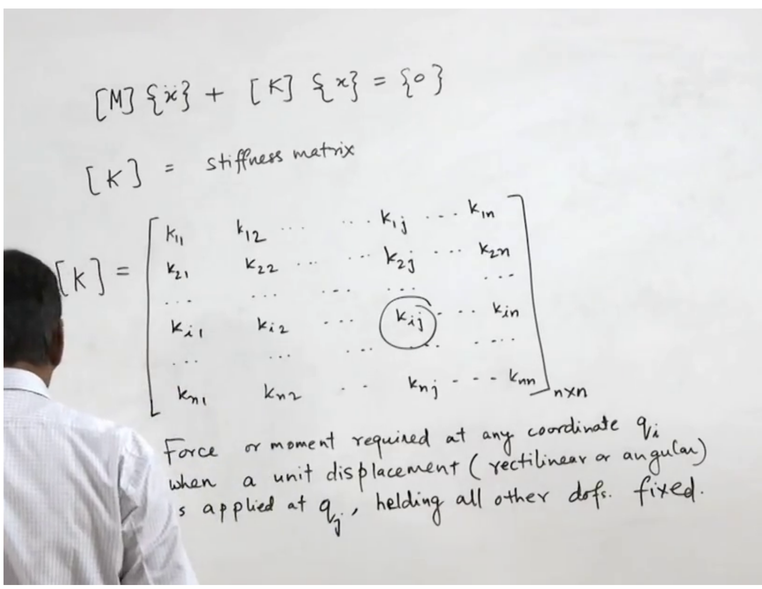


So we can write here M x double dot + K x = 0, so here K is stiffness matrix and M is the mass matrix, so K is the stiffness matrix and the elements of the stiffness matrix or cover called the stiffness influence coefficients, so here K can be written as. So we can see that the stiffness matrix that is made from stiffness coefficients influence coefficients and it is n by n square matrix, so its elements k11 k12 and knn.

So this we can find and we can make this matrix and we can use directly in that equation, so we do not need to write the equation of motions and prepare this matrix we can directly prepare the stiffness matrix, so here we concentrate on one element kij, so here kij what is kij? So kij is defined as the force or moment required at any co-ordinate q i when a unit displacement, this displacement could be either rectilinear or angular is applied at co-ordinate j holding all other coordinates fixed or all other degree of freedom fixed.

So here the stiffness influence coefficient kij is defined as a force or a moment so it is force if it is the linear motion and moment if it could be an angular motion. So what is the force that is required at a coordinates i when unit load a unit displacement is applied at j while all other degree of freedom have zero or fixed so they have the zero moment. Similarly, kji can be defined as reverse like kji is we are measuring the force at j.

And we are giving a unit displacement at i and again with this reciprocal theorem this kji is kij. So therefore we will find these coefficients for certain example, so let us take one example here and we try to find out this.

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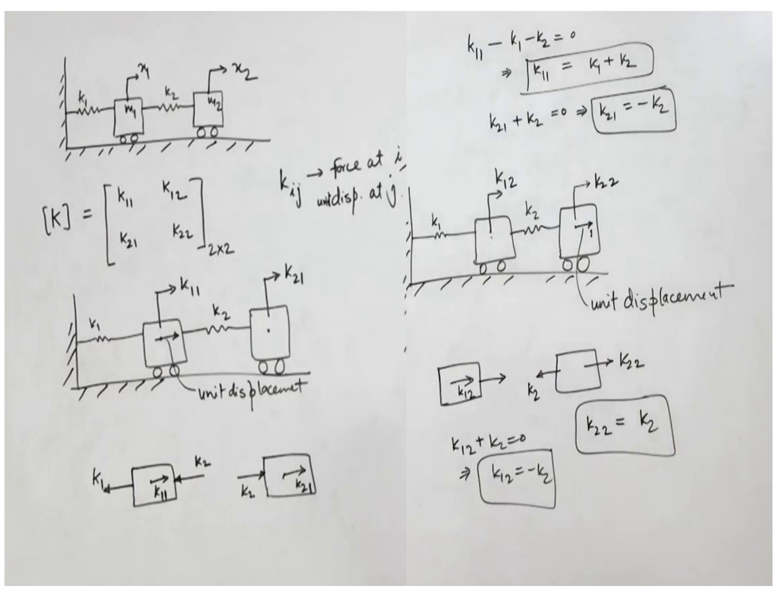


So we have this two degree of freedom system consisting of two masses m1 and m2 and to stiffness elements k1 and k2 now we have to find the stiffness influence coefficients and so the stiffness matrix K so it will have k11, k12, k21 and k22, because it is two degree of freedom system so this matrix will be square and 2 by 2 matrix and we will have total 4 influence coefficient that we have to find out.

So now as k11 or k12, k21, k22 so kij as we define that we want to measure the force that is developed as form of reaction when we apply a unit load at j, so here kij so the force at i when displacement unit, unit displacement at j so here now we give unit displacement here at so this is unit displacement so we are giving unit displacement to the first mass in this direction and we are keeping fixed other all the other degree of freedom must be fixed.

So this mass is fixed this is not moving, now when we give unit displacement we are going to measure the forces so we are going to measure here the force this and here the force, so this is k11 because we are giving unit displacement at 1 and measuring the force at 1, now we are giving unit displacement at 1 and measuring the force at 2 so this is k21, so now we will show the free body diagram for this system.

So the free body diagram, so this is my first mass and this is the second mass. so now this first mass is having unit displacement so when we are pulling so of course here the force k11 is here, k11 is acting on this mass when we give unit displacement with this spring will apply a force in this direction opposite direction that is k1 into displacement and displacement is 1, so the total force is k1.

Now we are compressing this spring so this spring will apply a force k2 into displacement so displacement is 1 so this force is k2, now this mass it is fixed so the same there is no any so here is a force k21 on this mass and then there is spring force that is k2 due to that spring and this will be opposite to this direction. So now if we write the equilibrium equations for these two free body diagram two masses.

So we will have for this first one k11 - k1 - k2 = 0, so this implies k11 = k1 + k2 so this is 1 stiffness influence coefficient that we have obtained. now for the second mass we have k21 + k2 = 0 and this implies that k21 = - k2, so we have got the second influence coefficient, so k11 is the force that is developed at 1 when we apply a unit displacement at 1 while in k21 is the force that is developed at 2 when we apply a unit displacement at 1.

Now for second other two elements so we take the case when we have this mass system, now we are giving in this case a unit displacement here so this is unit displacement to the second mass - unit displacement and we are measuring the forces at both masses so we here the force will be k22, because we are applying the displacement at 2 and measuring the force at 2 and here it will be k12, we are applying a displacement at 2 and measuring the force at 1.

Now we make the free body diagram so the free body diagram here, we will make for these two masses so for this one we will have here k22, now we are pulling this spring so with the unit displacement this will apply a force k2 into 1, so that is here and this spring the same will apply the same force on this mass then k12 is another force that is working here on this mass and this mass is fixed so there will not - no be any stretching or compression of the spring k1.

And therefore there will no be not be spring force by this spring on this mass, so we can write the equations here for equilibrium equations for these two masses, so here we will have k22 = k2 and from here we get k12 + k2 = 0 this means k12 = - k2.

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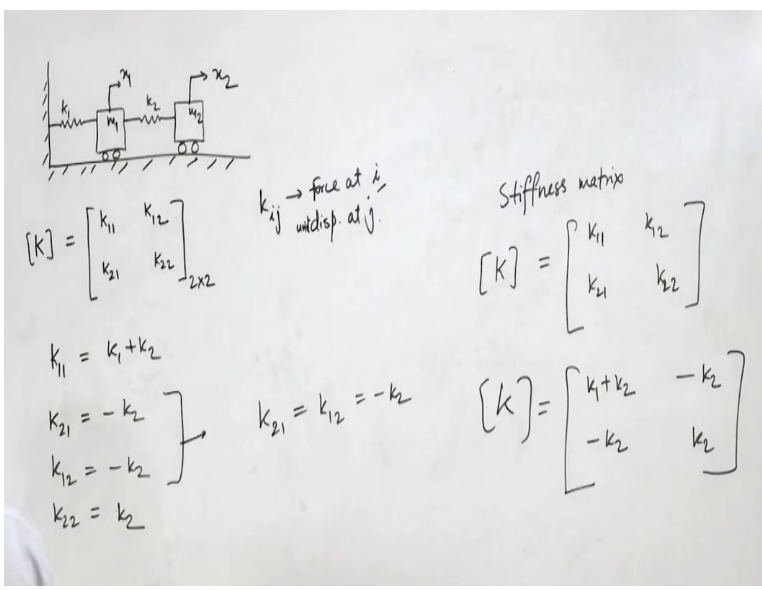

So we have obtained the all the four influence coefficients stiffness influence coefficients that is k11 that is k1 + k2, k21 that is - k2, k12 - k2 and k22 = k2, so again from here we see that k21 = k12 = - k2 due to the reciprocal theorem. Now we can write our stiffness matrix, so now we see that how we can find the stiffness influence coefficients for multi degree of freedom system and how the reciprocal theorem.

According to reciprocal theorem we can omit to calculate several terms and we can easily find complete this stiffness matrix. So this is what I discussed for the multi degree of freedom system that is undamped, so we only considered the mass and stiffness of the system, but we have also damped systems. So we have all the three elements that is mass stiffness and damping elements. So as for undamped system we wrote the equations for multi degree of freedom system.

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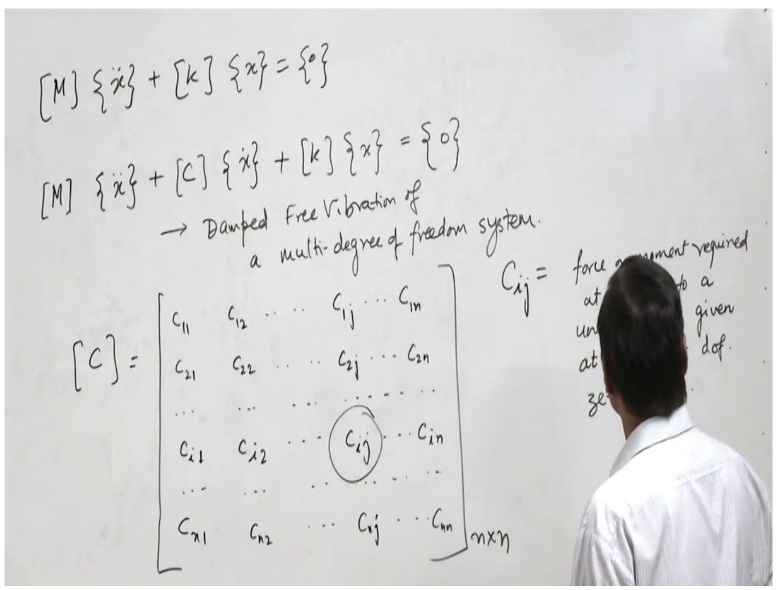


And we find represented in terms of matrices, so it was M x double dot + K x = 0, so this is this equation is for an undamped spring mass system multi degree of freedom system. now if we have damping and we introduced the damping in the system we have the C1 C2 C3 terms, then one more matrix will come into this equation and we call it damping matrix so we will we can write, so this is equation of damped free vibration of multi degree of freedom system.

So we discussed the flexibility influence coefficients, we discussed the stiffness influence coefficient. Now we have the damping matrix and therefore it has the damping influence coefficients so the damping matrix C so the damping matrix C can be written as, so this damping matrix C is also square matrix of having n by n elements if it is a n degree of freedom system and the elements of this matrix are called the influence the damping influence coefficients.

So if we take this cij, so cij, it is defined as the force or moment required at co-ordinate i due to a unit velocity given at j co-ordinate j while other degree of freedom have zero velocity they are at rest, so zero velocity. So similar to the stiffness coefficient we can also calculate the coefficients of this damping matrix and so we can find the damping influence coefficients. So I thank you for attending this lecture and see you in the next lecture, thank you.