Introduction to Mechanical Vibration
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Lecture - 36
Undamped Free Vibration

Welcome to the lecture on Multi Degree of Freedom System. We have discussed about the degree of freedom. So degree of freedom of any system is the minimum number of independent coordinates that are required to express the motion of the system and we have discussed Single Degree of Freedom System and Two Degree of Freedom System.

So Multi Degree of Freedom Systems or more than one Degree of Freedom System there are Multi Degree of Freedom System. However, we can easily handle the Two Degree of Freedom System manually we can write the equations and we can solve however when the degree of freedom increases it is difficult to handle manually this equation and therefore we form the matrix and we solve using the matrix laws.

So here we will discuss any system how many degree of freedom it should be expressed in.

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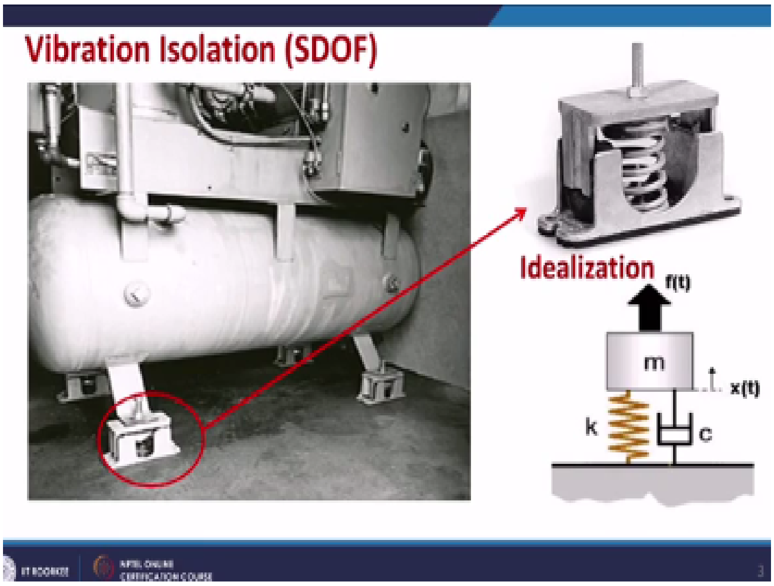

So here example is one vibration isolator so we can see here there is the spring and some damper system and it can be idealized as a Single Degree of Freedom System.

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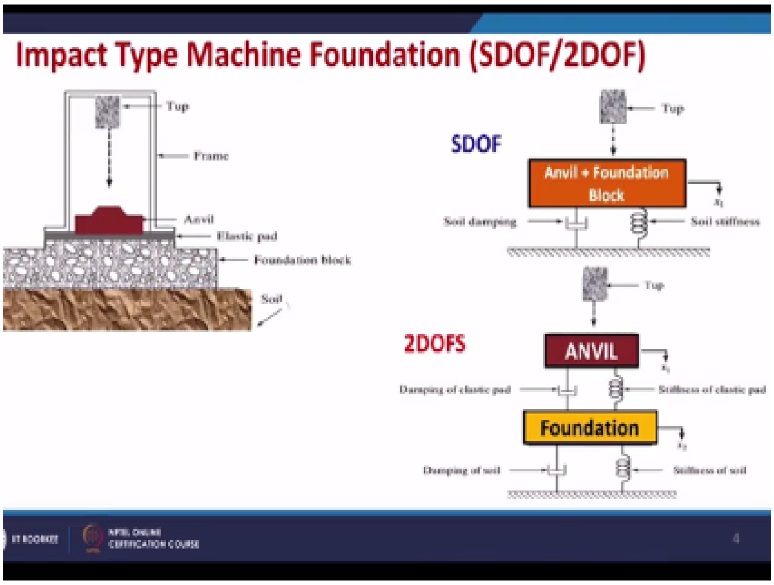

So here is a spring and damper and this mass so it is a Single Degree of Freedom System. However, if we have this system so we have this Anvil. Anvil is a high strength steel plate and there is some hammer that is coming from the top and it hits this Anvil. And the material that is on this Anvil that can be shaped and there is elastic pad to reduce the vibrations that is transmitted to the foundation.

And here is the foundation block that could be some concrete block and here is the soil. Now this is a real practical system and as we have already discussed that if we model a system with less number of Degree of Freedoms like Single Degree of Freedom. So these equations are easy to solve and they give us quick insight into the problem. Although when we increase the Degree of Freedom we are adding the complexity of the system.

And so we are complicating the equations. We are adding more parameters. However, we are doing this to get more accurate model. So we can start a system to model with some low Degree of Freedom, may be Single Degree of Freedom and then we can gradually we can upgrade that model to a higher Degree of Freedom System.

For Example, here we have this system and we have model is here as a Single Degree of Freedom. So here is the mass we have consider of Anvil and the foundation block. We have neglected the mass of the elastic pad here and the soil are damping and stiffness is here taken and so this is the Single Degree of Freedom system. When this mass is, the hammer is hitting this Anvil what is the response of this when we have this Damping and stiffness

Now we can increase the Degree of Freedoms by more components adding more details. So, for example, here we have introduced the Damping and Stiffness of the elastic Pad where elastic Pad will have some stiffness and some damping and therefore we have introduced this because this is between the Anvil and foundation block so we have introduced stiffness and damping of this.

Moreover, here there is foundation that is now another mass and then damping of soil and stiffness of soil. So we have modelled a system in term of two different Degree of Freedom System and from this model we cannot tell very accurately that what will be the response of Anvil and foundation block. We cannot differentiate their responses, but here we can differentiate. We can have a response of Anvil and different response of foundation as it can be in the practical situations

Now we have some another model may be here we have some three storey building and this three storey building. So this building have some columns or the walls they work as a stiffness element and then there are floors, the mass. So we can model this system like some mass stiffness system and this stiffness is representing a spiring property so we can again model it as a K1 M1, K2 M2 and K3 M3.

So three storey building can modeled here as a K3 this spring and mass system of the three Degree of Freedom System. So this is the objective of studying the Multi Degree of Freedom System that the more accurate models, more complex models are the multi degree of Freedom System models.

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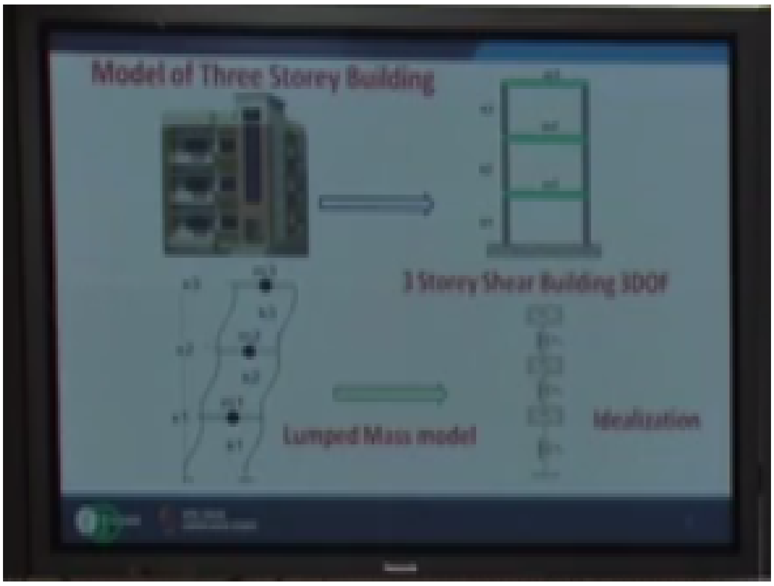

Now we take the general case of a spring mass system that is Multi Degree System and we derive the equations of motion for that system.

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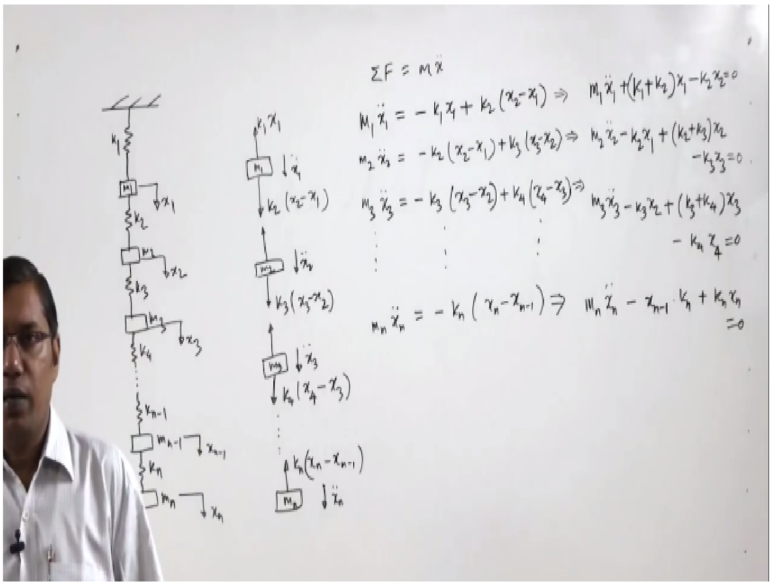

So let us take this system. So this is spring mass system. So here we have this K1 M1, K2 M2, K3 M3 and this is K4 and this is MN, MN-1, KN, KN-1. So this is a Multi Degree of Freedom spring mass system and we are interested to know how to write the equation of motion for such a system. We have N number of masses and stiffness. So here we have M1 and we can show the free body diagram.

So we have a force that is K1 X1 and if we assume X2 > X1 so this is pulling this spring force K2 X2 - X1. Now we have M2 so M2 is here applied with the same force K2 X2 - X1, but from M3 it is being pulled. So here K3 X3 - X1 and then this M3.is being pulled here with the same force K3 X3 - X1 and to the down side it is K4. So this is X1 double dot this is X2, so this is X2 and this is K4 and this is X3. Now we have this last spring. So here we will have MN KN XN - XN - one.

Now the free body diagrams of all the systems from M1 to MN has been shown. Now we will apply the Newton's law to write the equations. So we will have Sigma F = MX double dot. So we will write this M1 X1 double dot = - K1 X1 + K2 X2 - X1. So for Mass M1 we have written the equations. So MX1 double dot M1 and then the forces that is K1 X1 that is upward and K2 X2 - X1 is in the direction of X1 double dot so it is positive.

Similarly, for M2 we have this force, but opposite to the direction on X2 double dot so it is negative and this force is in the same direction of X2 double dot so it is positive. Now for M3 so we have this force that is opposite. So X3 - X2 + K4 X4 - X3 and so we can write for this Mass MN so MN XN double dot = - KN XN - XN - XN - 1.

So because here is the Mass N - 1, so the motion relative motion XN - XN - 1 into KN. Now we have to adjust this term. So here we can write M1 X1 double dot + K1 X1 so here K1 X1 and then here is K2 X1. So we will have K1 + K2. So we will write K1 + K2 X1.and then if we take this other side so - K2 X2 = 0. Then for this M2 X2 double dot now we have X1 K2 X1, so we have - K1 K2 so we have - K2 X1.

So it will go other side + so K2 X2 and K3 X2 they will go other side so it will be K2 + K3 X2 and then K3 X3 so - K3 X3 = 0. Similarly, for this equation so we have M3 X3 double dot, now X2 term so it is - K3 X2 then K3 X3 and K4 X3. So we will have + K3 + K4 X3 and - K4 X4 = 0. Then here MN XN double dot and then here - XN - 1 into KN and then + KN XN = 0.

So we have adjusted these equations So here we can write M1 X1 double dot + K1 + K2 into X1 - K2 X2 = 0 then M2 X2 double dot - K2 X1 + K2 + K3 X2 - K3 X3 = 0. We can again write this M3 X3 double dot so I am adjusting this term. So this X1 is known here then X2 - K3 X2 + K3 + K4 X3 - K4 X4 = 0. And here this terms are 0 X1 and here is + 0 X3 + 0 X4 + 0 X4 and so on and other terms are 0.

Again here we can write for MN XN double dot and so all the other terms 0 X1 0 X2 and so on and here we have - KN XN - 1 and + KN XN = 0. Now we have a set of N Algebraic equations and we have N variables so we have X1 and XN. So now we have this N equation we can adjust in terms of matrix.

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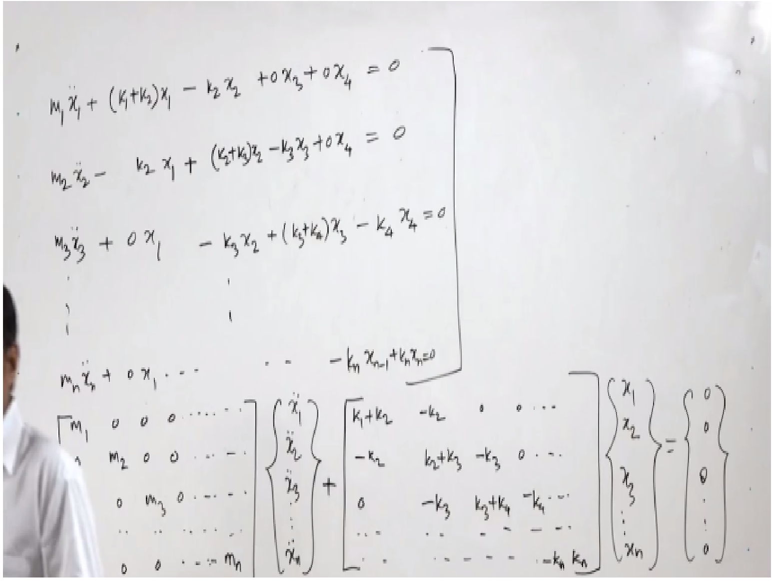

So we can write here so we can write M1 and then 000 so on. Then 0 M2 so this M term X1 dot term we can write in this form in the matrix. So M1 X1 double dot and this is M2 X2 double dot this is M3 X3 double dot and then this is MN XN double dot. Now the second term stiffness terms we can write +. So here we have K1 + K2 and then - K2 then 0 and then 0 and so on then - K2 then K2 + K3 then - K3 then 0 then 0.

And then - K3 then K3 + K4 and then - K4 and so on. So this we can write so we have here - KN. So we have written these algebraic equations into matrix form. So if we want to get the first equation so we have M1 X1 double dot because other terms are 0 + K1 + K2 X1 - K2 X2 and other terms are 0 = 0. So we will get the first equation. So all these N equations we can get from only this matrix and therefore we can write this as M X double dot + K X = 0.

So here M is N into N square matrix and it is known as the Mass matrix. K that is again N into N square matrix and this is known as the Stiffness matrix. So we see that how Multi Degree of Freedom undamped system. The equations can be adjusted into a matrix form and now we have this just a matrix equation can be solved and we can solve for the response of this system Multi Degree of Freedom system.

So we need to use the rules of matrixes and so when we deal with Multi Degree of Freedom

System we need the Mathematics of Matrixes, we need to work with Matrixes and several software are based on these matrixes. So it is easy to deal Multi Degree of Freedom system in software. Now I stop here and see you in the next lecture. Thank you.