

Introduction to Mechanical Vibration
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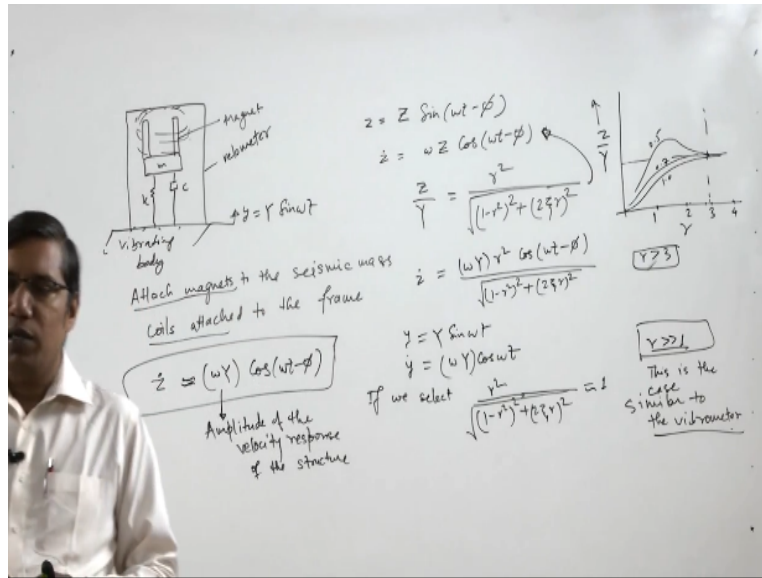
Lecture - 24
Velocity Pickup or Velometer

So, welcome to the lecture on Vibration measuring Instruments. So, we have discussed vibrometer and accelerometers. As we know that vibrometer measures the displacement response of a vibrating structure. Accelerometer meter measures acceleration response of the structure then if we want to measure the velocity response of vibrating structure we can use the velometer or velocity pickups.

And as we know that these picks are the instruments that contain some seismic mass some spring and damper element and they are used in combination with some transducers. So, if we want to measure the velocity what can be do as a transducer? So, transducer as remember that we said that for accelerometers we can use the piezoelectric materials that after application of the force they will give the charge and voltage and that we can measure.

So, now here velocity as we know that velocity if you want to measure we can use the transducer that is based on the change of the magnetic field or magnetic flux because the magnetic flux –so there is the voltage that is proportional to the change in the magnetic flux and so what we will do if we have this instrument. So, we have here this body

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That is vibrating so this is vibrating body $y = y \sin \omega t$ and this is our instrument and this is seismic mass and this is the frame of our instrument and so let us say this is we have to use the like velometer. So, we attach some magnet here. So, these are the magnet. So, we attach magnet with seismic mass. So, we attach magnet to the seismic mass and then we attach some coils.

So, these are some these binding the coils they are attached to the frame so the conductor coils they are attached to the frame of the instrument. Now, when there will be the movement of this seismic mass the magnets there will be change in the magnetic field in the coil and so there will proportionally there will be the voltage change that will be sensed by the transducer. And because here we are measuring the change in the magnetic field.

So this will measure the relative velocity $Z \dot{}$. So it is going to measure the $Z \dot{}$ the relative velocity between the excitation and the seismic mass. Thus we can measure the velocity response. Okay, Z is $Z \sin \omega T - \phi$. What is $Z \dot{}$? $Z \dot{}$ is ω into Z into $\cos \omega t - \phi$. Now, we know that Z by $y = r^2$ by $1 - r^2$ whole square + $2 \zeta r$ whole square under root.

So, we can keep this into this equation and so $Z \dot{}$ = ω into $y r^2$ square $\cos \omega t - \phi$ upon $1 - r^2$ whole square + $2 \zeta r$ whole square. So, $\omega y r^2$ square. Now, if we have $y = y \sin \omega t$ so what is $y \dot{}$? $y \dot{}$ = $\omega y \cos \omega t$. So, ω into y is the

amplitude of the velocity response of the vibrating body and so if we select if we select r square by $1 - r^2$ whole square + $2 \zeta r$ whole square approximately 1.

And this is possible for r much greater than. Because r much greater than 1. So we will have here r^4 because we can neglect one so this is r^4 and also we can neglect this and so r^2 by r^2 is about 1. So, it is possible for r much greater than 1 and this is the similar case of the vibrometer because this is the case similar to the vibrometer. As in case of vibrometer we had taken r much greater than 1 while in case of accelerometers we took r much less than 1.

So, if we select r much greater than 1, r^2 by this term will be about 1 so what will be $\dot{Z}$? So, the $\dot{Z}$ will be $\omega y \cos \omega t - \phi$. So, we see that the $\dot{Z}$ will give us the amplitude so it is approximate amplitude of the velocity response of the structure of the structure. Because ωy is the amplitude response amplitude of the response of this velocity of the structure so if we see this frequency response curve for velometer.

So the frequency response curve for the velometer will be same as the vibrometer or accelerometer. But we will be in the range of r much greater than 1. So, as we plotted here Z by Y and this was r and we show that here so this is $\zeta = 0.5, 0.7$ and this is 1.0 and so we are like 1, 2, 3, 4 so usually in case of vibrometer we said that r should be greater than 3 and similarly we will follow for the velocity pickup or velometer.

In this case, r must be greater than 3 and in this case we can have the –we can by measuring the relative velocity we can directly measure the amplitude of the velocity of the vibration body. And here we have this response curve. So, here we can see that although we are getting the amplitude but except phase because some phase will be introduced in the response because of this relative motion between y and z .

There are some damper so the relative –there will be some phase and so there will be some time lag between the response or y and z . Now, let us do one numerical example. Okay, so it is said that there is a vibration pickup that has a natural frequency of 5.75.

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$f_n = 5.75 \text{ Hz}$
 $\zeta = 0.65$
 a) 1% error b) 2% error

$$\frac{z}{y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$z = 1.01y$
 $1.01 = \frac{r^2}{\sqrt{(1-r^2)^2 + (2 \times 0.65 \times r)^2}}$
 $(1-r^2)^2 + (1.3r)^2 = \left(\frac{r^2}{1.01}\right)^2$
 $1 + r^4 - 2r^2 + 1.69r^2 = 0.98r^4$
 $0.02r^4 - 0.31r^2 + 1 = 0$

$$r^2 = \frac{0.31 \pm \sqrt{(-0.31)^2 - 4 \times 1 \times 0.02}}{2 \times 0.02}$$

$$= \frac{0.31 \pm 0.127}{0.04}$$

$$r^2 = \frac{0.31 + 0.127}{0.04} = 10.925 \Rightarrow r = 3.31$$

$$r^2 = \frac{0.31 - 0.127}{0.04} = 4.575 \Rightarrow r = 2.14$$

$r = 3.31 = \frac{f}{f_n}$
 $f = 3.31 \times f_n$
 $= 3.31 \times 5.75$
 $f = 19 \text{ Hz}$

So, this is f_n is 5.75 hertz and a damping factor of .65. So, ζ is 0.65. What is the lowest frequency beyond which the amplitude can be measured? So, we want the lowest frequency beyond which the amplitude can be measured within 1 percent error. So, case A is 1 percent error and case B is 2 percent. So, we know that $z/y = r^2 / \sqrt{1 - r^2 \text{ whole square} + 2 \zeta r \text{ whole square under root}}$.

Now, if we say this 1 percent error so it means that if y is 1 it could be 1.01 or it could be .99 so with 1 percent error. So, let us take $z = 1.01$ of y because we are solving case A so here $1.01 = r^2 / \sqrt{1 - r^2 \text{ whole square} + 2 \text{ into } 0.65 \text{ into } r \text{ whole square under root}}$. Now, we can solve $1 \text{ mins } r^2 \text{ whole square} + 1.3 r \text{ whole square} = r^2 \text{ upon } 1.01 \text{ whole square}$. So, we need a calculator to solve this. So, we can open out this.

So, this is $1 + r^4 - 2r^2 +$. So, it is 2.69 sorry 1.69 r^2 and this is equal to r^4 and there is 1.01 square. So, it is 0.98 r^4 . So, now we have to rearrange these terms and when we rearrange these terms so here r^4 this is .98 so it is 0.02 r^4 and this is - 2 and this is 1.69. So, it is 0.31 r^2 and $+ 1 = 0$. So, we see that this equation is quadratic equation in r^2 . So, we will get value of r^2 .

So, square equal to, so it is $-b \pm \sqrt{b^2 - 4ac} / 2a$. So, $-b$ is it is 0.31 $\pm \sqrt{b^2 - 4ac} / 2a$. So, we will get 0.31 $\pm$ and this is 0.04 and under root

quantity. So, we will take both + and - sign. So, we are getting $.31 + .127$. So, it is 10.925 and this is $.31 - .127$ it is 4.575. So, if we take under root so r , from here we will get r . This we get 2.14 and here 3.31.

So, we are getting these two values and we have to select which one is okay and that is correct. This is our frequency response curve and this is 1 and we know that we have curve like this. So, if we have this curve this is 2.14 and this is 3.31. So, of course if we say this curve is because it is asked that what is the lowest frequency beyond which the amplitude is measured with 1 percent error?

So, if we say beyond this so it is not possible because after here it is 1 percent error but because the curve will rise up and then it will fall to again come to 1 percent error at this point. So, in between these two points this curve is having more than 1 percent error and therefore we will not take this frequency. So, we will take this frequency because beyond this curve this is going to follow 1 so it is always less than 1 percent error beyond this frequency.

So r we will select as 3.31 and that is f upon f_n . So, $f = 3.31$ into natural frequency and natural frequency is 5.75. So, 3.31 into 5.75. So, it is 19 hertz. Okay. So here we have based on the given error condition that we want this error we have designed our instrument. Now the 2 percent error.

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$f_n = 5.75 \text{ Hz}$
 $f = 0.65$
 a) 1% error b) 2% error
 $\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$
 $\zeta = 0.65$
 $\frac{Z}{Y} = 0.98$
 $0.98 = \frac{r^2}{\sqrt{(1-r^2)^2 + (2 \times 0.65 \times r)^2}}$
 $(1-r^2)^2 + (1.3r)^2 = \left(\frac{r^2}{0.98}\right)^2$
 $1 + r^4 - 2r^2 + 1.69r^2 = 1.04r^4$
 $-0.31r^2$
 $0.14r^4 + 0.31r^2 - 1 = 0$
 $r^2 = \frac{-0.31 \pm \sqrt{0.31^2 - 4 \times 0.14 \times (-1)}}{2 \times 0.14}$
 $= \frac{-0.31 \pm 0.506}{0.08}$
 $= \frac{-0.31 + 0.506}{0.08} \approx 2.45$
 $r = 1.57 = \frac{f}{f_n}$
 $f = 1.57 \times 5.75$
 $= 9 \text{ Hz}$

So if we take 2 percent error. So, we can take here 1.02 or 0.98 if we check we will get here the r values they are imaginary. So, therefor we will take 0.98. So, 0.98 it means that $Z = 0.98$ of y . So, we will use $r^2 - r^2 \text{ whole square} + 2$ into 0.65 into r whole square under root. So, $1 - r^2 \text{ whole square} + 1.3 r^2 \text{ whole square} = r^2$ by 0.98 whole square. So, it is $1.04 r^4$. So, we can write $0.04 r^4$ and this is $- 0.31 r^2$ so we take other side.

So, it is $+ 0.31 r^2$ and then $- 1 = 0$. So, $r^2 = - b + \text{minus under root } b^2 - 4$ into a into c where c is $- 1$ by 2 into 0.04 and this is 0.08. So, we can do $0.31 + 0.506$ by 0.08 this is only valid because you take $-$ sign it is negative and so we cannot we get again imaginary. So, this is $- 0.31 =$ that is $-$ So, it is 2.45 and so $r = 1.56$. So, we are getting the value of $r = 1.5$ in fact, it is 57 and so $r = f$ by f_n . So, $f = 1.57$ into f_n and f_n is 5.75 so we will get 9 hertz.

So, we are on this response curve we are –so we were earlier 19 hertz and now we are for 2 percent error we can come here so we are $r = 1.57$ earlier we were at 3.31. So, we are here so if we come a bit before here so we are going to have 2 percent of error and here we are 1 percent error and this is r and this is 1. So, stop here and thank you for attending the lecture and see you in the next lecture.