Introduction to Mechanical Vibration

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Lecture – 34
Damped Dynamic Vibration Absorber

So welcome to the lecture on dynamic vibration absorber. Today we will discuss the damped dynamic vibration absorber. So in previous lectures, we discussed about the theory of the dynamic vibration absorber and we discussed that dynamic vibration absorber is an auxiliary system that is attached to the main system. So the main system subjected to some harmonic force and therefore the main system vibrates with certain amplitude.

Now when we attached the vibration absorber system, then the objective is to reduce the vibration of the main system, although the absorber system will keep vibrating. So in that case, when we used undamped vibration absorber, when it was just a spring mass system and we saw that due to proper design of the vibration absorber that is the proper selection of stiffness and mass of the vibration absorber.

We can bring the main system vibrations to zero and that was the role of that undamped vibration absorber. Now the problem with undamped vibration absorber is that it was at only a particular frequency because it is tuned to that frequency. Moreover, we can see that when we introduce dynamic vibration absorber, then the system resonance frequencies, because system becomes 2-degree system and therefore it has 2 natural frequencies.

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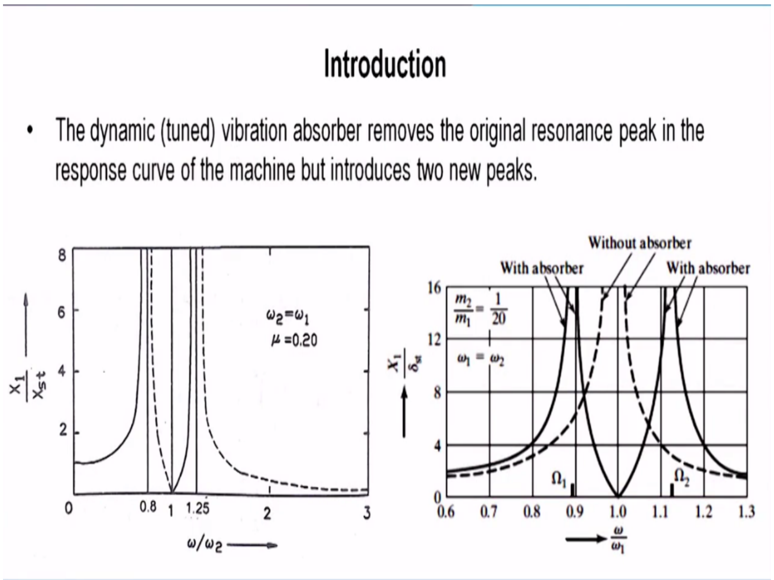

So these 2 natural frequencies; for example, if we have earlier omega/ omega 2 = 1, now we have at 0.8 and 1.25. So these are the 2 natural frequencies and we can see that, we can see that when we start any machine, it is start from some low speed and it raises to; its operating a speed. Therefore, if it passes through the lower resonance frequency of the system for example here, if it passes through 0.8, although its operating frequency is here.

But it has to pass through this frequency. So at this frequency, the amplitude of vibration are quite high; quiet large and therefore this; we can also see from here that without absorber we have here at omega = omega / omega 1 =1; we have this peak without absorber. When we introduce the absorber, we have these 2 peaks. So this be the absorber here and here, so therefore when we start a machine from low speed to reach to that omega / omega 1 = 1.

It will pass through this in this case 0.9. And in the other case here in this diagram 0.8. So it will pass from the lower resonance frequency and therefore at those frequencies the vibration amplitude will be quiet high and the undamped vibration absorber will be able to control this amplitude. Therefore, we will introduce some damping and therefore the vibration absorber will be called as damped vibration absorber.

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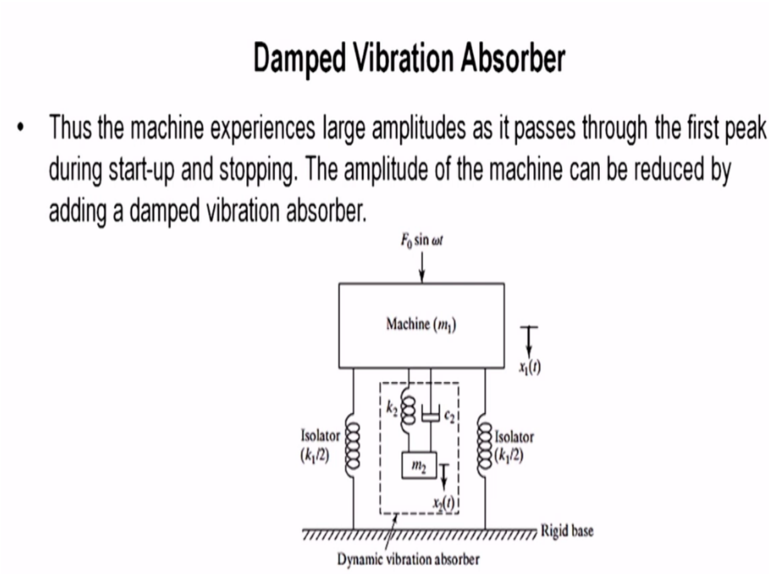

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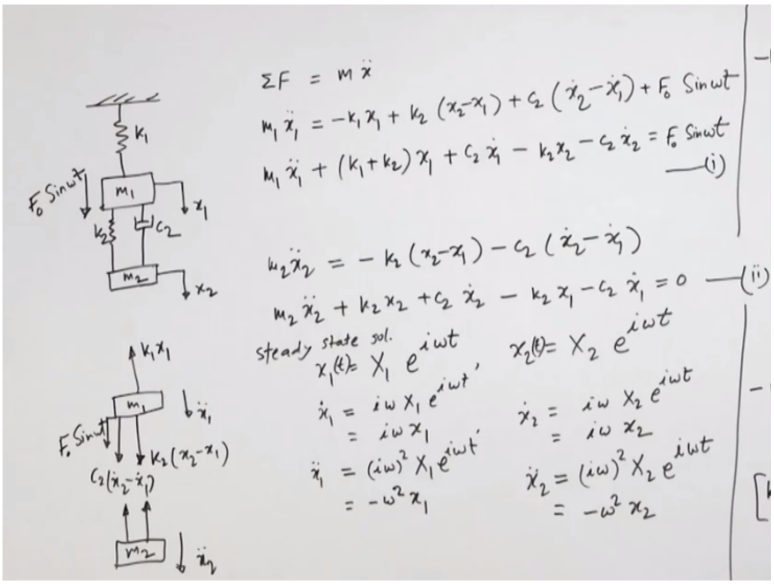

So here we can see there is a machine and that is supported on some isolator and the stiffness is k because these 2 springs are in parallel, so the equivalent (()) (05:47) is k1 and we are attaching an auxiliary system that is the vibration absorber; damped vibration absorber, k2, c2 and m2. So here we have one more damping parameter and so this system we have to study.

So this machine is, we can this machine and damped absorber system we can make like; so we have this support. And this is the stiffness of the isolator of the machine and this is the mass of the machine. So this is k1 and this is m1 and from this mass, we are attaching the vibration absorber; damped vibration absorber. So we have this spring and then this damper and then this is the mass of the absorber, so it is m2, this is k2 and this is c2.

And here we are applying some load on the main mass that is F0 sin omega t. And the displacement we are taking positive downsides, so it is x1 x2 here. So that, system is presented like this, okay now we will go to write the equations of motion applying the free body diagram and Newton's law. So we make the free body diagram for m1, so here we have m1.

Now when we are pulling, so this is the direction, we are pulling it will apply, outward; upward load k1 x1. Now here if you assume x2 > x1, we will have this k2 x2 - x1 and then due to damper, it is c2 x2 dot - x1 dot. So this is the spring force due to k2 and damping force and then there is one F0, the applied force. Now when we make the diagram of m2.

So it will have the same c2 and k2, these 2 forces but in the upward direction. So here, we have shown the free body diagram of this 2 degree of freedom system where the vibration absorber contains some damping c2. Now to write the equations of motion, we have to apply the Newton's law that is sigma F = m x double dot, so for the first mass; the main mass, we have m1 x1 double dot = so the force is; that is - k1 x1 + k2 x2 - x1 + c2 x2 dot - x1 dot and then + F0 sin omega t.

Now we can adjust these terms, so we can write m1 x1 double dot + k1; so we take the other side, so k1 + k2 x1 + c2 x1 dot - k2 x2 - c2 x2 dot that is equal to F0 sin omega t. So this is; let us say equation number one. So now we write for the second mass the equation motion, so for m2, we will write m2 x2 double dot = the force is; they are upward, so they will be opposite to the x2 double dot, so there will be -, so; - k2 x2 -x1-c2 x2 dot - x1 dot.

So now we can write, m2 x2 double dot +; so here we have k2 x2 + c2 x2 dot = so we can have here, - k2 x1 and this is - c2 x1dot that is equal to 0, this is equation number 2. So we have obtained the equations of motion for the both masses. How we have to solve this equation? So we assume some solution, so let us say that x1 = x1 e power i omega t in the steady state solution.

So these are the steady state solution, so x1 = x1 and x2 = x2 e power i omega t. So because here, frequency is omega and amplitude with x1and here for x2, it is amplitude is capital X2 and frequency of the same omega. Because they are in the steady state, so they will vibrate with the same frequency as the applied force. So now we have; if we write x1 dot that is equal to; so we can differentiate it, so i omega into x1 e i omega t, so that is i omega into x1 and similarly here, x2 dot = i omega x2 e power i omega t.

Because the differentiation of e theta is e theta and this is these term the I omega. So now this is equal to i omega x2. So a small x is the motion that is function of time here and this is function of time and x2 is the amplitude. So here we are writing. Now x1 double dot that is; if we differentiate these again, so it will be i omega whole square into x1 e power i omega t. So this we can write; I square is - 1, because I is imaginary these number under root - 1.

So we can write it - so, - omega square and x1 e i omega t is again x1. So we can write here x1 and similarly here x2 double dot = i omega square into x2 e power i omega t and so we can write - omega square and x2. Now we have x1 x1 dot and x1 double dot, now we can put these values of x1, x1 dot and x1 double dot, in these 2 equations and we can find the x1 and x2. So now if we put in these equations, so let us first, we put here in equation number 2.

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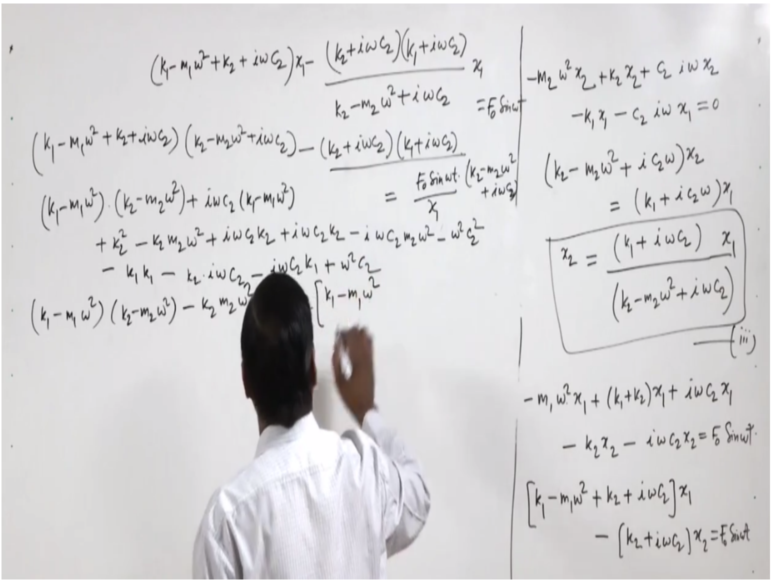

So we will have here m2 x2 double dot is - omega square x2, so it is - omega square x2 + k2 x2, so + k2 x2, + c2 x2 dot, so c2 x2 dot we have c2 into; x2 dot is i omega x2; i omega x2 and here - k1 x1 and - c2 x1 dot; x1 dot is i omega x1, so - c2 i omega x1 that is equal to 0. So we can write here k2 - m2 omega square + I c2 omega x2 that is equal to k1 + I c2 omega x1.

Now we can express x2 in terms of x1, so x2 = k1 + i omega c2 upon k2 - m2 omega square + i omega c2 x1, so this equation number 3. So we find x2 in terms of x1. Now we put the same these values x1 dot; x1 double dot in equation number one, so we can have; so m1 x1 double dot it will be - m1 omega square, because x1 double dot is - omega square x1+ k2 + k2 x1 +; so here, i omega c2 x1 and - k2 x2 - i omega c2 x2 = F0 sin omega t.

Okay, so we have written this. Now we have to write the equation in terms of x1 and x2, so we can have k1 - m1 omega square + k2; so k1 - m1 omega square + k2 + i omega c2 and this is x1 - k2 + i omega c2 x2 = F0 sin omega t. So we can have; now we can put x2 from here, so it will this equation will come in terms of x1, so here we have k1 - m1 omega square +k2 + i omega c2 x1 - k2 + i omega c2.

Now x2 we can put these terms, so it is k1 + i omega c2 upon this term, so it is k2 - m2 omega square + i omega c2 into x1 that is equal to F0 sin omega t. So now we can multiply these terms. So we multiply here these term, so we will get, so k1 - m1 omega square + k2 + i omega c2, I will be multiplied this term, so it is k2 - m2 omega square + i omega c2, -; so x1 we can take out.

So this is k2 + i omega c2 into k1 + i omega c2 that is equal to; so F0 sin omega t into these term will go here, so we have k2- m2 omega square + i omega c2, so this term will multiply here by x1; x1 we can bring in the denominator later we will take other side. Now we multiply it, so we multiply k1 - m1 omega square into k2 - m2 omega square. So we will have k1 - m1 omega square and k2 - m2 omega square + i omega c2 into k1 - m1 omega square.

So we will multiply k1 - omega square, with these and these; now we have to multiply these terms with these terms, so we multiply with k2, so it is +, k2 is square - k2 m2 omega square + i omega c2 k2, now we multiply with this, so it is + i omega c2 k2 - i omega c2 m2 omega square or and here + I square omega square c2 square, so I square is - 1; I square is - 1, so here we can write - omega square c2 square.

So this is only the left hand side term. Now we can open this one, so we can open this. So i omega c2 k1 - I omega; so we can collect the i omega c2 terms here, so here k1 - m1 omega square k2 - m2 omega square + so here we have - k2 m2 omega square + i omega c2 terms, so i omega c2 terms here is k1 - m1 omega square. Now we have this terms also, so we have; we can write here -, so here k2 k1 and - k2 m2 i omega c2 -.

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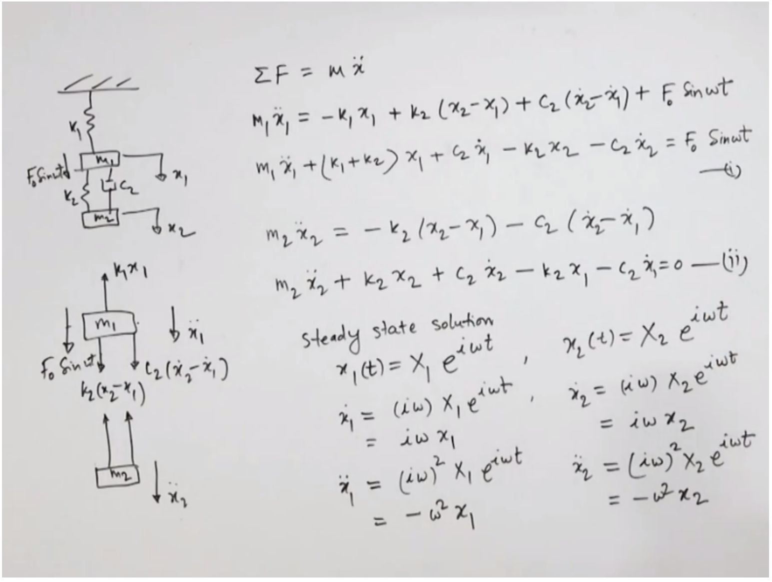

So here - k2 k1 and -; then here - i omega c2 k1 and then it is +; - I square so it is + omega square c2. So we can represent that system that in these forms, so we have this machine m1 that is supported with some spring k1 and we can have the vibration absorber that is k2, c2 and m2. Now we have to write the equation of motion and so we have to make the free body diagram.

And therefore we make the free body diagram, so here we have m1 and we assume this direction positive, so we have; if we pull it downward so it will apply a force k1 x1 then here there is a spring force; if we assume x2 > x1, so it is k2 x2 - x1 and here the damping force, c2 x2 dot - x1 dot and there is also one force F0 sin omega t, so the external source that is what.

Now for m2, we can have the same forces but in the opposite direction, the k2 due to the spring k2 and damper c2 and this is the direction of x2 double dot. So we have made the free body diagram, now we can write the equation of motion, so we can have sigma F = mx double dot, so for the first mass, we can write m1 x1 double dot =; we have the forces - k1 x1, because this is opposite to the x1 double dot.

Now we have + k2 x2 - x1 + c2 x2 dot - x1 dot and then + F0 sin omega t, the external force. Now we can readjust the term, so m1 x1 double dot and we have + k1 + k2 x1 and then + c2 x1 dot, then - k2 x2 - c2 x2 dot = F0 sin omega t. So this is equation number one. So we have this equation of motion for this first mass. Now the second mass m2 that is the absorber mass, we can write the equation of motion.

So for this, we have m2 x2 double dot = the forces they are opposite, so - k2 x2 -x1 - c2 x2 dot - x1 dot. So we can adjust these terms, so m2 x2 double dot + k2 x2 + c2 x2 dot - k2 x1 - c2 x1 dot =0, so this is equation number 2. So now we need to solve these 2 equations to find the values of x1 and x2, so we assume some solution. So the steady state; steady state solution, we assume that x1 t = x1 e power i omega t and x2 t = x2 e power i omega t.

So we assume that the solution will have the amplitude capital X1 and frequency omega and the second mass will have the amplitude X2 and frequency omega, now we find x1 dot, so we differentiate these with respect to time. So we will get i omega x1 e power i omega t, so i omega and x1 e power i omega t is, this x1, so x1. Similarly, here x2 dot = i omega x2 e power i omega t and so we; i omega and x2 e power i omega t is x2, so we can write x2.

Now to find x1 double dot, so x1 double dot =; we have; we can differentiate x1 dot, so if it is differentiate this again, this is i omega square and x1 e power i omega t, so this will be I square is - 1, so it is - omega square and x1 e i omega t is x1, so we can write x1 and here x2 double dot = i omega square x2 e power i omega t, so that is - omega square x2 omega t is x2, so x2.

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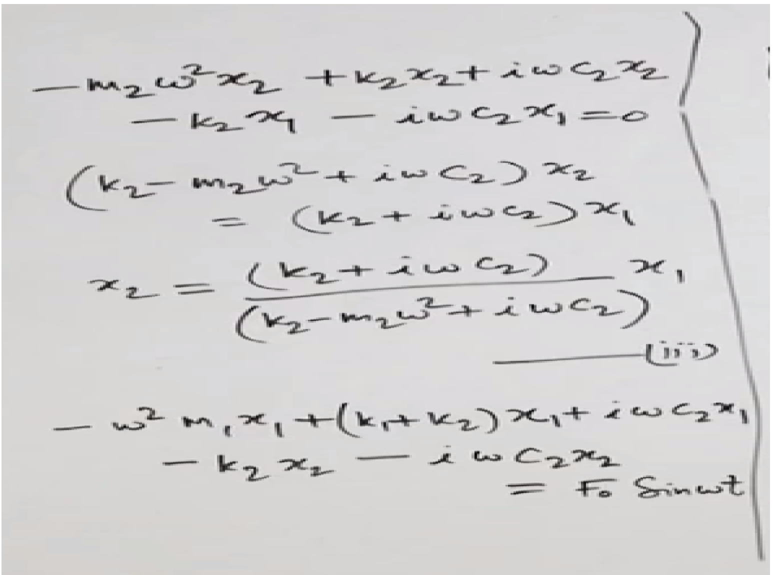

Now we can put these values of x1, x1 dot, x1 double dot and x2 x2 dot and x2 double dot in equations 1 and 2 and we can find; we can solve for x1 and x2. So if we put; so let us put it, so here, so now we put the values in eqaution2, so here we will have m2 x2 double dot, so x2 double dot is - omega square x2 so m2 - omega square x2 + k2 x2 + c2 x2 dot; x2 dot is i omega x2, so i omega c2 x2 then - k2 x1.

So, x1 is x1 and then - c2, so x1 dot is i omega x1. So we will have i omega c2 x1, so i omega x1, so we will - i omega c2 x1, that is equal to 0. So we can write this equation as k2 - m2 omega square + i omega c2 x2 = k2 + i omega c2 x1. So we can write x2 = k2 + i omega c2 upon k2 - m2 omega square + i omega c2 x1 that is equation number 3. So we have got the expression of x2 in terms of x1.

Now we put these values of x1, x1 dot and x1 double dot, x2, x2 dot and these in equation number 1. So we will have here -, so m1 x1 double dot, so x1 double dot is - omega square x1, so - omega square m1 x1 + k1 + k2 x1 + c2 x1 dot; x1 dot is i omega x1 so it is + c2, so it is i omega c2 x1 and then - k2 x2, so it is - k2 x2 and - c2 x2 dot, x2 dot is i omega x2.

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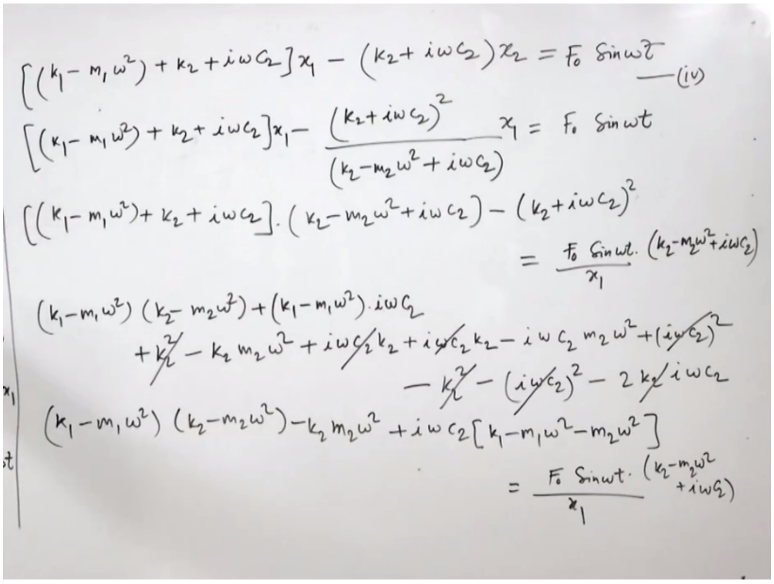

So it is - i omega c2 x2 and that is equal to F0 sin omega t. So now we can rewrite these terms, so we can write k1 - m1 omega square x1, so this is k1 - m1 omega square + k2 + i omega c2 and this is x1 and then - k2 + i omega c2 x2 that is equal to F0 sin omega t. So this is equation number 4. Now we can solve these 2 equations because they are the unknown x1 and x2, so we can solve them.

So we put the value of x2 in terms of x1 in equation 4, so here k1 - m1 omega square + k2 + i omega c2 x1 -; here k2 + i omega c2 into x2, so if we put these values so k2 + i omega c2 whole square because here we have this terms, so it will be multiplied and denominated term k2 - m2 omega square + i omega c2 into x1 = F0 sin omega t. Now we multiply this denominator term here and here and here.

So we will have; we will have k1 - m1 omega square + k2 + i omega c2 into k2 - m2 omega square + i omega c2 - k2 + i omega c2 whole square and this is equal to F0 sin omega t *; so this term will multiply here, so k2 - m2 omega square + i omega c2 and we can take for moment x1 here, so that we can solve this left side and then we can write x1 in terms of these parameters.

So we multiply these, so we will have k1 - m1 omega square into k2 – m2 omega square + k1 - m1 omega square into i omega c2. So we will multiply this, with these term and this term. Now we multiply these terms to these terms, so + k2 we multiply, k2 square - k2 m2 omega square + i omega c2 k2 and here i omega c2 will multiply then all term, so I omega; so k2 square, - k2 m2 omega square + i omega c2 k2.

Now we will multiply these term to all, so this is + i omega c2 k2 - i omega c2 m2 omega square and then i omega c2 whole square, so this is + i omega c2 whole square. Because this is multiply with this term, then -, so - this square, so it is k2 square - i omega c2 whole square and - 2 k2 i omega c2. So this is just left hand side terms.

Now we can see k2 square here + and here - they will cancel out. Here i omega c2 k2 and i omega c2 k2, there are 2 i omega c2 k2 and here it is 2 i omega c2 k2 in -, so these 2 terms will cancel out with this term. Moreover, i omega c2 whole square + and i omega c2 whole square -, they will also cancel out, so now we can rewrite these, so k1 - m1 omega square k2 - m2 omega square - k2 m2 omega square + i omega c2.

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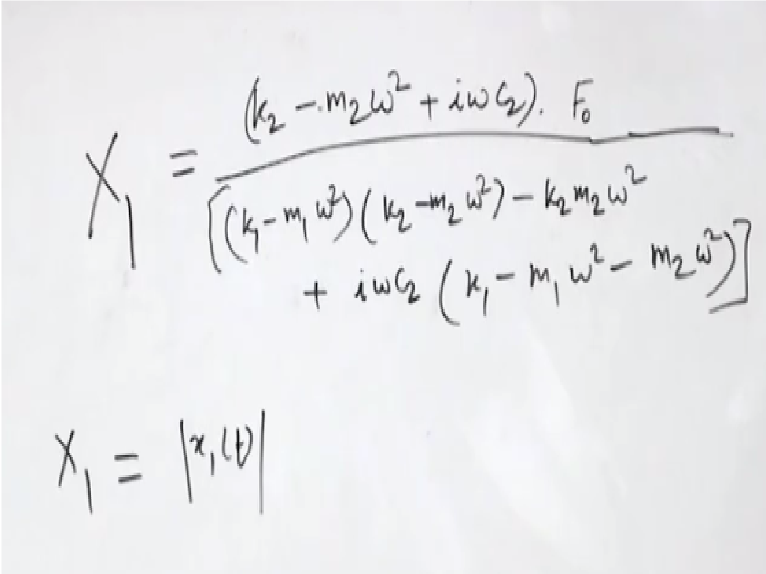

So i omega c2, we have k1 - m1 omega square and then m2 omega square, - m2 omega square because we can take this out. So this is the left hand side and that is equal to F0 sin omega t into k2 - m2 omega square + i omega c2 by x1. So now we can write x1, so we can write here. So from here we can write x1, so x1 t =; so we can write x1 = k2 - m2 omega square + i omega c2 into F0 sin omega t by this term.

So this is k1 - m1 omega square into k2 - m2 omega square - k2 m2 omega square + i omega c2 k1 - m1 omega square - m2 omega square. So we have got x1. Okay so we have got x1 and from this equation 3, we can get x2. Now the amplitude x1 equal to; we have the mod of this; so that is we have only these terms, so we have X1 equal to this term, okay this is the amplitude X1. So we have obtained the amplitude X1 equal to this expression.

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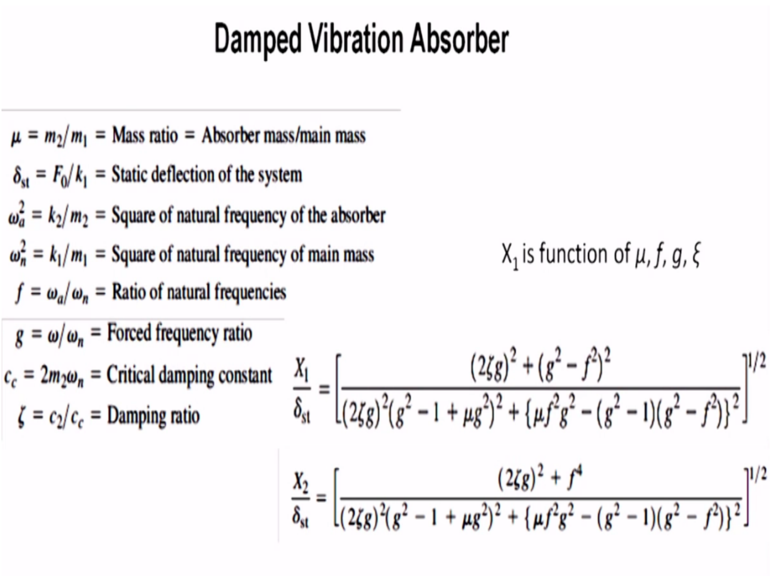

Now we can define some parameters, so these parameters we defined like mass ratio m2/m1 and delta st and omega a square and omega n square, f, g and cc and zeta and we can express these equations in this terms, so we can write these the same function of x1 in terms of these parameters that is mu is the mass ratio, m2/m1, m2 is the mass of absorber and m1 is the mass of the main system.

Then the static deflection that is F0/k1 and omega a; the absorber natural frequency that is k2/ m2, omega n is the main functions in natural frequency that is root k1/m1. F is the ratio of natural frequency of absorber and the main system, this is the forced frequency by main system frequency, cc is the critical damping constant 2 m2 omega n and zeta is c2/cc. So we can write these equations, x1/ delta st and x2 / delta st.

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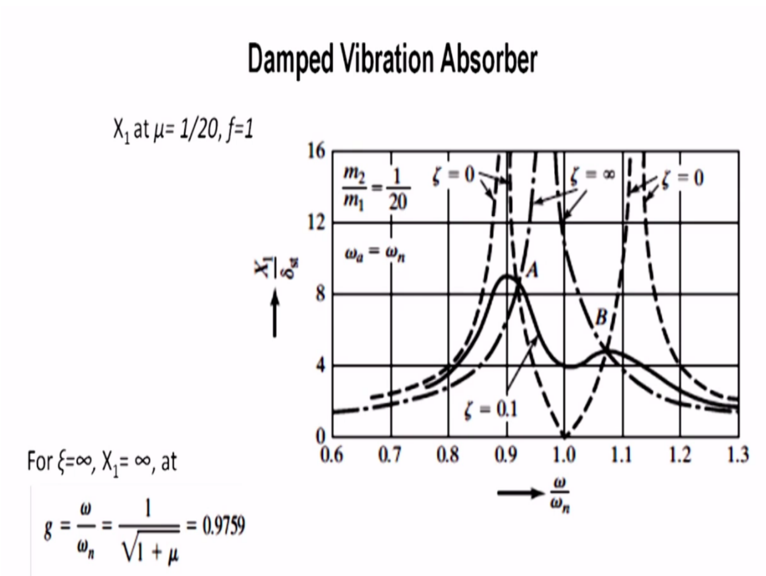

And here we can; for some given mass ratio 1/ 20 and f =1, we can see that for zeta =; so this is this equation is plotted for zeta = infinite and zeta =0, so we see that the; for zeta = infinite, x1 is again infinite and corresponding g is 0.9759. Okay, so it means the; for 0 damping, the x1 is infinite. Because without 0 damping means there is no any damping in the system and we are getting these natural resonance frequencies.

And when we put the very high damping that is zeta = infinite, we are getting shift in the natural frequency and again the x1 is very high. For other frequencies between 0 and infinite, we will get some minimum amplitude so this damped absorber will for certain values of damping between 0 and infinite, will give some finite values of the amplitude of x1. For example, here for zeta= 0.1, it is plotted.

And we can see that it is giving some reduction in the amplitude of x1. Okay. So we see that the damped vibration absorber for certain value of damping, it can be reduced amplitude from infinite to some finite value and so I stop here and we will discuss more in the next lecture. See you in the next lecture. Thank you.