Introduction to Mechanical Vibration
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Lecture - 20
Numerical problems

So welcome to this lecture. So, we have been discussing the forced vibration of single degree of freedom system. We have already discussed the forced vibration due to the harmonic function, harmonic forced function. Forced vibration due to the rotating and balance in the machines to support exact distance. So today we will discuss some numerical problems. Okay. That are some specific problems that I would like to discuss to you.

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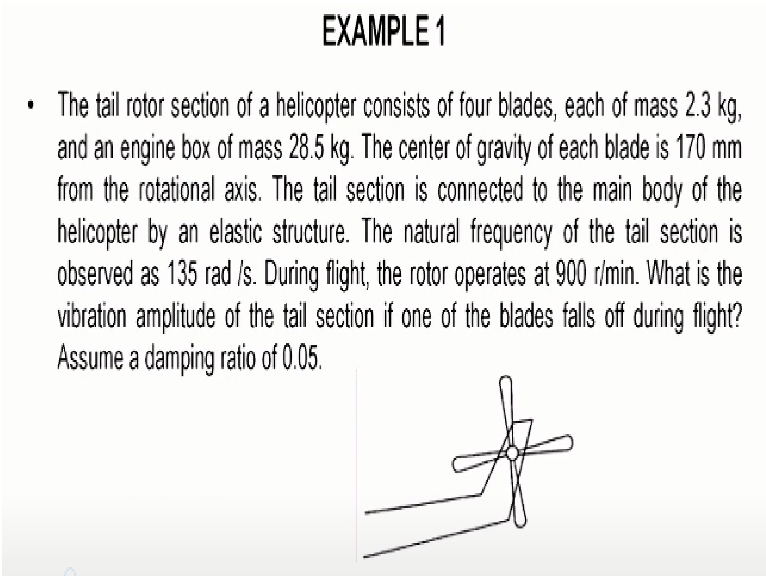

So, let us start with the first problem. So, this first problem. So, there is a helicopter a router, So the tail router of section of a helicopter that consists of four blades. So, there is a tail router of helicopter and there are four blades, each of mass 2.3kg. So, the blades has a mass of 2.3kg and engine box of mass 28.5kg. And there is an engine box with mass 28.5kg. The centre of gravity of each blade is 170mm from the rotational axis.

So here we see that the blades are connected to the rotational axis and they are rotating about that axis. But their mass a centre of mass is 170mm from the axis of rotation. Now this tail section is connected to the main body of the helicopter by some elastical structure. And the natural frequency of the section is 135 radian per second. So now during the flight, the router operates at 900 radian per min.

Then what is the vibration amplitude of the tail section if one of the blades falls off during flight? So, during the flight when the router is operating with the 900rpm then one of the blades falls then it is asked that what will be the vibration amplitude now when there is only three blades, there are only three blades, assume some damping ratio of 0.05. okay. So, we have, so it is clear that when there are four blades, okay, so this is my-

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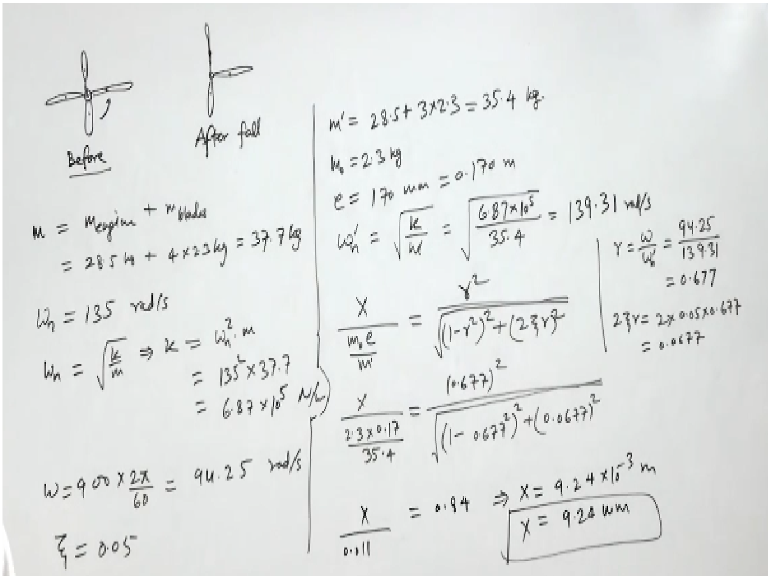

So here are the let us say this is there are four blades and their centre of mass is here. one point, 170mm from the rotational axis and this is the axis of rotation. Okay. And they are rotating. Now if one blade is fall down so we have like this condition. Okay, so here is the centre of mass. now we can see that when we have these four blades they are balanced because their centre of marks is balanced, but if one is falls so this one blade will be unbalanced.

This blade is balanced because they are combined centre of mass will fall on the rotational axis. but this blade centre of mass now is unbalanced after fall, before and after, after fall. Okay. So, this is the problem of rotational unbalanced mass. And so, we have like, so we have the mass m = mass of the engine and + mass of the blades and mass of the engine is 28.5kg and this is four into two-point three kg.

So, this is before and, so it comes out to be 37.7 kg. so, this is the total mass of the system and it is given that the natural frequency of this section is 135. So mega n is 135 radian per second. So mega n = root k by m. So, we can find the stiffness of this section. So, k = mega n square into m and so it is 135 square into 37.7. So that is equal to, so it comes to be 6.87 into 10 power 5. This is in newton per meter.so we have got the stiffness per meter.

So now when mega is 900rpm. So, 900 into 2 phi by 60. We have 94.25 radian per second. So, one blade is falling and damping ratio zeta is given 0.05. So, when one blade is fall, so we have, now the mass after the falling of the blade, so m is now engine mass + now the it is 3 into 2.3. Because now we have only three blades and, so we have, we can calculate 35.4. So, this is the mass of the system.

Now unbalanced mass, that is m0 = 2.3kg and e = 170 mm that is 0.170 meter. Okay. So, this is the mass, total mass. So, and this is the unbalanced mass. And this is the distance of that unbalanced centre of gravity of that unbalanced mass from the axis of rotation. Now the mass is changed after the falling of the blade, the mass of the system is changed. And you know that the natural frequency of mega n = root k by m.

And when mass is changed definitely the natural frequency will change. Here we assume that the stiffness is not changed after falling of the blade. So, we assume the stiffness to be constant, but because the mass is changed now the natural frequency will be changed. So, the natural frequency of mega n now will be root k by m dash. So, we know that K is 6.87 into 10 power five and M dash is 35.4 so = so it is 139.31 radian per second.

So earlier natural frequency was 135 radian per second and you know that it is inversely proportional to route M so if mass is reduced the natural frequency is increased here. Now we have to find the Vibration amplitude, so we know that the vibration amplitude in case of rotating unbalance is X by M0 E by M = R square upon 1 - R square whole square + 2 zeta R whole square under root, so this is the formula for X.

Here M0 is now M dash so I put here as M dash. So, X upon M0 is 2.3 into E is 0.17 upon M dash M dash is 35.4 and = R square. So, we have to calculate R and if I calculate R R = Omega by omega M dash and omega is 94.25 and omega M dash is 139.31 so we have 0.677 so this is R and two Zeta R = 2 into zeta 0.05 into R is 0.677. So equal to 0.0677. so, we can put in this equation.

So 0.677 square upon 1 - 0.677 square whole square + two zeta R is 0.0677 whole square under root. So now we can calculate X = 9.24 into 10 power - 3 meter so it is 9.24 millimeter. So it means that the system will vibrate with harmonic motion and the amplitude of that motion will be 9.24 mm so here we now we go for next example.

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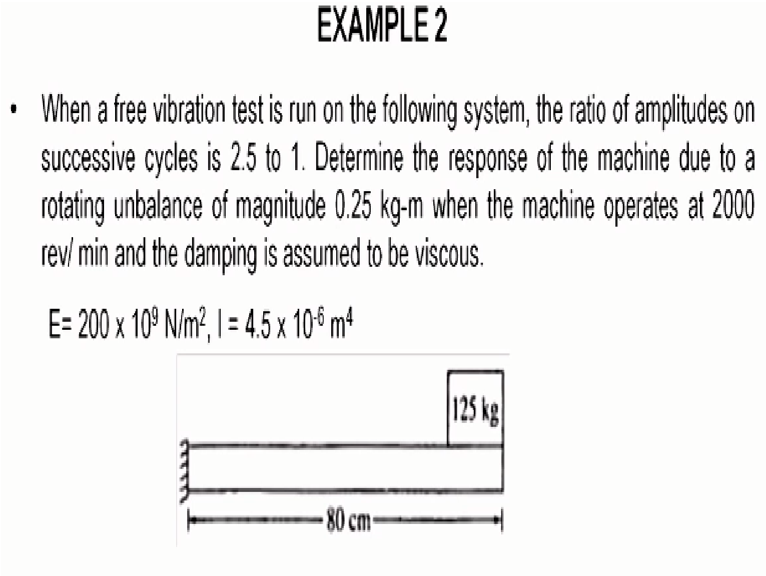

So, example 2, here in example 2 it is said that when a free vibration test is run on following system. The ratio of amplitude on successive cycles is 2.5 to 1. Determine the response of the machine due to a rotating unbalance of magnitude 0.25 kg M when the machine operates at 2000 rpm and the damping is assumed to be viscous. Here we have a system having viscous damping and we know that if there is a viscous damping and free vibration test is performed.

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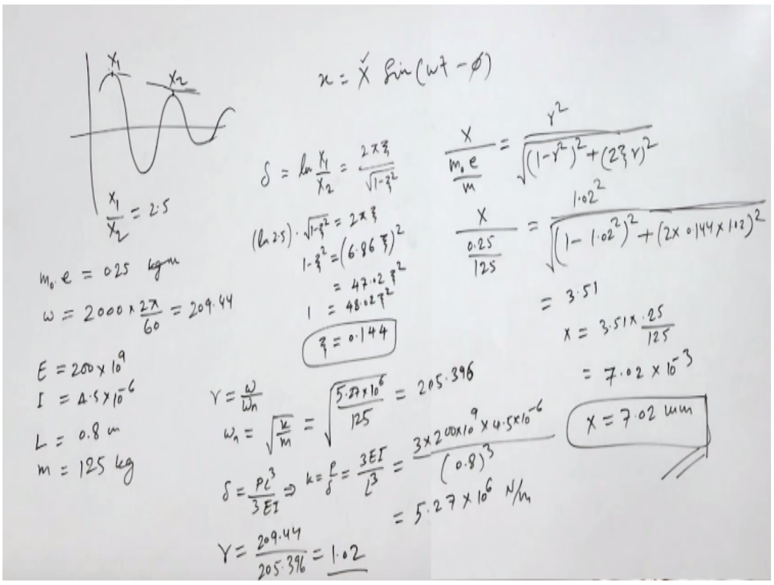

So, and there is some amplitude ratio so free vibration test when we perform, we have some amplitude ratio here decay like this. So here let us say this X1 and this is X2, so it is given that x1 by x2 = 2.5. 2.5 to 1. So it is 2.5 now we to find there is a rotating unbalance in the machine so there is a machine of 125 kg on the cantilever beam and the machine in the machine.

There is some rotating unbalance M0 into E that is 0.25 kg in two-meter Kg into meter kg meter. When the machine operates at 2000 rpm so omega = 2000 rpm, so it is 2 5 by 60 radian per second and dampen it viscous and we have given some information about beam so E = 200 into 10 power 9 newton per meter square and I = 4.5 into 10 power - 6 meter per 4 and length of the beam is 80 cm it is 0.8 meter and mass is 125kg

Mass of the machine. So, we have to find the response of the machine due to rotating unbalance. Ok so we know that we have the rotating unbalance so there is the force and Zero E omega square sin omega T on that system and so the response X is something X Sin Omega T - Phi so this is the response, so we have to find, let us find X the amplitude of the response to find the response.

First let us find the damping because here we know the logarithmic decrement Delta = log X1 by X2 and that is equal to 2 phi zeta upon root 1 - zeta square. So here align X1 by X2 is 2.5 into route 1 - zeta square = 2 phi zeta. So, we can find 1 - zeta square = so let us calculate this, so we get from here the damping ratio that is 0.144. And we calculate omega so omega is 209.44 radian per second.

So, because we have to find the amplitude and we know that X by M Zero E by M = R square by 1 - R square whole square + two zeta R whole square and route. So, we know M Zero E and we know M which is 125Kg. RV to find Zeta we know so R = Omega by Omega n, but we do not know omega N. So, what is the natural frequency of the system, so Omega N = route K by M. M we know but we do not know the stiffness.

So now stiffness we have a cantilever beam so and at the end the machine is there, and it is giving the transverse vibration. So, we know that K for a cantilever beam is so we can have so this formula comes like this, 3 into E 200 into 10 power 9 to 4.5 into 10 power - 6. L is 0.8

cube. So, we get K = 5.27 into 10 power 6. Now we can calculate natural frequency. So natural frequency so 5.27 into 10 power 6 by 125kg.

So, we get omega in 205.396. so, we can get R, so we have R. R = Omega by Omega N so 209.44 upon 205.396. So, it is 1.02 so R is 1.02. so, we have all other things and we can one more term, so X by M Zero E is 0.25 and M is 125 Kg and R is 1.02 square so 1 - 1.02 square whole square + 2 into. So, zeta is 0.144 into R. R we have 1.02 whole square and then under root.

So we will get so we get here that displacement amplitude of this system due to the unbalance of the mass is coming to be 7.02 milli meter. So, I think with these examples we can understand the how to identify and apply the theory of rotating unbalance in the system and find the vibration response amplitude of the system. So, I thank you to you for this lecture for attending the lecture and let us see in some next lectures. Thank you.