Introduction to Mechanical Vibration
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Lecture - 32
Tuned Absorber

So welcome to the lecture on Tuned Absorber. So, we discussed the dynamic the theory of dynamic vibration absorber and we found that if there is a main system and it is subjected to some external excitation, harmonic excitation that is f zero sign omega t. And we attach another system whose frequency is equal to the excitation frequency. The vibration response of the main system will be zero. So, the main system will come at rest. So, we had this system discussed.

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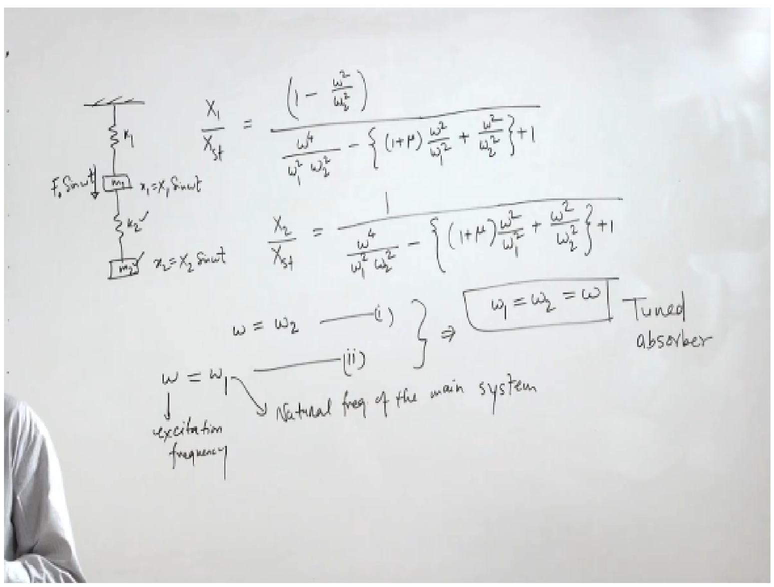

So, it has this k1, m1, k2, m2. So, this is the main system and this k2, m2 is the absorber system and this system is subjected to some harmonic load with frequency omega. And we found that x1 by Xst = 1 - omega square by omega 2 square upon omega 4, omega 1 square -. So, this is the response of the mass this x1.

So, that is x1 = x1 sign omega t and this is x2 = x2 sign omega t. So, the amplitude of the response and similarly x2 by Xst is 1 by omega 4 by omega 1 square omega 2 square 1 + mu omega square by omega 1 square + omega square by omega 2 square + 1. So, this is the response that we found and then we said that if omega = omega 2 the x1 equal to, this x1 is zero. So, it means that the main system will have no vibration it will be at rest.

So, we will have omega 2 is root k by m. So, if we select this k to an m2 such that the under root of k2 m2 = omega then the x1 is zero. Now, in this case what happens to x2 what is the response of x2? So, in this case x2 by Xst = 1 by so here if we put equal to omega 2 so it will be omega 2 square by so omega 2 square by omega 1 square - 1 + mu omega 2 square by omega 1 square + 1 + 1. So, because here omega = omega 2.

So, here we can write so omega 2 square by omega 1 square and - here is omega 2 square by omega n square that will cancel out and this is + 1 and this is – 1, this will cancel out. So, we will find mu omega 2 square by omega 1 square. So, here we have mu, mu is m2 by m1 and omega 2 square is k2 by m2 and omega 1 square is k1 by m1 so we can write m1 by k1. So, here m2 cancel out, m1 cancel out so we can write x2 = Xst into k1 by k2 and here is the - sign.

So, we will have - X st, the zero frequency deflection of the main mass is subjected to the f zero load. So, Xst is F0 by k1 into k1 by k2. So, k1 is canceled out. So, we have - F0 by k2. So, from here we write that F0 = - k2, x2. So, from this equation we see that x2 will not have the zero response but x2 will have some finite. So, the second mass will have some finite vibration and that will be equal to F0 by k2.

So, F0 is the amplitude of the force and divided by the stiffness of this absorber spring. Then one thing we can notice that there is negative sign so x2 will have out of phase response. So, what we see that if we have x2, x2 will be such that it is going to transfer this force F0 to m2 through this spring and this mass will vibrate so it will take the vibration of the main system. Now, we will discuss the tuned when we call it tuned absorber.

So, omega = omega 2, we have x1 = 0. Now, when do we need the absorber? We know that any systems response is extremely high when there is resonance condition means when the excitation force frequency, coincides or it is near to the natural frequency of the system then the vibration response amplitude is quite high. Therefore, if we want to design an effective vibration absorber we should definitely design for that frequency and therefore we have to keep the omega = omega 1.

So, the excitation frequency, this is excitation frequency and this is the natural frequency of the main system. So, when omega = omega 1 this is the resonance condition. Because in this condition the response of the system will be maximum. Therefore, we have to design the absorber for this frequency when omega = omega 1. But we know that to have zero vibration for the main system omega 2 = omega.

So, it means that from this is our condition number 1 and this is our condition number 2. So, from this two conditions we find that omega 1 = omega 2 = omega and this we call as the tuned vibration absorber. Because this absorber is tuned for the particular frequency that is the natural frequency of the main system. Now when we have omega 1 = omega 2 equal omega, then what will be the response? x1 and x2. So, we can write here.

So, when we write here omega. So, we write omega 1 = omega 2.

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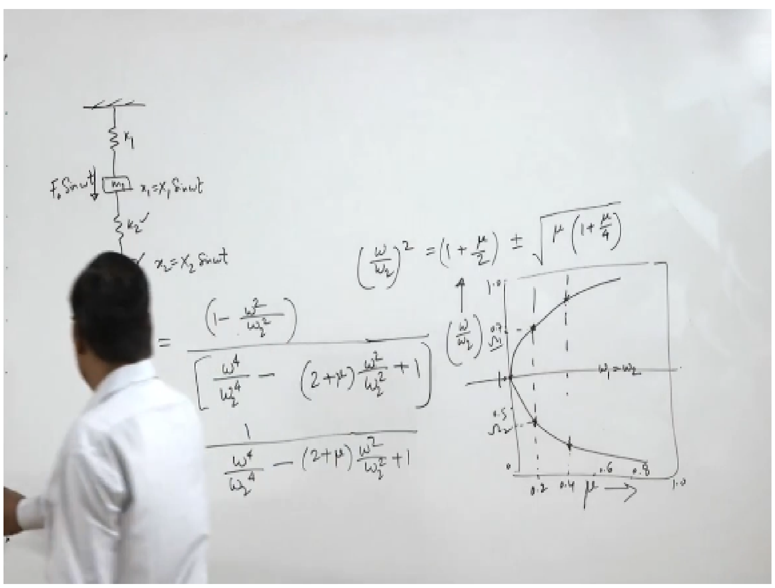

So, we will have x1 by Xst = 1 - omega square by omega 2 square and this denominator we put omega 1 = omega 2. So, we will have omega 4 by omega 2 power 4 -. So, we will have this is omega 2 as omega 1 = omega 2. So, we will have 2 + mu omega 2, omega square by omega 2 square + 1. Similarly, x2 by x t that is equal to 1 upon omega 4 by omega 2 4 - 2 + mu omega square by omega 2 square + 1.

So, now our equations for the tuned absorber I have reduced in this form. Now, what is important again here we see that of course in this condition with x1 is at rest because omega = omega 2. Now, this denominator term this is nothing but the frequency equation and when we equate this equal to zero and solve for omega that will give us the resonant frequency of the system. Because this system having, the two degree of freedom it will now have two frequencies.

Because earlier they were two separate systems having the natural frequencies omega 1 and omega 2. But now these two systems are combined together and they are now the two degree of freedom system and a two degree of freedom system has the two natural frequencies. So, if we solve this equation we will find the two natural frequencies of the system. So, we have omega by omega 2 power 4 - 2 + mu omega square by omega 2 square + 1 = zero.

So, we have to solve this so we will get omega because it is quadratic in omega by omega 2 square. So, this is a quadratic equation in omega by omega 2 square. So, we will write - b so this is 2 + mu so square - 2 square so we will write mu + 4 into 2 + mu - 2 so into mu so we can rewrite. So, we can take it inside. So, that is mu by 4 + 1 into mu. So, we will have the two resonance frequencies, one for the - and one for this + sign and therefore we can see that we can have these frequencies.

So, for different values of mu we will get the two values of omega. So, that is omega = omega 1 and so first one omega then second one. So, for example if we have mu = 0.2 we are getting these frequencies and this frequency. We have mu = 0.4. We are getting these frequencies and this frequency. So, for different values of mu we will get this omega and we see that when we are increasing the value of mu the separation between the two frequencies is increasing.

So, if we have mu = 0, they are the same so they are omega 1 = omega 2 here they are the same frequencies. They coincide but once we are adding this we are increasing this mass. So what does it mean? It means that our earlier system was the main system was a single degree of freedom system then our vibration absorber system it was also a single degree of freedom system.

They have some frequencies their own natural frequencies omega 1 and omega 2 but now when they become the two degree of freedom system, now the resonance frequencies are changed and what they are and what will be their values that will be governed by the mu. Mu means the ratio of the mass of the absorber. So, it will be governed by m2 by m1. Now, one thing is clear that if we have the observer with lower mass what will happen?

It will have more vibrations the x2 will be more because we know that x2 = F0 by k2 and omega = root k2 by m2. So, if we select low mass we have to select this k2 smaller so that we can satisfy this condition omega 2 = omega and then we will have the x2 will be more if k2 is lower. If we have more mass we will need more k2 and although x2 will be less. But if add more mass to the system that is not desirable.

Because the vibration absorber is an auxiliary system that is attached to the main system and it may change the functionality of the main system. Therefor it is not desired to have a large mass however the m2 can be 5 to 25 percent. So, we can select mu value 0.05 to 0.25 so in this range. So, it is 5 percent to 25 percent. Let us take one example we have selected mu = 0.2. So we have mu = 0.2. So, if we select mu = 0.2 we will get some frequencies omega 1 and omega 2 here.

So, this is let us say some omega 1 and this is omega 2 so these are the two resonance frequencies.

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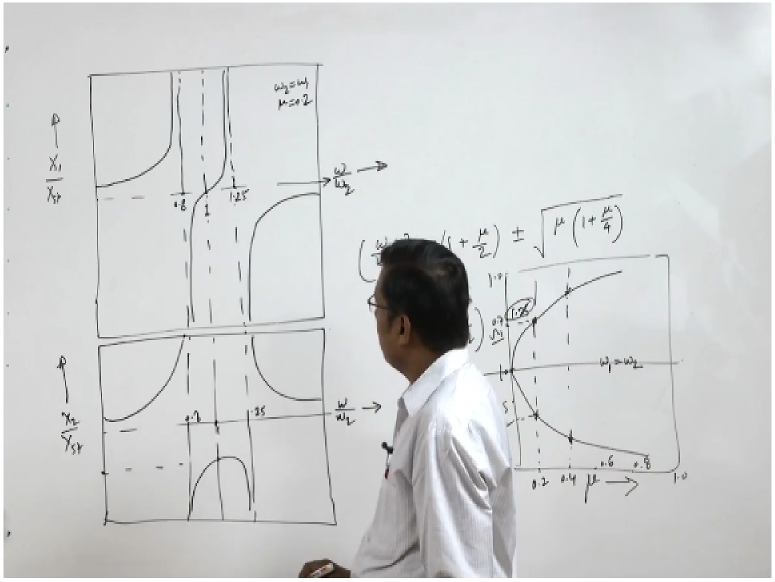

So when keep mu = 0.2 we will get omega by omega 2 = 0.8 and 1.25. So, this is 0.8 and this is 1.25 and that is here. And this is a tuned absorber so omega 2 = omega 1 in this case. So it means that this is omega by omega 1 = 1 so this is the resonance condition of the main mass. Now, when we have added a mass with mu = 0.2 here we have added a mass with mu = 0.2. So, we have the response of x1 like this.

So, what is going on the system the main system is here at rest and it is resonance frequency has shifted so now the vibration because now the systems natural frequencies it is two degree of freedom system and it is natural frequencies are now .8 and 1.25 and therefor the resonance condition is sifted to these frequencies and therefore we can see that the amplitude is maximum at these frequencies but not here.

So, therefore the system is protected the main system is protected. What happen to x2? So, this is the response of the absorber and we can see it is response and we see that the absorber response here is not zero it has some response here that is some x2 value. However, we see that from here to the left side the absorber response in phase with the response of the main system because this curve is going in the positive side and the same here negative, here negative.

But once we shift to the other side of this we see that this curve is negative side but here it is positive side and here it positive and this is negative. So, that is out of phase here. So, from here we understand that how the effect of the observer mass affect a response of the system and its own response. So, now we have tuned this system and if there is little variation of the frequencies the amplitude is lower.

But if it reaches to here this omega rises to the limits then again we have to retune this system. So, from here we show that the tuned absorber, in case of tuned absorber we have the excitation frequency we design it for the excitation frequency equal to the natural frequency of the system and for the absorber to the functional the omega is already to omega 2. So, the omega 2 = omega and we see that we can select some mass ratio that could be between 5 percent to 25 percent that is usually taken.

And based on these we can spread out the resonance frequencies and the system will work. The main system is safe for its own natural frequency. So we stop here and thank you for attending the lecture. See you in the next lecture.