Introduction to Mechanical Vibration

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Lecture – 23
Accelerometer

Welcome to the lecture on vibration measuring instruments, so we will discuss about accelerometers today, so as we have already discussed about transducers, the vibration pickups and vibrometers, so here what is I just want to recall, what is vibration pickup, so vibration pickup, they are the vibration measuring instruments that contain some seismic mass, spring and some damper element and they are used in the combination with some transducers.

So, here, and accelerometers that is type of vibration pickup that measures the acceleration.

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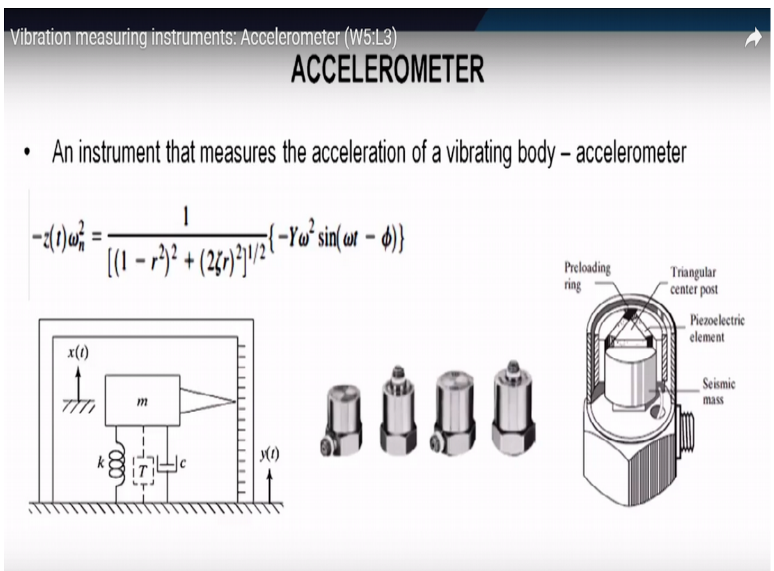

so here we can see that if we want to measure the acceleration we have to attach the, so this is our, this would be our vibration pickup or this is our accelerometer and we would like to attach the system to body that is vibrating and we will measure the relative response of the system. So, here we can see, so there are some examples of the accelerometers and this is a piezoelectric based accelerometer.

So, as I said the piezoelectric materials, they are materials such that if some force is applied on the surface of those materials, there is development of some charge and so the voltage and when we remove that force, the charges disappear. So, piezoelectric is a concept of transducer and we can use that piezoelectric material, so here we can see, we can put a piezoelectric material between the seismic mass and the frame because frame is attached to the vibrating body.

And this is the seismic mass, so the signal that we will get from the piezoelectric material will be the relative of the two. So, here is this a accelerometer that is based on the a piezoelectric material and we can see here the seismic mass and there will be some other stiffness or elements and this is a piezoelectric element that is in triangular form and that is placed between the seismic mass and the frame.

So this is the frame and so these piezoelectric accelerometers, they are widely used in the measurement of vibration.

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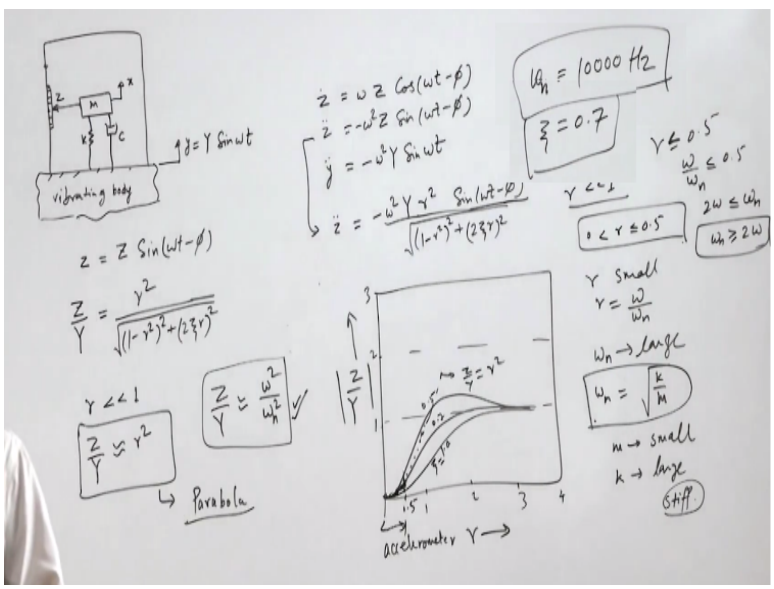

so we have this system, so this is our vibrating body, so this is vibrating body or vibrating structure. So this is vibrating with some Y=Y sign, omega T, and we are placing the seismic instrument here so seismic instrument has some stiffness and damping element. And then it has some seismic mass, M, K and C and of course seismic mass will have some X and here we will have Z and we have this frame and here we have some scale.

Now, so we will get the Z and our objective so from the derivation that we did that if some system, single degree of freedom system, is subjective to the base excitation or support excitation, Y=Y sign omega T, then we have Z, that is relative motion, Z sign, omega T - phi, where phi is face lag, omega is the frequency and Z is the magnitude or amplitude of the relative motion and we have already got this relation Z/Y = r square/1- square, whole square + 2 zeta R whole square under root.

So, this is the amplitude ratio of the relative and the support, now if we have this R much less than 1, then what will happen. If R is much less than 1, so this is 1 and of course this is much less so in compared to 1, this is also we neglect, so it is Z/Y is approximate R square. Moreover, so we see that here this Z/Y = R square is omega square/omega and square, but R is the frequency ratio, omega/omega.

Now, if see the curve of Z/Y, we will see the next slide, but before that let us calculate here Z double dot, so Z dot = Z omega into cos, omega T - phi and Z dot = omega square Z - sign omega T - phi and what is Y double dot, so Y double dot is - omega square Y, sign omega T. Now, in this equation, we can write, Z double dot = - omega square and Z, we can write here like Y x R square, upon 1- R square whole square + 2 zeta R whole square under root, and here is sign, omega T - phi.

We know that this omega square Y, here omega square Y is the amplitude of the acceleration of the support or the vibrating body, so we can write here, so if we take 1/1 - R square whole square, + 2 zeta R whole square, this is equal to 1, so we select R such way that this term is equal to 1 then we will have Z dot that is - omega 2 Y, R square sign omega T - phi =, so the amplitude Z double dot = Y double dot amplitude x R square.

This means that, so what is the amplitude Z double dot that is omega 2 Z and that is omega 2 Y and R that is omega 2 Y omega in square, so acceleration and amplitude, so omega N2 x Z double dot = - omega 2 x Y. So, it means that if we know the acceleration amplitude of Z, relative acceleration amplitude and we multiply with omega and square, we will get the acceleration amplitude of seismic mass.

Now, we can see this equation, so this equation is representing parabola, so this is equation of parabola and if we see the frequency response curve, so here is the frequency response curve.

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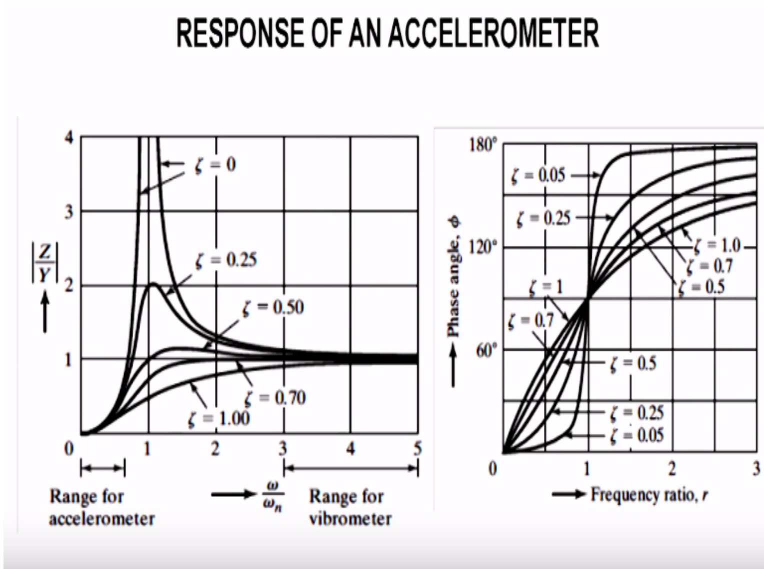

And we can see the frequency response curve. So this frequency response curve, Z/Y, it is Z/Y and this is R and so this 1 and this is 2 and 3 and this is 1, 2, 3. So, if we brought this curve, so this curve is, okay, so for different values of damping, this curve is plotted and if we have, now if we plot this curve, so it is parabola, so this is a parabola, Z =, so this is Z/Y= R square. So what we see that this condition is satisfied when we have R much less than 1.

So this curve Z/Y=R square is passing through the other curves for different damping for the initial values of R, so here we have these values, we see here for let us say 0.5, so $0 < R < 0.5$, in this range we are going to satisfy that other curves also have the similar behavior in this range for different damping, so therefore, this is the range of accelerometer. So, this is the range of accelerometers.

So usually for accelerometer, we keep the value of R less and usually between 0 and 0.5. Now, if we want to keep the value of r small, and r is omega/omega N, so it means we have to keep omega in large. If we want to keep r small, so range of accelerometers, we have to keep omega enlarge and omega enlarge and omega N = root k/m and so to keep omega enlarge, we have to keep m small and k, also k we can keep large.

So m is small means low mass of the instrument, lower mass and k is large means it is stiff. So, accelerometers, if we keep based on the requirement, the accelerometers have less mass and large stiffness, so they can be available in very compact sizes, small sizes and therefore they are one of the perfect instruments that are used for the measurement of vibration or acceleration, so now if we have r = 0.5 and so omega/omega N <= 0.5, so 2 omega <= omega N or omega N >= 2 omega.

So it means that the natural frequency of the instrument could be at least = twice the frequency that we want to measure; however, there are several frequencies, which fall in the higher hormonic range, so if we have omega, we have 2 omega, 3 omega, so the higher order frequencies, if we want to capture them, then definitely the omega and the natural frequency of the instrument should be even higher.

So, therefore, the natural frequency of accelerometer is kept quite large and so that if we have quite large natural frequency, we are more following the condition r much less than 1, so usually in case of accelerometers, omega N, we keep about 10,000 Hz, which is quite large natural frequency and so we are able to capture several higher hormonic of our excitations or the vibrations.

Now if we see the phase curve here, we can see that these are the phase plot and in the phase angle plot, they start from 0 and at resonance they are 90-degree and then they are increasing and tending to 180 degrees. So, because we are in this range, okay. So, we are our phase angle that is close to 0, however, if we keep zeta = 0.7 because this is zeta = 0.7, we get a linear variation of the phase angle in this range between 0 to r = 0 to 1.

We get a linear phase variation in this range and therefore we usually keep the value of zeta or damping factor for accelerometer = 0.7 or little less than 0.7, also for 0.7, our parabola, this curve is more close when zeta = 0.7, so when here zeta is 0.7, our parabola is more close here, so it follows perfectly when zeta is 0.7, so usually in accelerometer, we keep zeta value = 0.7.

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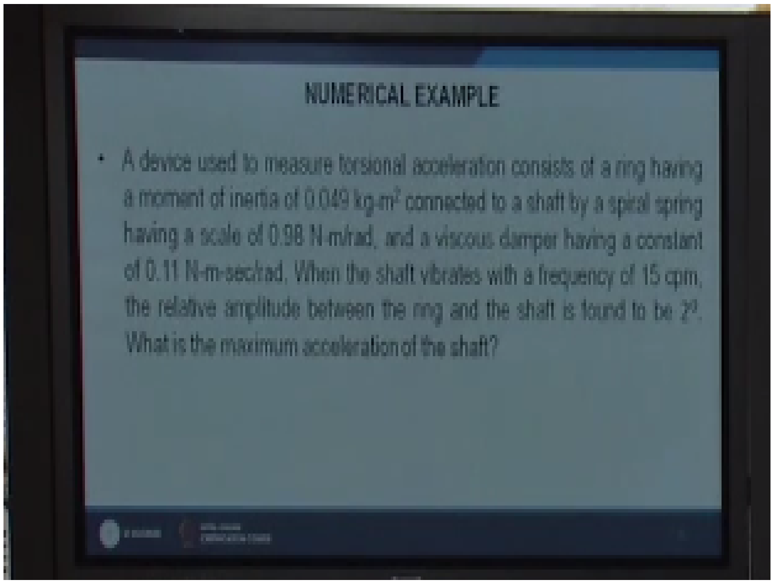

So we will now do one numerical problem, okay, so a device is used to measure torsional acceleration so we are going to measure torsional acceleration, although we have discussed the theory for the linear accelerometers, but we will only change the degree of freedom from linear to torsion and we can use the same formula that we have derived.

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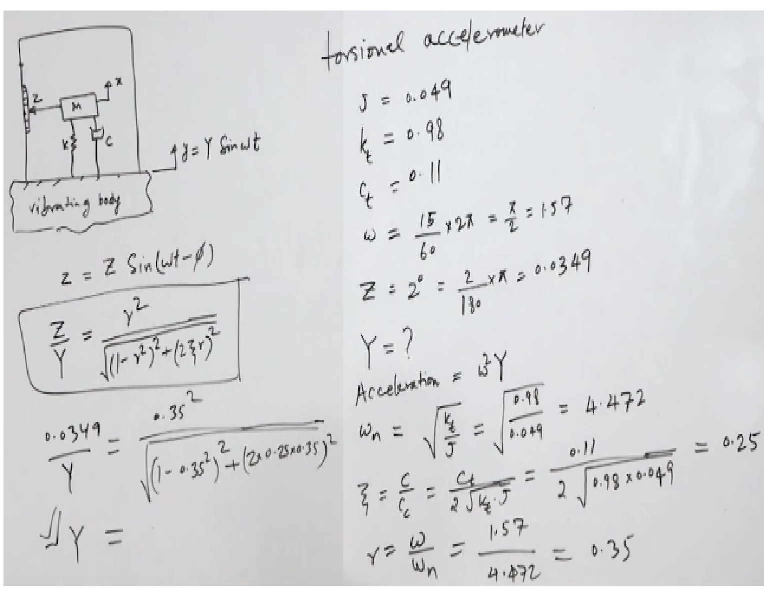

So we are talking about torsional accelerometer, so here it is torsional accelerometer. so we want to measure some acceleration consisting of a ring having a movement of inertia of 0.049, so here movement of inertia J is 0.49 kg/m2 connected to a shaft by a spiral spring, because we are dealing with torsion, so here the spring will be spiral spring, not a linear spring and it has a scale of 0.98.

So here stiffness spring scale means stiffness of the spring that is 0.98 and a viscous damper having a constant of 0.11 Newton meter second per reading, so here CT = 0.11 when the shaft vibrates with the frequency of 15 cycles per minute, so it is saying that the vibration of the shaft is this, so this is omega because here this is the vibration of the body, so we are taking omega, so omega is 15 cycle per minute, so 15 x 60, so it is cycle per second x 2 pie.

So it is radian per second and the relative amplitude between the ring and the shaft is found to be 2 degrees, so here relative amplitude that is z, so z is 2 degree and we can convert into radian so 2/180 x pi, so we can convert into radian. What is the maximum acceleration of the shaft, so we want to know the maximum acceleration of the shaft, so first we have to find y and then we will find the maximum acceleration corresponding to this y.

Because this is the maximum amplitude so maximum acceleration that will be omega square x y. So first, we have to find y. So let us do of course to find y, we have to use this formula and to get this. We have to first find r and zeta, so omega here we find omega and we have root k/j because omega n is root k by m, here m is j and it is kt. So 0.98 x 0.049 and zeta is c/cc = c/2 root km. So kt x n, msj, so this is ct. So we can find is 0.11, 2 root, kt is 0.98 x j, j is 0.049.

So let us calculate these terms, so omega that equal to pi/2, so it is pi/2 and so it is 1.57, then this one, so it is 0.034, so it is 0.0349 and omega n is 4.47 and zeta 49, so it is 0.25, so we have all got these terms, so we can keep here, so we put it is 0.0349/y, so r, r is we have to get r, so r = omega y, omega n and omega is 1.57, omega n is 4.472. So it is 0.35. So we can keep all these terms here, 0.35 square 1.035 square whole square + 2 x 0.25 x 0.35 whole square.

From here, we can find y and once we get y, we can calculate y and so let us calculate it.

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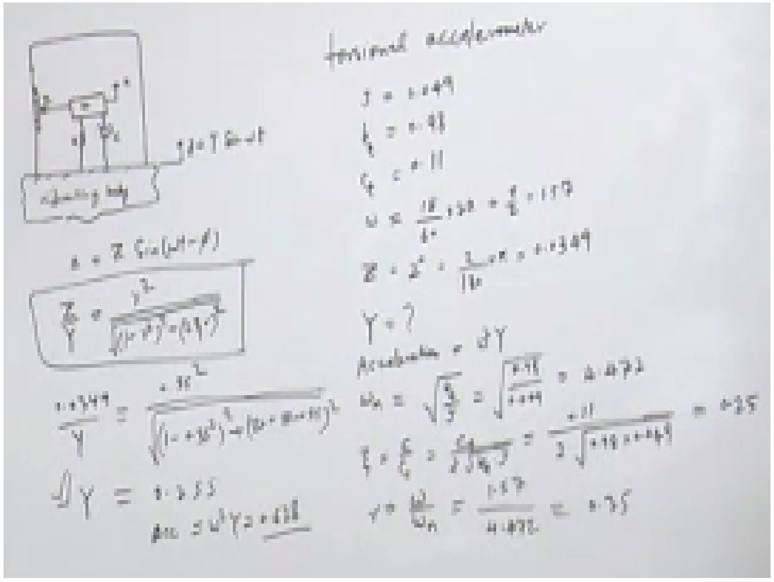

So, it is coming about 0.255 and so omega square y that is excitation amplitude and so we can take omega 1.57. So, it is about 0.628. So we can see how to calculate these acceleration values for any system. Here we have considered the torsion system. Okay, so we stop here and thank you for attending this lecture and see you in next lecture.