Introduction to Mechanical Vibration
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Lecture - 22
Vibrometer

So welcome to this lecture and we will discuss the theory of vibrometers. So we have discussed that the vibration measurement is necessary to know the response of structure or any system and those responses can be represented in terms of displacement response, velocity response, acceleration response and based on these responses, the instruments are named like if we say displacement response, we call it vibrometer. So we are going to discuss the theory of vibrometer.

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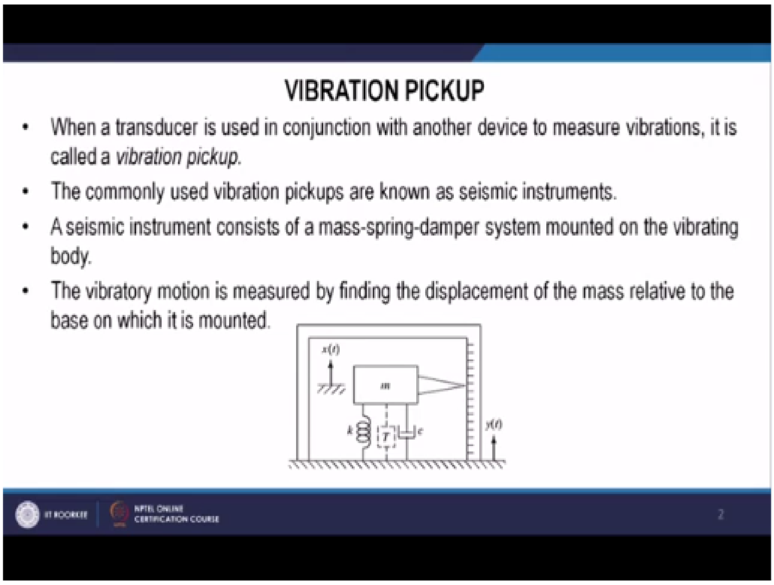

So as we say that usually these vibration pickups, they are called the seismic instruments and they consist mass, spring and damping elements.

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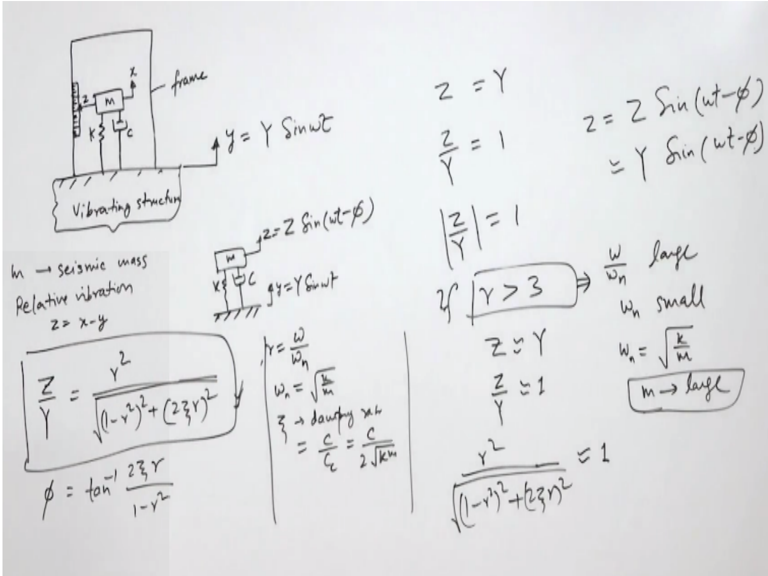

Now suppose, I have some structure that is vibrating, so I have a structure here. So, this is my structure and this is vibrating. So this is vibrating structure or surface, so this is vibrating with certain, we assume for the simple analysis just one harmonics, so we assume that this is vibrating as y = Y sin omega t. Now I have to measure, we do not know what is Y sin omega t.

We know that it is vibrating with certain harmonic way, but we do not know Y and we do not know omega. So we have to measure this y and for this as I said we use some vibration pickup. So we will apply these vibration pickups. So vibration pickups, so let this is my instrument frame, so this is the instrument so this is the frame and my vibration pickup contains, it is a compact instrument including this frame.

And this frame is installed on this vibrating surface, so it is rigidly fixed on the surface. So it means that the frame is vibrating with the same vibration as this structure and as I said this seismic instrument or vibrometer, there are vibration pickups, they contain basic spring, mass and damper element. So we have this spring, we have this damper and we have this mass and this mass m, k and c.

This mass m is called seismic mass and you see this is complete this frame and that is fixed on the surface. Now this due to the vibration this y and it is attached to the surface and this will have some vibration behavior like x and there will be some relative of this x and y that is z, z = x - y. So the relative vibration of seismic mass with respect to the surface y and so can we measure x?

We cannot measure x because it is inside the frame of the instrument and it is vibrating relative to the y. So we can measure the relative displacement, relative vibration of this and that is let us say it is z and here suppose we have some scale so we can measure the z that is relative and by measurement of z we have to know what is the y. So now if you see this you relate with the theory that I have already discussed earlier.

So it is nothing but the theory of the vibration of a single degree of freedom system under the forced condition due to vibration of the base or vibration of the support. So here, we have if we simplify this, this is my system and this is the support and this support has this vibration y = y sin omega t and so here we have z and z = z sin omega t - phi and these are m, k and c. So we have already discussed and developed the response formula for this kind of system.

And if we remember, we have this z by y = r square on 1 - r square whole square + 2 zeta r whole square under root and phi = tan inverse 2 zeta r upon 1 - r square. So the amplitude of the relative motion upon amplitude of the support motion are here is the vibrating structure is this formula, r is the omega by omega n, omega is the frequency here and omega n is the natural frequency that is root k by m and zeta is the damping ratio.

It is damping ratio and that is = c upon cc and c upon 2 root km. So all these things we have already discussed. Now we are going to use this. Now what is a good instrument I mean what is our expectation that the z that we are measuring that should precisely follow or precisely represent y. So what do we want, we want that z should be = y or z by y should be = 1 or we can say z by y should be 1.

Now in which condition, so when z by y = 1, we measure z and it means we are measuring the y so in which condition we will have this formula tend to z by y = 1.

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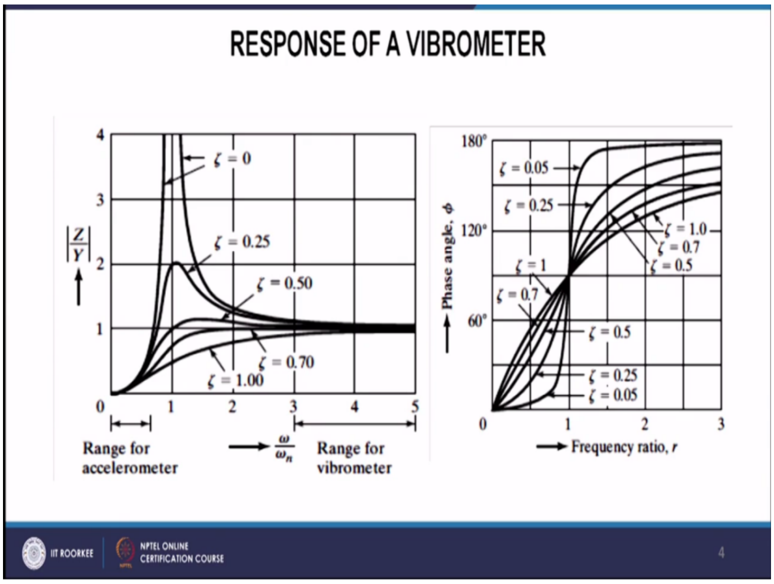

So for this to understand this, we have to see the frequency response curves and let us see the frequency response curves. So these are the frequency response curves and if we see the frequency response curves z by y, we see that z by y is tending to 1 because z by y is 1 when r that is the ratio of omega by omega n is greater extending to greater than 3 Hz, r is greater than 3, my z is almost = y or z by y = 1.

So it means that, r square 1 - r square whole square + 2 zeta r whole square under root = 1. So if we want to design a vibrometer, we should select values of k, values of m, values of c in such a way that the omega by omega n falls in the range greater than 3 and that is the design criteria for the vibrometers and when we are in this range we can directly measure.

So my z is z sin omega t - phi and so it is equal to y sin omega t - phi. Now if we have just to have this r > 3, we must have because r is omega by omega n and it should be large so to have this large omega n should be small. Okay if want omega by omega n large, it means we must have omega n small then only for most of the natural forced frequency, we will have large value of r.

And omega n = root k by m, it means we will have m should be large. So if we want m large, it means my instrument will have more mass, so it will be a bulky instrument and usually we like to avoid bulky instrument. Therefore, we have to select maybe some smaller mass, but in that case if we have to get the relation, we have to use this formula, z by y is equal to this expression.

Now second point that I would like to discuss is the phase because we are measuring z = y sin omega t - phi.

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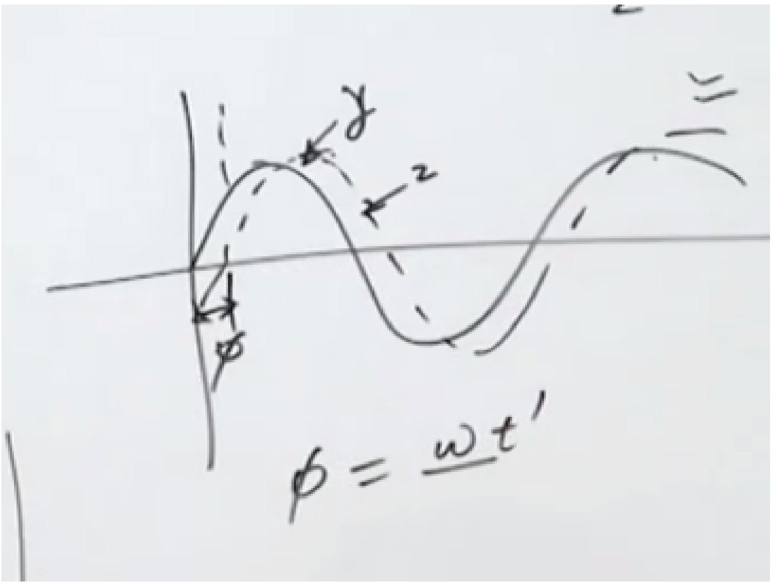

So if we have single harmonic vibration, then there is no problem because if this is my y, I may get some phase lag of z so it will be like this. So this is z, so there will be some phase lag phi and phi = omega t. Now, if we have several harmonic functions say that excitation of the base or the structure contains may be a periodic function and the periodic function may have various higher harmonics.

Because we know that, that can be decomposed in harmonics having the higher terms. So in this case if we are for higher harmonics, my r is even greater so we are more precise for z. However, there will be the phase shift and phase shift will also increase so if my one is here and other is starting from here and so if you combine them, they are not going to give you the exact resultant because there is a phase shift for each signal.

So in that case, if we take zeta = 0 so for zeta = 0, you can see we have the phase shift that is 180 degree. So for zeta = 0, we have for r > 3, I mean this is 180 degree phase shift. So the phase shift for each harmonics even higher harmonics will be the same and they will start from the same point.

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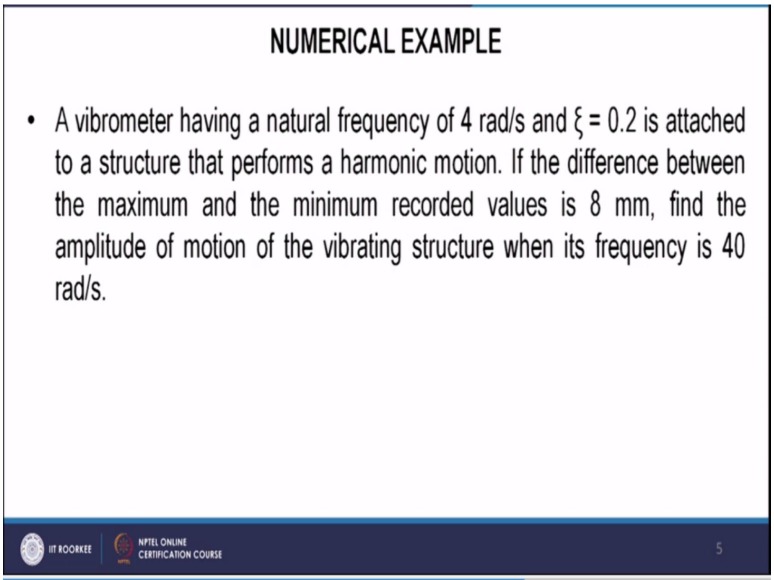

And so now let us take one example. Okay so let us take numerical example. So there is a vibrometer that has a natural frequency of 4 radian per second and damping factor is 0.2 and that is attached to a structure that performs a harmonic motion. If the difference between the maximum and the minimum recorded values is 8 mm, find the amplitude of the motion of the vibrating structure when its frequency is 14 radian per second.

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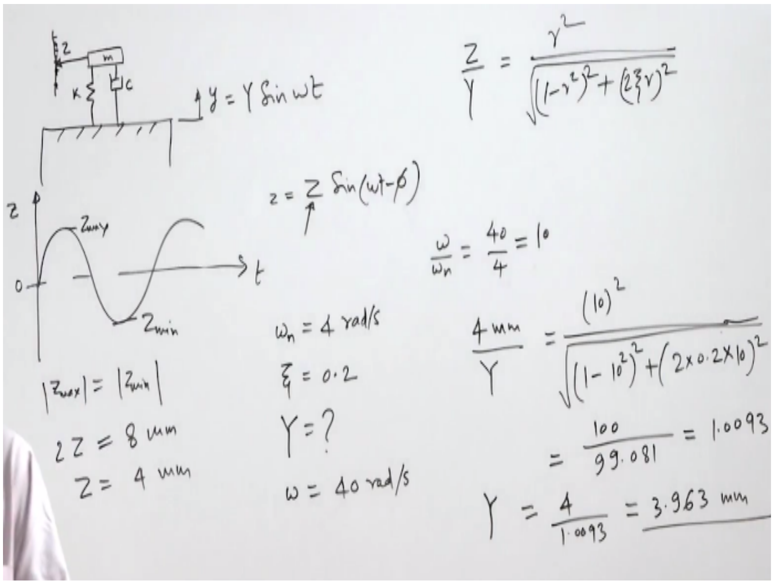

So we have this structure and it is vibrating with y = y sin omega t and we have this seismic instrument. So it is m, k and c and here we are measuring our z, so here is our scale and we are measuring z. Now when we are measuring z, so this is z max and this is the z mean. So the maximum amplitude and minimum amplitude and if it is a sinusoidal, so here we have this is 0, this is t.

So z max = z mean and so it is said that the difference between the maximum and the minimum recorded value is 8 mm. So it means difference between maximum and minimum is 8 mm means, 2 times z max or z max is nothing but z because if we write z = z sin omega t - phi. So it is this amplitude z that is z max 2 times z is 8 mm, which means the z is 4 mm and the natural frequency is 4 radian per second.

So omega n = 4 radian per second and zeta is 0.2. So we have the damping factor that is 0.2. Now we have to find the amplitude of motion of the vibrating structure. So we have to find Y when the frequency is 14 radian per second. So the omega is given, so this is omega and omega is given as 40 radian per second. So if we want to find this, we have to use this formula of z by y = r square because if we calculate omega by omega n.

What is omega by omega n? It is 40 by 4 that is 10. So r square by 1 - r square whole square + 2 zeta r whole square under root. So z is 4 mm, so it is in mm and y, so r is 10 square 1 - 10 square whole square + 2 into 0.2 into r, r is 10 square under root. So we have to solve this so this is 99.081 and that is equal to 1.0093. So if we calculate y so we will get the amplitude y that is equal to 3.963 mm.

So here we see the effect of this parameters zeta**,** m and k that what we are measuring that is 4 mm and the actual is 3.963. It is close, but not exact. Now here again, I would like to stress this frequency response curve. In this frequency response curve, when we design as I said that when r > 3, z by y is closer to 1 and therefore this gives the range of the omega by omega n for the vibrometers.

So for the displacement vibration pickups or seismometer, this is the range of the omega by omega n we have to keep. So we discussed today this concept of vibrometer and we see that how the theory that we discussed of the support excitations that helps us to design and understand the theory of vibrometers and vibrometers are also known as seismometer or seismic instrument.

They contain some seismic mass, spring and damper and they are used to measure the displacement of the vibration response and their valid frequency range omega by omega n > 3 and so I thank you for this lecture and I stop here and we will see you in the next lecture.