Introduction to Mechanical Vibration
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Lecture - 38
Flexibility Influence Coefficients

So welcome to the lecture on multi-degree of freedom system. We will discuss today the flexibility influence coefficients. So what is flexibility as flexibility is reciprocal of the stiffness and we know that in any vibrating system, we have mainly three elements spring, mass and damper system. So for spring, mass system that is undamped multi-degree of freedom system, we have already derived the equation of motion that comes into the form of a matrix.

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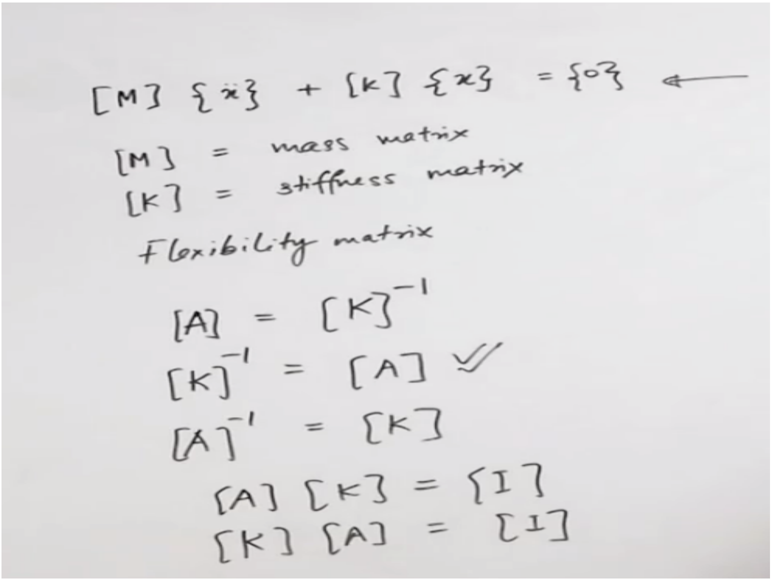

So the equation of this motion that is M x double dot + k x = 0 and here I said that this M is mass matrix and here this k is stiffness matrix. Now what is the flexibility matrix? So here because flexibility is inversely proportional that is the reciprocal of the stiffness. So here the flexibility matrix let us say A that is equal k inverse and similarly k inverse = A or A inverse = k.

So here we can also write that A into k that is equal to an identity matrix or k into A is equal to identity matrix. So here we have this equation M x double dot + k x = 0.

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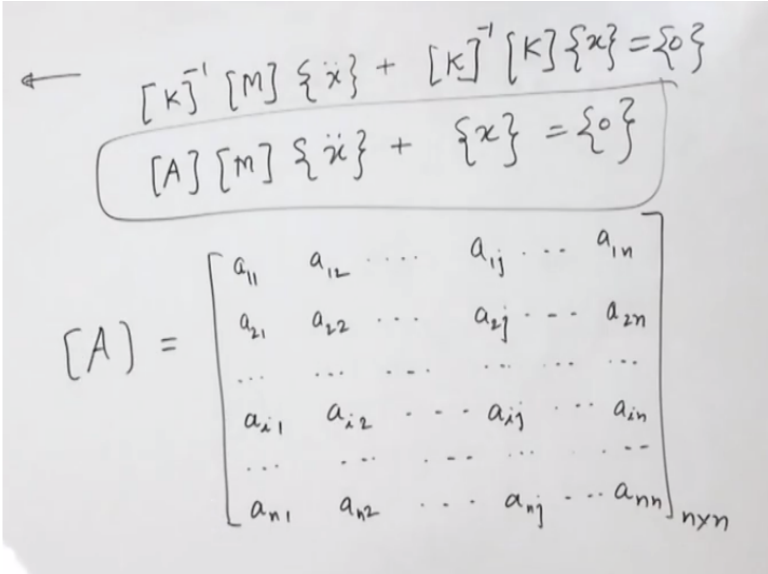

Now we multiply with k inverse so we will have k inverse M x double dot + k inverse k x = 0. Now here k inverse is we can write the flexibility matrix so this is A m x double dot + here k inverse into k is i and when we multiply i with x it is x so that is 0. So finally our equation of motion results to b in the form of mass matrix and flexibility matrix. So therefore if we can make we can prepare the flexibility matrix.

We can also write the equation of motion directly using the flexibility matrix. So flexibility matrix A has its coefficients. So here we have like a11, a12, a1j, a1n, if it is n into n; here a21, a22, a2j, a2n; we have here ai1, ai2, aij, ain; here we have an1, an2, anj, ann. So we have this is n into n matrix that is flexibility matrix and the elements of this matrix are known as flexibility influence coefficients.

So if we take this aij, one element aij, so how we will define this aij and how we will obtain aij because if we know all these elements we can prepare a flexibility matrix for that problem. Now aij means what? So we will define here aij.

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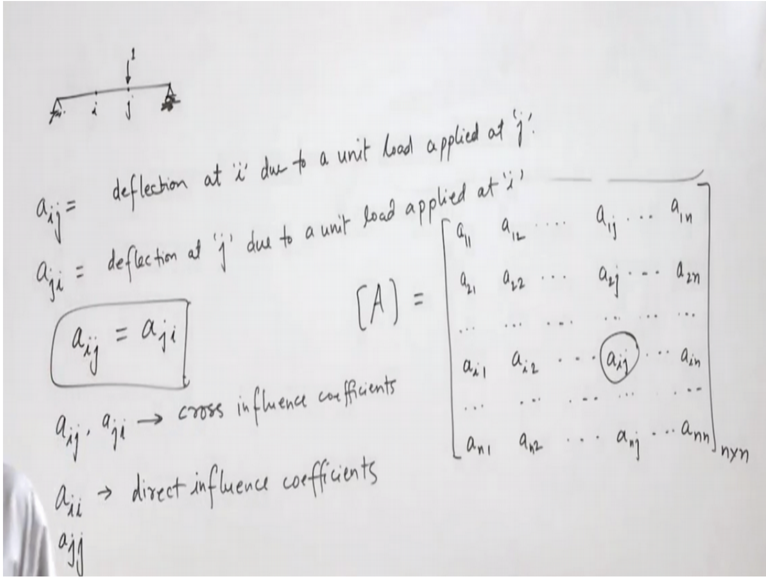

So suppose we have some member here and here are two points that is i and j. So aij is defined as the value of deflection at i due to a unit load applied at j. So if we have this flexibility influence coefficient aij defined as the deflection at i when we apply a unit load j at j that is unit load and we measure what is the deflection that is coming here and that is aij. Similarly, aji is the deflection at j due to a unit load applied at i, so here we are applying a unit load at i and measuring deflection at j.

So now the Maxwell's reciprocal theorem states that this aij = aji. So it means that the deflection at i due to unit load at j is equal to the deflection at j due to unit load at i. Now here we have aij, aji and we have also aii so ii means we have the deflection at i and unit load at i so at the same point. So we are applying the load at the same point unit load and measuring the deflection at the same point so aii are ajj.

So these are called the direct influence coefficients and these are called the cross influence coefficients. Now as we have defined what is the flexibility matrix? What are the flexibility influence coefficients? Now if we want to prepare this flexibility, how we will do it. So I take one example and we can apply these rules that we have already studied these equilibrium equations and we will find these coefficients.

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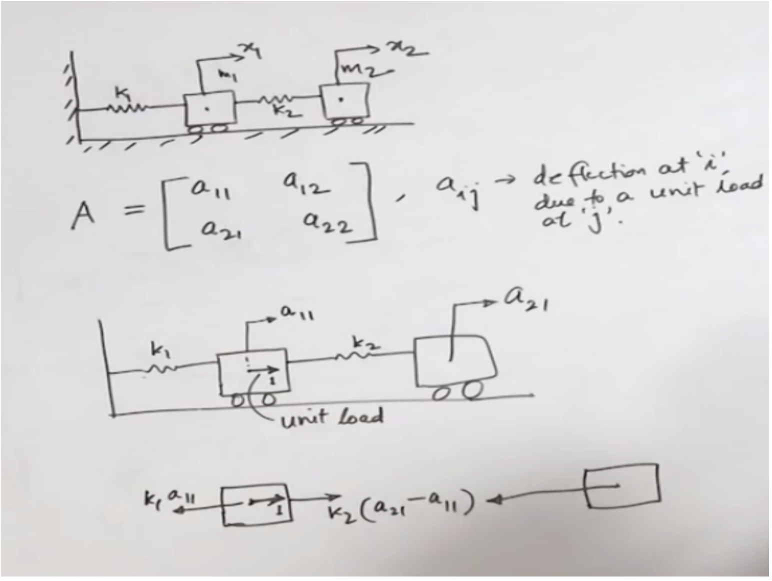

So now we take one problem and we try to derive these flexibility coefficients. So here we have two degree of freedom system and comprising the two masses m1 and m2 and two stiffness elements that is k1 and k2. We assume that the surface on which these masses are resting, the friction is negligible or 0. So we are not considering the friction. And now we have to find the flexibility matrix.

So because we have this two degree of freedom system, so our flexibility matrix will be square matrix having 2 into 2. So we will have A, the flexibility matrix that is having a11 and a12 and a21 and a22. So now we have to find the values of these coefficients and so we will find the flexibility matrix. So now if we want to find this a11, a21 so let us now we do like this. So now what we are doing because the flexibility coefficients are defined as the deflection, so aij is defined as deflection at i due to a unit load at j.

So it means, we have to apply a unit load at one point and measure the deflection at other points. So if I apply here so this is one and this is two so this is first mass, so I apply a unit load here like this so this is unit load. So this is unit load here. And now I want to measure the deflection so deflection here and deflection here so what will be deflection here that will be a11.

Because I am applying a unit load at 1 and I am measuring deflection at 1 so I am applying the load at j so here i = j is 1. Now what will be the displacement at mass two, so I am applying load 1 so here I am applying and measuring at 2 so a21 so I am applying load at 1 and measuring deflection at 2. Now we have to make the free body diagram to find the equilibrium equations.

So here we have this mass first mass and this first mass we have load here unit load then because we have some deflection here we will have spring force here that is k1 because this is k1 and this is k2. So k1 into a11 because a11 is the deflection and k1 is the stiffness. So the spring force acting due to the movement is k1 into a11 in the opposite direction. Now there is spring k2 and this will apply a force, so k2 into this is moving a21 this is a11.

So here a21 - a11 so the net force that is due to spring is the relative displacement of this spring so that is a21 if we assume greater than a11. So a21 - a11 into k2. Now on the second mass, we have to make this free body diagram of this mass so this mass is m2. On this mass we do not have any external load as in this case we have unit load. So this mass is only subjected to force due to the spring k2 and that is the same magnitude, but the direction is opposite one.

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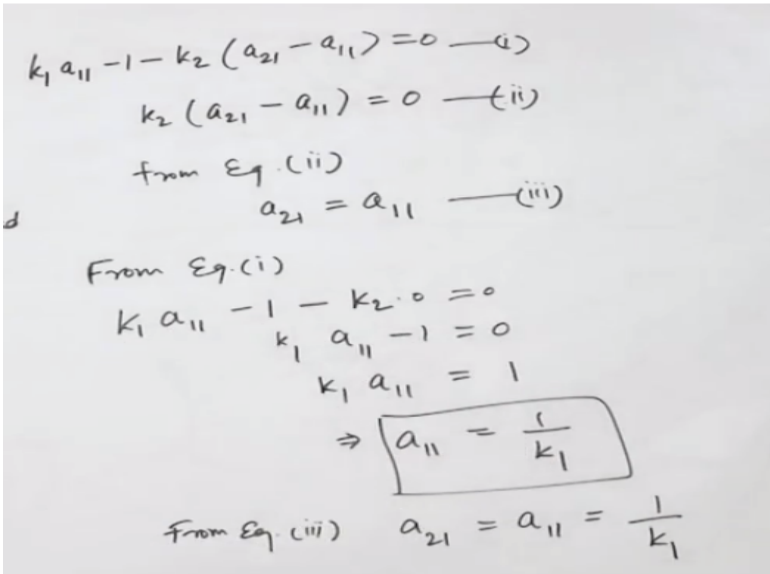

So now we have to balance these equations so for this equation number one, these two forces in this one direction and this is opposite direction. So we will have k1 a11 - 1 - k2 a21 - a11 = 0. So the net total sum of the forces are 0. Similarly, for the second mass, we will have k2 a21 - a11 so only this force and so that is equal to 0. So from this equation so this is equation number one and this is equation number two.

So from equation number two, we will have a21 = a11 because here this is 0 and k2 cannot be 0 because it is finite stiffness of this spring two and therefore we will have a21 = a11, let us say this is equation number three. Now from equation number one, k1 a11 - 1 - k2 into a21 - a11, a21 - a11 is 0 so into 0 and equal to 0. So we will have k1 a11 - 1 = 0. This means k1 a11 = 1 and this implies that a11 = 1 by k1.

So we have got this influence coefficient a11 that is when the unit load is applied at one and the displacement is being measured at 1. Now from equation three, we have a21 = a11 and therefore it is also equal to 1 by k1. So we have got the second influence coefficient a21 that is when the load is applied at 1 and displacement is measured at 2 and they are equal. So we have got these two influence coefficients.

Now we have other coefficients that we need to find out and therefore we will apply the similar process. So now in the previous case, we applied a unit load at 1, now we will apply a unit load at mass two.

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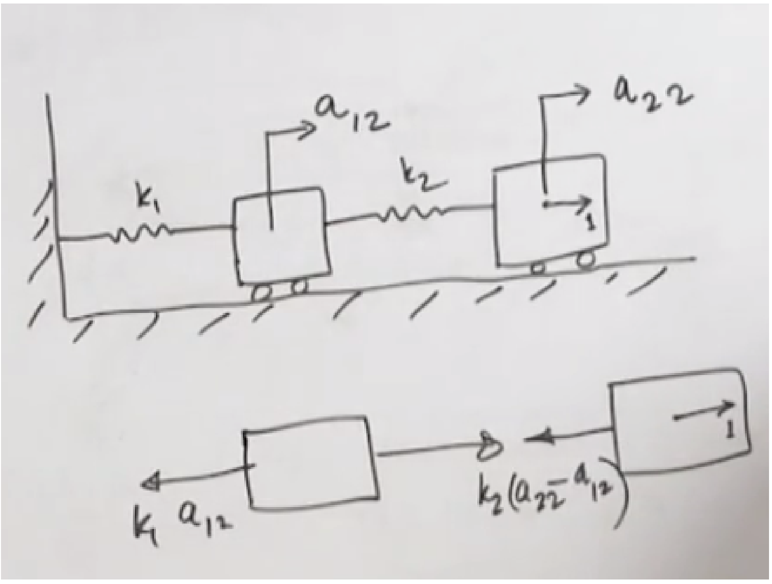

So we will have this spring k1 k2 and this is mass m1 and m2. Now in this case, we are applying a unit load here so let us we apply a unit load here and we measure the deflection at other places. So here deflection at this place will be a22 means we are applying a load at mass two and we are measuring deflection at two and this one will be a12. So we are applying unit load at 2 and measuring the deflection at 1.

Now we make the free body diagram of this load system. So we will have this first mass, so we will have and this is the second mass. So on the second mass, we have this unit load + this is spring force so we will have this spring force k2 into a22 - a12. So this is k2 into a22 - a12 and similarly this force will also work on this mass because the spring is connected to both, only the direction will be different. Now we have here due to a12 movement, there will be a force k1 into a12.

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$$k_1 a_{12} = k_2 (a_{22} - a_{12}) \quad \text{—(i)}$$

$$k_2 (a_{22} - a_{12}) = 1 \quad \text{—(ii)}$$

$$a_{22} - a_{12} = \frac{1}{k_2}$$

$$k_1 a_{12} = \cancel{k_2} \cdot \frac{1}{\cancel{k_2}} = 1$$

$$\Rightarrow \boxed{a_{12} = \frac{1}{k_1}}$$

Now we apply the equilibrium equations, so here k1 a12 = k2 a22 - a12, this is equation number one and then for this k2 a22 - a12 = 1. So here a22 - a12 so this is equation number two and from here we will get 1 by k2, this a22 - a12. Now we put this value a22 - a12 into this equation number one, so this a22 - a12, so we write k1 a12 = k2 into a22 - a12 is 1 by k2 and so this will cancel out and it is equal to 1.

So this implies a12 = 1 by k1. So we have got this influence coefficient a12 when a unit load is applied at 2 and we are measuring deflection at 1.

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From Eq (ii)

$$k_2 \left(a_{22} - \frac{1}{k_1}\right) = 1$$

$$a_{22} = \frac{1}{k_2} + \frac{1}{k_1}$$

$$\boxed{a_{22} = \frac{k_1 + k_2}{k_1 k_2}}$$

Now we put the value of a12 in equation number two so from equation two, if we put this value a12 here so k2 a22 - 1 by k1 = 1. So now we will have a22 = 1 by k2 + 1 by k1 and so it is k1 + k2 upon k1 k2. So this is another influence coefficient.

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$$\boxed{a_{21} = a_{12} = \frac{1}{k_1}}$$

$$a_{11} = \frac{1}{k_1}$$

$$a_{22} = \frac{k_1 + k_2}{k_1 k_2}$$

So we know that from the reciprocal theorem, a21 = a12 and so that is we see a12 is 1 by k1 and a21 is 1 by k1. So even if we know from here a21, we do not need to calculate a12, we already can use this as the value of a12 and here a11 = 1 by k1 and a22 = k1 + k2 by k1 k2.

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$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad a_{ij}$$

$$[A] = \begin{bmatrix} \frac{1}{k_1} & \frac{1}{k_1} \\ \frac{1}{k_1} & \frac{k_1+k_2}{k_1 k_2} \end{bmatrix}$$

So now we know all the elements of this matrix so we can write this matrix, so A = a11 that is 1 by k1 and a12 that is 1 by k1 and a21 is also 1 by k1 and a22 is k1 + k2 by k1 k2. So this is the flexibility matrix that we can obtain. So I stop here and I thank you for attending this lecture, see you in the next lecture.