Introduction to Mechanical Vibration
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Lecture - 19
Motion transmissibility

So welcome to the lecture on motion transmissibility. So, we already discussed that we use the vibration isolators, to reduce the transmission of the forces or motion, from a machine into the foundation or if to talk to reduce the motion, when some sensitive instrument or sensitive item is put on some base, some where there is some vibration from the base.

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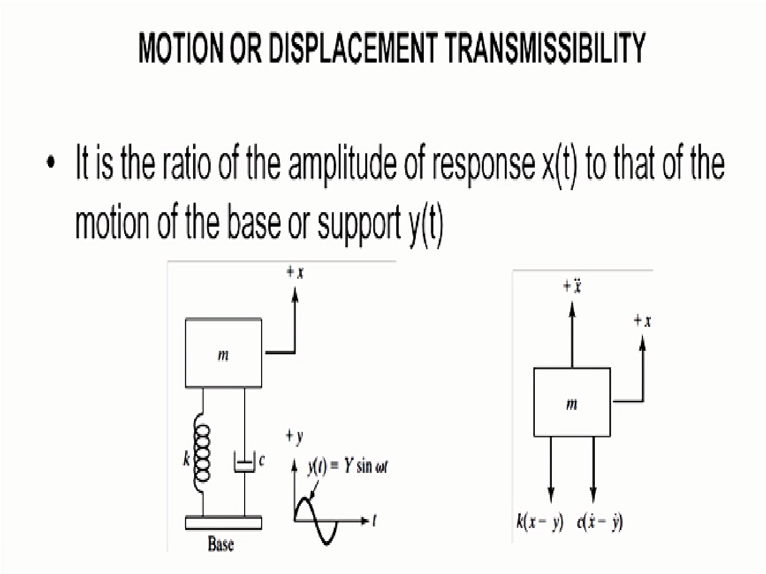

So, we can see that, this motion transmissibility also we call it displacement transmissibility. So, it is the ratio of the amplitude of the response x to that of the amplitude of the support okay. So, let us see here there is some sensitive instrument m is kept on some surface ok, and that surface is vibrating with some harmonic way and so this motion is going to affect the response of this mass.

And, so we have to keep these isolators, so that the motion from the space is least transmitted to this mass. And so, the here the role of the displacement transmissibility will come. So, let us discuss this, so we have this system.

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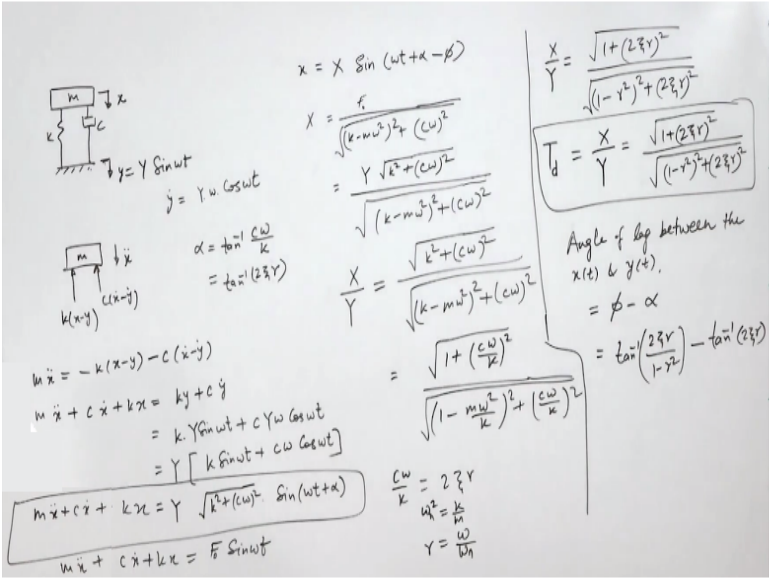

So we have a mass and here we have we say this is the support, but this support has some motion, it has some motion. So okay, so the support is moving, it is vibrating, so this is a case of the support similar the case of the support vibration and we can do the analysis, by making the free body diagram of this mass. So, this is the mass and we have, because this spring will be compressed by x - y.

So, the net force on this spring is, k times x - y and similarly the damping force is cx dot - y dot and here this is x double dot. So, we can write the equations of motion so here mx double dot = - kx -, y - c x dot - y dot, so we can write mx double dot + c x dot + kx = ky + cy dot. Now here y = y sin Omega t, so y dot =.

We have to differentiate this. so, we can have y into Omega into cos Omega t. So here we can write k into y sin Omega T + C into y Omega Cos Omega T. So, we can write k sin Omega T + C Omega Cos Omega T. So here they are the two harmonic motions of the same frequency, and we know that we can combine them. So, we have y root k square + c Omega whole square into sin Omega t + alpha.

So, where this alpha = tan inverse c Omega by k, and this is tan inverse c Omega by k 2 Zeta r, okay. So now we have this system and if we compare this system with another system mx double dot + cx dot + kx = f0 sin Omega t. So, we see that it is this f0 is equivalent to this term, that is the amplitude, and this is the harmonic motion. So, there is some harmonic force equivalence of this term that is applied to this system.

And when there is for the single degree of freedom system, which is subjected to some harmonic force, we have already derived the response and the response so x = steady state response x = x sin Omega t + alpha - Phi. So, because this is the force and some phase lag will be there, here and x =, if we remember it was f0 upon k - m Omega square whole square + c Omega whole square under root.

Now here f0 is equivalent to this term y root k square + c Omega square, so it is y root k square + c Omega square upon under root k - m Omega square whole square + c Omega whole square. So, from here we can find x by y = root k square + c Omega whole square upon k - m Omega square whole square, + C Omega whole square under root.

So, we take k outside and we will have under root 1 + c Omega by k whole square upon 1 - m Omega square by k whole square + c Omega by k whole Square and this is under root. So, we can write c Omega by k = 2 Zeta r and Omega n equal square = k by m and r = Omega by Omega n. So, if we put these terms here we will find, x by y = root 1 + 2 Zeta r whole square upon.

1 - r square whole square + 2 ETA r whole square under root. And according to the definition of the task the displacement or motion transmissibility, we say that it is the ratio of the amplitude of the response xt. So, amplitude of this response xt is capital x and m that of the motion of the base so motion of the base is y sin Omega 2, so amplitude is y so x by y. So, this is the if we say transmissibility displacement transmissibility

That is x by y and this is equal to root 1 + 2 Zeta r whole square upon, under root 1 - r square whole square + 2 ETA r whole square. So, this is the formula for displacement transmissibility and we see that the formula is same as the we obtained for the force transmissibility. Now the angle of lag between the xt and yt. So how much this motion ex is lagging the y.

So that is equal to Phi - alpha, and phi we know that Phi is tan inverse 2 Zeta r upon 1 - r square and alpha is, tan inverse 2 Zeta r. okay so this is the angle of lag between the two motions. So, we see that the angular flag similar to what we obtained in the force transmissibility, that how much the transmitted force will lack the impressed force. So, it is the same and so, there is some relation between these terms.

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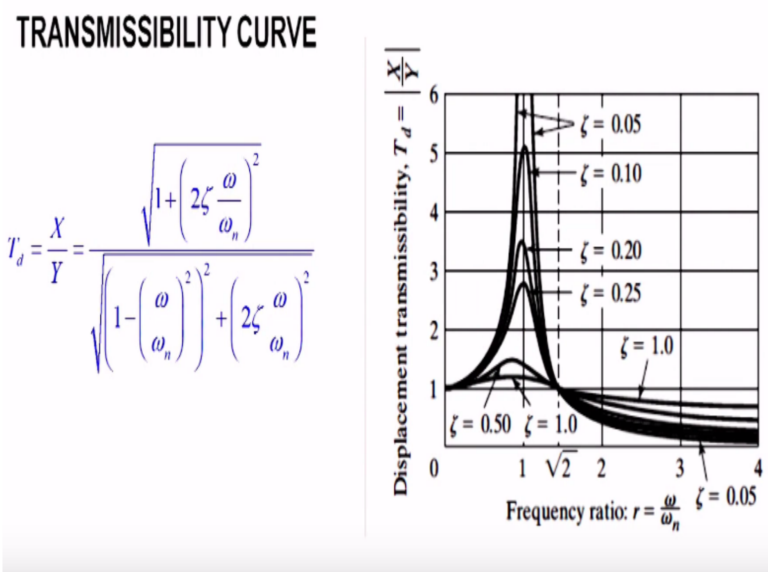

So here we can see the transmissibility curve again for the displacement transmissibility. Of course, the curve the nature of the curve will be same as, we obtained in the case of force transmissibility. So here again we see that the curve we start from one at R = 0 and it goes to Maximas near, r = 1 and again it drops and passes cross the transmissibility 1 or = 1 at under root 2.

So, root 2 it is again being unity and going to the lower values of displacement transmissibility. And when we have R that is large the transmissibility is lower, so this is the region where we are interested in this region, and as we discussed that this region is mass controlled. And so, we have to stay in this region for lower transmission of the displacement to our mass m.

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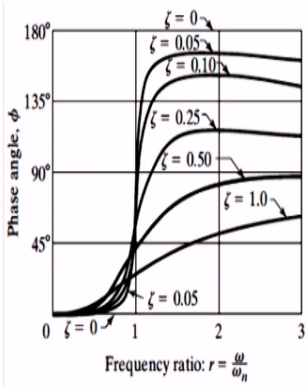
PHASE LAG

$$\phi - \alpha = \tan^{-1}\left[\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right] - \tan^{-1}[2\zeta(\omega/\omega_n)]$$



Similarly, if we see the phase lag, so phase lag, this phase is starting from zero and for Zeta less than 0.5, it is going to some maxima and then coming and tending towards 90 degree while, for Zeta > 0.5 there is no maximas but directly they are increasing to 90 degrees for r greater than for higher values of r okay. So now we will do one problem based on the displacement transmissibility.

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NUMERICAL EXAMPLE

- A radio set of 20 kg mass must be isolated from a machine vibrating with an amplitude of 0.05 mm at 500 cpm. The set is mounted on four isolators, each having a spring scale of 31400 N/m and damping coefficient of 392 N-sec/m.

(a) what is the amplitude of vibration of the radio?

(b) what is the dynamic load on each isolator due to vibration ?

So, we can see this example, we see that a radio set of 20 kg mass must be isolated. So, there is a radio set and it is kept on some vibrating machine, so there is some machine that is vibrating and on some that machine, the radio set is kept, and that machine is vibrating with an amplitude point 0 5 mm at 500 cycle per minute. So, it is given the vibration that is why the set is mounted on four isolators, so that set is put on some isolators.

Each having a spring scale of 31400 Newton per meter and damping coefficient of 392 Newton second per meter. So now it is asked that what is the amplitude of vibration of the radio and what is the dynamic load on each isolator due to the vibration. So, let us do this problem.

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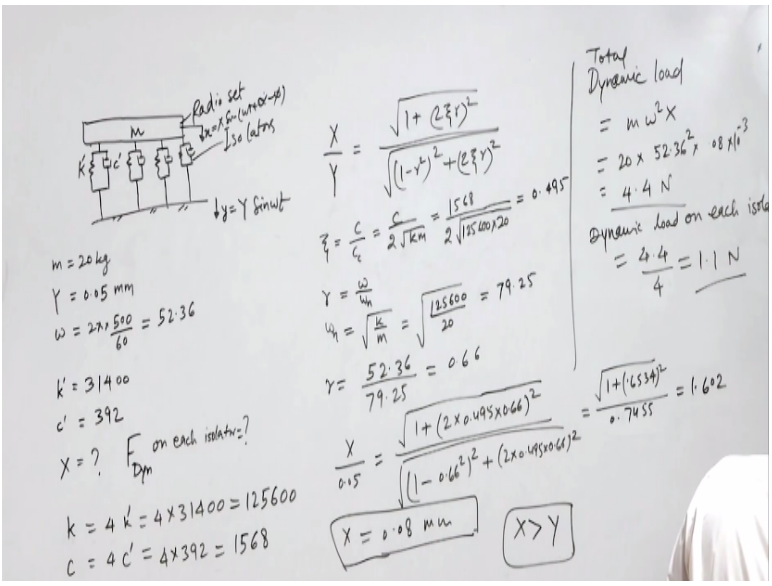

So, we have a radio set, So, it is like this system, so we have this is a radio set and it is put on four isolators, so we have four isolators okay. So, we have four identical isolators here, and this in mass m this is radio, so this is a radio set, and these are our isolators. So, four identical Isolators and each has some k and let us say k dash and c dash some values each, so m is given 20 kg and the y = y sin Omega t, so y is given 0.05 mm and then it is 500 cycles per minute.

So, Omega = 2 phi into 500 by 60 this is Radian per second. So, we can calculate and write here then the set is mounted on four isolators, so we have four isolators each, having a spring scale so k dash, that is the spring constant for one spring is 3 1 4 double zero and damping coefficient C dash is 392. What is the amplitude of vibration of the, so we want so let us say this has some motion like xt,

So, we want x, so because x = x sin Omega t - + alpha - Phi. So, we want x and we want the dynamic load on each isolator, so these are isolators and there will be some load coming from the mass, so what is the force on each isolator, so we want f dynamic on each on each isolator. k dash that is spring constant for 1 spring is 31400 and damping coefficient c dash is 392.

What is amplitude of vibration of the. So, we want let us say this has some motion like x. We want x, so x = X Sin Omega t + alpha - pie. We want x and we want the dynamic load on each isolator, these are the isolators and there will be some load coming from the mass so what is the force on each isolator. So, we want F Dynamic on each isolator. So now we have four each springs in parallel and four dampers in parallel.

So, we can arrive so combined stiffness at four times, k dash so 4 into 31400 and c = 4 times c dash so it is 4 into 392. So, we can calculate, So, first we calculate Omega, so it is 52.36 radium per second and k is 125600 newton per metre and then the damping is 4 into 392 is 1568 newton second per metre. So, we have find to X as we have already derived this relation.

X by Y = root 1 + 2 Zeta r whole square upon 1 - r square whole square + 2 Zeta r whole square under root. So, to find X, we know Y we need to find Zeta and r. So, Zeta = c by cc that is c by 2 root k into m, so it is c is 1568 by 2 root k, k is 125600 into m and m is 20 kg. So, we can compute this, so we find Zeta 0.495.

Okay So we have this damping factor. Now we have to find r.r = Omega by Omega n and what is Omega n so Omega n is root k by m. So, root k, k we have 125600 and m is 20. So, we find, it is 79.25 radium per second. So, r = Omega, Omega we calculated it is 52.36 by 79.25, so r is, it is 0.66. So Now we have r and we have Zeta and we can put in this equation X by Y, Y is 0.05 mm, so we will get X in mm = and the root 1 + 2 into 0.495 into r.

So, r is 0.66 square upon 1 - 0.66 square whole square + 2 into 0.495 into 0.66 square under root so = 1.602. So, X = 0.08 mm. Okay So we are getting X = 0.08 mm. Now, yet we see that the X > Y means the more Displacement Amplitude Motion is transmitted, more Displacement is transmitted to the Mass than what is coming to the base.

Why because we are in r = 0.66 and r < 1. And when we are r < 1, we are in the damping control region. And They are and usually we are above transmissibility more than 1. So now want to know what is the Dynamic load on each isolator. So first what is the Dynamic load, so the Dynamic load, so the total Dynamic load. First, we compute, then we will find for each isolator.

So Dynamic load comes due to the inertia force of the mass. So, this mass has inertia force m Omega Square x, so that is m Omega Square x. m is 20 and Omega we have 52.36 square into x, x we have got 0.08 into 10 power - 3. So, we will get, so it is coming about 4.4 Newton. So, because there are 4 isolators, so the Dynamic load on each isolator = 4.4 by 4 = 1.1 Newton.

So here we see that the application of this concept in the Vibration Reduction or Vibration Isolation of some sensitive article. Like the Reduce rate of some instrument. So, I thank you for attending this lecture and let us see in the next lecture. Thanks.