FIntroduction to Mechanical Vibration
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Lecture – 26
Undamped Free Vibration

So welcome to the lecture on 2 degree of freedom systems. So today in this lecture, we will deal with undamped free vibration of 2 degree of freedom system. So we have already studied the single degree of freedom system and we defined that degree of freedom is the independent coordinates that a system can be defined in terms of those coordinates. So in case of single degree of freedom system, we had one coordinates.

But in 2 degree of freedom system it will be 2 coordinates.

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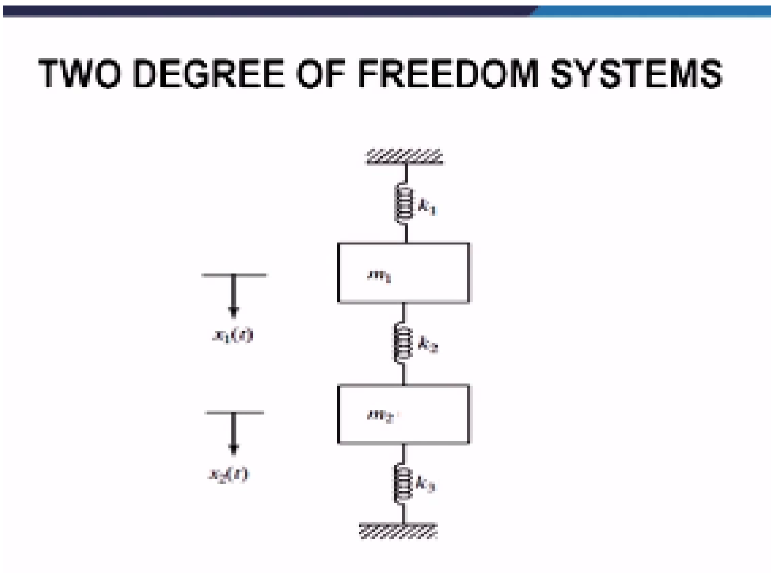

So here let us see we have the system, so this system there is 3 spring and 2 masses and each mass has one translational motion so it has x1 and x2, so 2 degree of freedom. So how to study the undamped free vibration of 2 degree of freedom system, so that we will study now.

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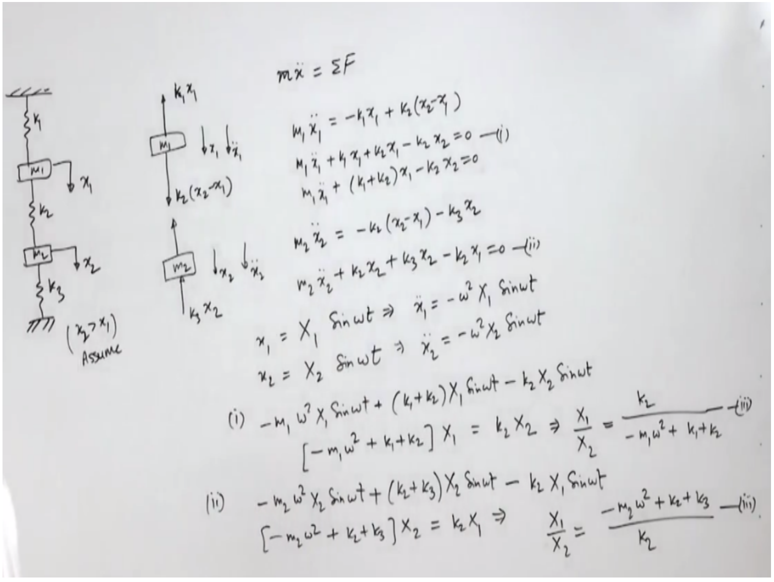

So this is our system, it contains 2 masses; m1 and m2 and they are connected with 3 springs; k1, k2 and k3. So each mass has one degree of freedom, so there are 2 masses, they will have 2 degree of freedom, so that is why we called it 2 degree of freedom system. Now how to study these as; we have already discussed that what is the vibrational analysis procedure that if we have any system, we have to isolate the masses.

And we have to show all the action force, reaction force or an inertia forces. And then we apply the Newton's second law to write the equations of motion and then we solve those equations. So here we will let us first take m1, so here we have m1 and this is the direction of x1, and this is x1 double dot we; at that instant. So here, we have; if we pull this mass with x1, so there will be a spring force that will work opposite to the direction of x1.

So we will have one spring force here that is k1 into x1. Now from a spring k2, there will again be a spring force working on this mass so what is the; that will force will depend on the extension or compression of the spring k2. Now a spring k2 has, from here it is getting compression x1, from here it is getting extension x2. So if we assume that x2 > x1.

So we assume that x2 > x1, so it means that there will be force net extension of the spring k2 by the mass m2, so the spring will apply a force in this direction that is k2 x2 - x1. Similarly, on this second mass, m2, here will be the same force is spring force but in an opposite direction on the mass m2 and it is here x2 and x2 double dot. Moreover, when mass x2 mass m2 is coming down, it is compressing k3.

So it will apply an upward spring force that is k3 x2. So this will be the free body diagram showing the forces on these 2 masses. Now we have to apply Newton's second law, so Newton's law says that sigma; so mx double dot equal to summation of the forces, so for first mass m1, x1 double dot, so the forces; so this force is opposite, so it is - k1 x1. So because its direction is opposite to direction of x1 double dot so - k1 x1.

And this is in the same direction so it is + k2 x2 - x1. Similarly, for so we can adjust this so m1 x1 double dot, so we bring the x1 terms here, so + k1 x1 and this is k2 x1 so + k2 x1 and - k2 x2, because here is k2 x2 and that is equal to 0. This is my equation number one. Now for the second mass, m2 x2 double dot equal to, so we have forces or both the forces opposite to x2 double dot.

So we will have - of k2 x2 - x1and - k3 x2. So we will have m2 x2 double dot and this is + k2 x2 and + k3 x2 and this is +; and so that side - k2 x1 = 0. So this is equation number 2. So now we are dealing with the undamped free vibration of the system. So now we are dealing with the undamped free vibration of the system. So the thing is; if we disturbed the system, it will vibrate with harmonic motions.

As we know that, if we have a single degree of freedom system without damping, so undamped system. If we disturbed it and we release it, it will vibrate with harmonic motions and the frequency of that motion is natural frequency. Now we have a system that has 2 masses and 2 degree of freedom, so we have to assume that the whole system will vibrate with one frequency at one time. So x1 = x1 sin omega t and x2 = x2 sin omega t.

So omega is the frequency of vibration so each this both masses will vibrate at the same frequency omega but the amplitude of vibration that is x1 and x2 that would be different. So we assume this assumption. Now we have to put this values in equation number 1 and 2. So from here, we will have x1 double dot equal to; so we differentiate 2 times here, so it is - omega square x1 sin omega t.

And here x2 double dot - omega square x2 sin omega t. Now we put these values here, so what we will get for this equation? Anyway we can write here, so for equation number 1, we get m1, so - m1 omega square x1 sin omega t + k1 into x1 sin omega t, so here we can write, m1 x1 double dot + k1 + k2, so there k1 + k2 x1 - k2 x2 and that is equal to 0. So here also we can write, k1+k2, these are k1+k2 into x1; at x1; so it is x1 sin omega t -k2 x2 sin omega t.

So now we can write these as; - m1 omega square + k1 + k2 x1 = k2 x2. So this implies that we can write x1/x2 = k2 upon - m1 omega square + k1 + k2. Let us say this is equation number 3, similarly for equation number 2, we put these values in equation number 2, so it is - m2 omega square into x2 sin omega t. Again here it is k2 + k3 into x2, so x2 is x2 sin omega t, so x2 sin omega t - k2 x1.

So - k2 x1, so - k2 and x1 is x1 sin omega t. So we can write here, -m2 omega square + k2+k3 x2 = k2 x1. So this implies that x1/x2= -m2 omega square +k2 + k3/ k2. Okay so if we see the equations number 3 and 4, so this is equation number 4, so we have got the amplitude ratio of the 2 masses that is x1/x2 and they look different because one each k2/ -m1 omega square + k1 + k2, other is -m2 omegas square + k2 + k3 /k2.

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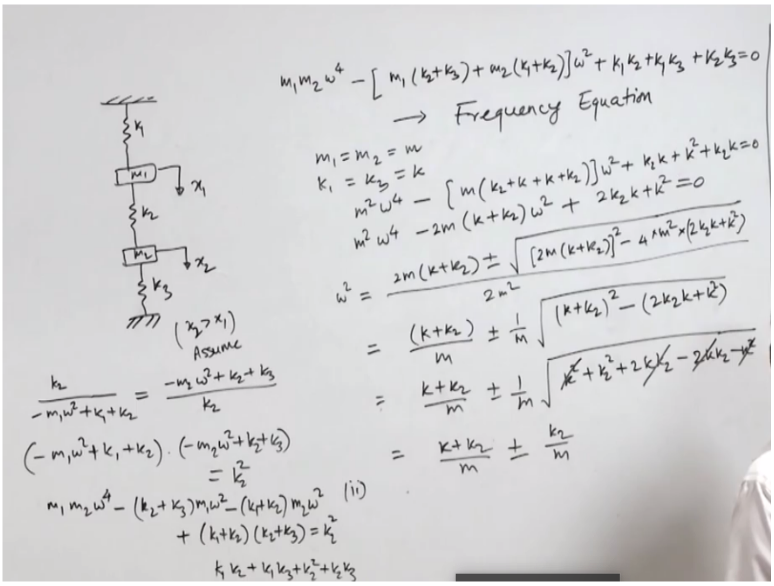

But from the left hand side, they are same. Because they both are; their amplitude ratio, x1/x2. So we can equate these 2 left side, so let us equate it, so it means that k2 / - m1 omega square + k1 +k2 = -m2 omega square + k2 + k3 upon k2. Now we can write, so it is -m1 omegas square + k1 + k2 into - m2 omega square + k2 + k3 = k2 square. Now we have to solve this.

So we can multiply, so this is m1 m2 omega power 4 +, - k2 m1 omega square - k3; so it is we can write like k2 + k3 m1 omega square then we can multiply these, - k1 + k2 m2 omega square, so we multiply these k1 + k2 to these side, so - k1 + k2 m2 omega square and + k1 + k2 into k2 + k3 and that is equal to k2 square. So we can further simplify this. So it is m1 m2 omega power 4 and - m1 k2 +k3 + m2 k1 +k2 omega square +.

So we can multiply this, so it is, if you multiply this; this is k1 k2 + k1 k3 + k2 square + k2 k3. So this k2 square and this k2 square will cancel out, so we will have k1 k2 + k1 k3 + k2 k3 that is equal to 0. So we have this expression. So this equation as we say that it is equation for omega. So if we solve this equation, we will get the values of omega that is; that will satisfy this condition.

So that omega that we will get that will be the natural frequencies of the systems. So we have to; we have this; we call it frequency equation, this is frequency equation and when we solve the frequency equation, we will get the natural frequency and so of course let us take some assumptions to solve those equations. So the equation is that we assume that m1 =m2=m and k1= k2 = k, so if take this assumption, what will happen?

So this is m square omega 4 - m; so this is k2 +k because here k3 sorry, k1 = k3 = k; k2 is k2. So k2 +k + k +k2 and omega square + k2 k + k square + k2 k =0. So we further simplify and we get m square omega power 4 -; so m; so we have here; k2 +k + k + k2, so it is twice k + k2 omega square + twice k2 k + k square that is equal to 0.

So we will now solve this equation. So here omega; because this is the quadratic equation in omega square, so we can solve it. So omega square equal to, so - p so it is; this + - the root; 4m and see, 2k, 2k + k square upon 2 m square. So we can solve it. So we will have k + k2 / m because here it is k + k2 / m + -; so we can take this 2 m; 1/m we keep here and we take other m inside, other 2m we take inside so what will be inside?

So if we take 2m here, so it will be 4m square and this 4m square so it will be cancel out so we will have k + k2 whole square -, so 4m square cancel out, we will have 2k 2k + k square so = k + k2 /m + - 1/m and here if we solve this, this is under root. So k square + k2 square + 2k k2 - 2k k2 - k square. So 2k k2 will cancel out and this k square will cancel out and this will be k2.

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$$\omega_2^2 = \frac{k+k_2}{m} + \frac{k_2}{m} = \frac{k+2k_2}{m}$$

$$\omega_1^2 = \frac{k+\cancel{k_2}-\cancel{k_2}}{m} = \frac{k}{m}$$

$$\omega_1 = \sqrt{\frac{k}{m}}$$

$$\omega_2 = \sqrt{\frac{k+2k_2}{m}}$$

So we will have k +k2/m + - k2 /m. Now if we take the + sign, so omega square = k + k2/m + k2/m, so it will be k + 2 k2/m and if we take negative sign, so it will be omega square = k + k2 - k2 /m so this is cancel out, it is k/m. So of course this is k/m, this is k+ 2 k2/ m so let us say this is omega 1 and this is omega 2. So omega 1= root k/m and omega 2 = root k + 2 k2/m.

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$$\omega = \omega_1$$

$$\left.\frac{X_1}{X_2}\right|_{\omega=\omega_1} = \frac{k_2}{-\cancel{m}\frac{k}{\cancel{m}} + k + k_2} = +1$$

$$\left.\frac{X_1}{X_2}\right|_{\omega=\omega_2} = \frac{k_2}{-\cancel{m}\left(\frac{k+2k_2}{\cancel{m}}\right) + k + k_2} = \frac{k_2}{-k_2} = -1$$

So we are getting the 2 natural frequencies of the system. So we are getting omega 1; that is root k/m and omega 2 that is equal to k + 2 k2 /m. Now if we want the amplitude ratio, x1/x2 for each of the omega, omega 1 and omega 2, then what will be? Now we put here omega = omega 1 in; so x1/x2 equal to, so it is for omega = omega 1 equal to; so k2 by, so - m1 omega square; so it is omega 1 square and that is k/m + k1 + k2.

So here this is m1 is m and k1 is k. So x1/x2 this m will cancel out and this k, k will cancel out and k2=+1. So we are getting x1/x2=+1. Similarly, x1/x2 at omega = omega 2 will be; we put any of these equations, so k2/-m1; so it is m into omega, so omega 2, it is k+2 k2/m + k + k. So m, m cancels out and this is k and k2, so it is - k, this will cancel out and this is k2, so it will be - k2. So it is k2/ - k2 and that is equal to - 1.

So we are getting at these natural frequencies the mode shape will be; for one frequency, it will be; the ratio will be + 1, and for other it will be - 1. So thank you we stop this lecture and see you in the next lecture.