

Introduction to Mechanical Vibration
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Lecture - 33
Numerical Problems

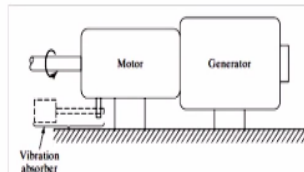
So, welcome to the Numerical Problems that we will solve regarding the vibration absorber theory that we discussed. So, we discussed about the vibration absorber and that this is a dynamic system that when attached to the main system which is vibrating under some excitation force. It can bring the main system to rest. So, we will have one numerical example.

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Numerical Example

A motor-generator set is designed to operate in the speed range of 2000 to 4000 rpm. However, the set is found to vibrate violently at a speed of 3000 rpm due to a slight unbalance in the rotor. It is proposed to attach a cantilever mounted lumped-mass absorber system to eliminate the problem. When a cantilever carrying a trial mass of 2 kg tuned to 3000 rpm is attached to the set, the resulting natural frequencies of the system are found to be 2500 rpm and 3500 rpm.

Design the absorber to be attached (by specifying its mass and stiffness) so that the natural frequencies of the total system fall outside the operating-speed range of the motor-generator set.



So, here we see that there is motor generator set and there is some unbalance in the motor and they have some vibrations due to those unbalanced forces and therefore we can attach some vibration absorber here. So, that the vibration of the main system that is motor generator set can be brought to the vibration observer. So, it is said that the motor generator set is designed to operate in the speed range is 2000 to 4000 rpm.

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speed range 2000 - 4000 rpm
 $\omega_1 = 3000 \text{ rpm}$
 $\omega_1 = \omega_2 = 3000 \text{ rpm}$
 $m_2 = 2 \text{ kg}$
 $\omega_1 = 2500 \text{ rpm}$
 $\omega_2 = 3500 \text{ rpm}$
 $\omega_1 = \omega_2 = 3000 \text{ rpm} = 3000 \times \frac{2\pi}{60} = 314.16 \text{ rad/s}$
 $\omega_1 = 2500 \text{ rpm} = 2500 \times \frac{2\pi}{60} = 261.75 \text{ rad/s}$
 $\omega_2 = 3500 \text{ rpm} = 3500 \times \frac{2\pi}{60} = 366.45 \text{ rad/s}$
 $\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{4}\right)\mu}$
 $\gamma^2 - 1 = \frac{\mu}{2} - \sqrt{\left(1 + \frac{\mu}{4}\right)\mu}$
 $(\gamma^2 - 1)^2 = \left(\frac{\mu}{2}\right)^2 + \left(1 + \frac{\mu}{4}\right)\mu - \mu \left[\frac{\mu}{4} + \left(1 + \frac{\mu}{4}\right) - \sqrt{\left(1 + \frac{\mu}{4}\right)\mu} \right]$
 $(\gamma^2 - 1)^2 = \mu \gamma^2$

So, its speed range is 2000 to 4000 rpm. However, the set is found to vibrate violently at speed of 3000 rpm due to a slight unbalance in the router. And it is proposed to attach a cantilever mounted lumped mass absorber system to eliminate the problem. So, at 3000 rpm the system is vibrating violently. It means this is the natural frequency of this system that is ω_1 and to reduce this vibration when there is some trial it is found that 2 kg mass can be tuned to 3000 rpm.

So, tune means it is tuned absorber so tuned absorber $\omega_1 = \omega_2$ and mass m_2 is 2 kg and it is tuned to 3000 rpm. The resulting natural frequencies of the system are found to be 2500 rpm and 3500 rpm. So, when we attach this 2 kg tuned mass absorber. The resonance frequencies are found to be 2500 rpm. So, $\omega_1 = 2500 \text{ rpm}$ and $\omega_2 = 3500 \text{ rpm}$. Now we have to design the absorber so that means we have to find the mass and stiffness.

So, that the natural frequencies of the total system fall outside the operating speed range. So, we have the operating speed range this but my system is designed for this range and this frequency. So, this is the natural frequency of the system. But we have to shift the range from 2500 to 3500 to 4000. So, we can now have so we have $\omega_1 = \omega_2$ that is 3000 rpm = 3000 into 2π by 60. So, we convert into radian per second and $\omega_1 = 2500 \text{ rpm}$.

We convert 2500 into 25 pi by 16. And $\omega_2 = 3500 \text{ rpm}$ so 3500 into 2 phi by 60. So, we can

convert them into radian per second. So, it is 314.16 radian per seconds. Now, we know that the frequencies ω_1 by ω_2 square that is equal to $1 + \mu/2 \pm \sqrt{1 + \mu/4}$ into μ . So, this is the resonance frequency and we can solve this so let us take first the ω_1 by ω_2 and we assume it as r_1 the ratio of these two frequencies.

So, r_1^2 square that is equal to $1 + \mu/2 - \sqrt{1 + \mu/4}$ into μ . So, we first take the lower frequency so $1 + \mu/4$ into μ so we can write $r_1^2 - 1 = \mu/2 - \sqrt{1 + \mu/4}$ into μ . So, $r_1^2 - 1$ whole square equal to, so we can do whole square here so $\mu/2$ whole square. So if we do whole square so $\mu/2$ whole $+ 1 + \mu/4$ into $\mu - 2$ into $\mu/2$ under root $1 + \mu/4$ into μ .

So, here we can write $\mu^2/4 +$ so we can take μ as common. So, we have $\mu/4$ because we take one μ outside so $\mu/4 + 1 + \mu/4$ --- So, this cancel out μ is outside so we have under root $1 + \mu/4$ into μ . So, we can have $\mu/4$ and $\mu/4$ and $\mu/4$ it is $\mu/2$. So, μ into $1 + \mu/2 - \sqrt{1 + \mu/4}$ into μ and this term is nothing but r_1^2 square so we can write μ into r_1^2 square.

Because here we have r_1^2 square for the lower frequency we are working so $1 + \mu/2 - \sqrt{1 + \mu/4}$ into μ . So, we see that if we solve this we find that μ into r_1 . So, now we have separated μ from this expression so we put in terms of r_1 because we know the r_1 . r_1 is ω_1 by ω_2 and ω_1 here we have this value 2500 rpm and ω_2 is 3000 rpm and we can find the μ . So, we finally find this $r_1^2 - 1$ whole square. So, $r_1^2 - 1$.

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speed range 2000-4000 rpm

$\omega_1 = 3000 \text{ rpm}$
 $\omega_1 = \omega_2 = 3000 \text{ rpm}$
 $m_2 = 2 \text{ kg} \checkmark$

$\omega_1 = 2500 \text{ rpm}$
 $\omega_2 = 3500 \text{ rpm}$

$\omega_1 = \omega_2 = 3000 \text{ rpm} = 3000 \times \frac{2\pi}{60} = 314.16 \text{ rad/s}$
 $\omega_1 = 2500 \text{ rpm} = 2500 \times \frac{2\pi}{60} = 261.75 \text{ rad/s} \checkmark$
 $\omega_2 = 3500 \text{ rpm} = 3500 \times \frac{2\pi}{60} = 366.45 \text{ rad/s}$

$\frac{\omega_1}{\omega_2} = \frac{261.75}{314.16} = 0.833$

$\left(\frac{\omega_1}{\omega_2}\right)^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{4}\right)\mu}$
 $\mu = \frac{(\gamma^2 - 1)^2}{\gamma^2} = \frac{[0.833^2 - 1]^2}{(0.833)^2} = 0.135$

$\frac{m_2}{m_1} = 0.135$
 $m_1 = \frac{m_2}{0.135} = \frac{2}{0.135} = 14.81 \text{ kg}$

$\gamma = \frac{\omega_1}{\omega_2} = \frac{2000}{3000} = 0.667$
 $\mu = \frac{[0.667^2 - 1]^2}{0.667^2} = 0.6926 \checkmark$

So, we can write $\mu = r_1^2 - 1$ whole square by r_1^2 square. So, we now calculate the mass ratio μ for the system. So, mass ratio we can calculate. So, what is r_1 first we calculate r_1 .

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speed range 2000-4000 rpm

$\omega_1 = 3000 \text{ rpm}$
 $\omega_1 = \omega_2 = 3000 \text{ rpm}$
 $m_2 = 2 \text{ kg} \checkmark$

$\omega_1 = 2500 \text{ rpm}$
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$\omega_1 = \omega_2 = 3000 \text{ rpm} = 3000 \times \frac{2\pi}{60} = 314.16 \text{ rad/s}$
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$\gamma = \frac{\omega_1}{\omega_2} = \frac{261.75}{314.16} = 0.833$

$\left(\frac{\omega_1}{\omega_2}\right)^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{4}\right)\mu}$
 $\mu = \frac{(\gamma^2 - 1)^2}{\gamma^2} = \frac{[0.833^2 - 1]^2}{(0.833)^2} = 0.135$

$\frac{m_2}{m_1} = 0.135$
 $m_1 = \frac{m_2}{0.135} = \frac{2}{0.135} = 14.81 \text{ kg}$

$\gamma = \frac{\omega_1}{\omega_2} = \frac{2000}{3000} = 0.667$
 $\mu = \frac{[0.667^2 - 1]^2}{0.667^2} = 0.6926 \checkmark$

So, $r_1 = \omega_1$ by ω_2 and ω_1 is 261.75 and ω_2 is 3000 rpm that is 314.16 and so if we calculate it is 261.75 divided by 314.16 and we get it as 0.833. So, we get r_1 now we put here we will get μ . So, $0.833^2 - 1$ whole square by 0.833^2 square. So, it is 0.135. So, this is the mass ratio that is 13.5 percent. So, the mass of the absorber 13.5 percent of the mass of the main system.

So, it is m_2 by $m_1 = 0.135$. So, $m_1 = m_2$ by 0.135 and what is m_2 , m_2 is 2 kg. So, we put 2 by

0.135 so it is 14.81 kg. So, the mass of the main system that is motor generator set that is 14.81 kg. Now, we have to shift this omega 1 to 2000 rpm. So, we want to know what will be the mu in this case so for $r_1 = \omega_1$ by ω_2 . So, omega 1 is 2000 and omega 2 is 3000 rpm.

So, we have this is 0.667 this r_1 if you put r_1 we can know what is the mu that is required. So, mu is $0.667^2 - 1$ whole square by 0.667^2 . So, this is 0.6926 means now we need 69 percent of the mass of the vibration observer. So, for this value of mu what will be the omega 2 so if we use this mass ratio what will be the omega 2 and so we can find omega 2. So, omega 2

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Handwritten notes on the whiteboard:

- Speed range 2000 - 4000 rpm
- $\omega_1 = 3000 \text{ rpm}$
- $\omega_1 = \omega_2 = 3000 \text{ rpm}$
- $m_1 = 2 \text{ kg} \checkmark$
- $\omega_1 = 2500 \text{ rpm}$
- $\omega_2 = 3500 \text{ rpm}$
- $\mu = 0.6926$
- $\frac{m_2}{m_1} = 0.6926$
- $\sqrt{m_2} = 10.26 \text{ kg}$
- $k_2 = m_2 \omega_2^2 = 10.26 \times (314.16)^2 = 1.01 \times 10^6 \text{ N/m}$
- $\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(1 + \frac{\mu}{2}\right) \mp \sqrt{\left(1 + \frac{\mu}{4}\right)^2}$
- $\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(1 + \frac{\mu}{2}\right) + \sqrt{\left(1 + \frac{\mu}{4}\right)^2}$
- $= 1 + \frac{0.6926}{2} + \sqrt{\left(1 + \frac{0.6926}{4}\right)^2}$
- $= 2.248 \Rightarrow \frac{\omega_2}{\omega_1} = 1.5$
- $\omega_2 = 1.5 \times 3000 = 4500 \text{ rpm}$
- $\gamma_1 = \frac{\omega_1}{\omega_2} = \frac{2000}{3000} = 0.667$
- $\mu = \frac{0.667^2 - 1}{0.667^2} = 0.6926 \checkmark$
- $m_1 = 14.81$

By omega 2 square = $1 + \mu$ by 2 + under root $1 + \mu$ by 4 into mu. So, we take + sign for the higher frequency so omega 2, we put $1 + \mu$, mu is .6926 by 2 + under root $1 + .6926$ by 4 into 0.6926. So, we can solve it. So, that is 2.248. So, we can find omega 2 by omega 2 = under root of this quantity. So, it is 1.5 and so therefor we find omega 2 = 1.5 into omega 2 and omega 2 is 3000. So, that is 4500 rpm. So, the higher resonance frequency is coming to 4500 rpm.

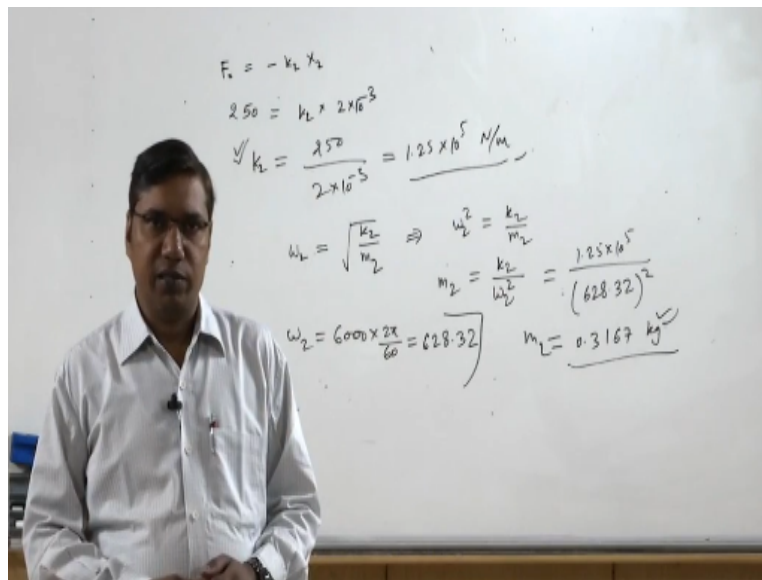
So, we are already away far from the required range 4000 rpm. So, this is a satisfactory acceptable solution. So, absorber we select $\mu = 0.6926$. We decide $\mu = 0.6926$ and m_2 by $m_1 = 0.6926$ and m_1 we have already calculated, m_1 was 14.81. So, $m_2 =$ so it is 10.26 kg. Now, $k_2 = m_2$ into omega 2 square. So, we have 10.26 into so omega 2 was 3000 rpm and it was 314.16 square. So, we find that it is 1.01 into 10 to power 6 Newton per meter.

So, we show that for the given problem we have selected a tuned mass damper, tuned absorber that is keeping the frequency range that is out of the 2000 to 4000 rpm and we find the m_2 and k_2 that is the design parameters of the vibration absorber. So, now we have another problem. So, there is a diesel engine that is supported on a mount and this engine is the operating speed is 6000 rpm.

Now, we have to determine the parameters of the vibration absorber that will reduce the vibration when mounted on the pedestal. The (()) (24:26) force is 215 Newton and the amplitude of motion of the auxiliary mass is to be limited to 2 mm. So, we have a diesel engine, weight is 3000 Newton and our operating speed is 6000 rpm. So, if we say determine the vibration absorber parameter.

So we have to find the m_2 and k_2 , the mass and stiffness of these parameters. So, here we can have this problem we can solve. So, we know that here

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The whiteboard contains the following handwritten equations:

$$F_0 = -k_2 x_2$$

$$250 = k_2 \times 2 \times 10^{-3}$$

$$\checkmark k_2 = \frac{250}{2 \times 10^{-3}} = 1.25 \times 10^5 \text{ N/m}$$

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} \Rightarrow \omega_2^2 = \frac{k_2}{m_2}$$

$$m_2 = \frac{k_2}{\omega_2^2} = \frac{1.25 \times 10^5}{(628.32)^2}$$

$$\omega_2 = 6000 \times \frac{2\pi}{60} = 628.32 \quad m_2 = 0.3167 \text{ kg}$$

the $F_0 = -k_2$ into x_2 . So, because here the force is 250 Newton the amplitude of the force that is 250 Newton and if we just take the magnitude so we will have k_2 into x_2 , x_2 is the motion of the amplitude of the axillary mass that is the mass of the vibration absorber. So, 2 mm so it is 2 into 10 per - 3 meter. So, we can find from here k_2 is 250 by 2 into 10 power - 3 and we find it. So,

this is 1.25×10^5 Newton per meter.

So, this is the stiffness parameter. Now, we have to find the mass and to find the mass as we know that we have $\omega^2 = k/m$. So, it means that $\omega^2 = k/m$. So, we can find $m = k/\omega^2$. Now, ω is the frequency that is the operating speed 6000 rpm. So, $\omega = 6000 \times 2\pi/60$. So, that is 628.32. So, we have k that is 1.25×10^5 and ω , 628.32 square.

So, that is 0.3167 kg. So, the mass of the absorber is 132 kg and so we find the design parameter of the vibration absorber. So, I stop here and I thank you for attending that lecture. See you in the next lecture.