Introduction to Mechanical Vibration
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Lecture – 18
Vibration Isolation and Force Transmissibility

So welcome to lecture on vibration isolation and force transmissibility. So what is vibration isolation and why it is important. So as we know that there are some machines, rotating machines or reciprocating machines, they have some unbalanced forces and so vibrations from that machines pass to the foundations where they are connected and so in order to stop these vibrations, we have to use the isolation.

There is are some sensitive instruments, they are kept on some measurement and due to the vibration of the base okay, so there is the vibration of the instrument or there is some error in the treating of the instrument okay. So the sensitive items they must be isolated from the foundation or where they have kept okay. Similarly, the vibrations coming from the machine should be isolated.

So that they do not reach to the foundations otherwise they will create some vibration on the n problem to the people working on the floor.

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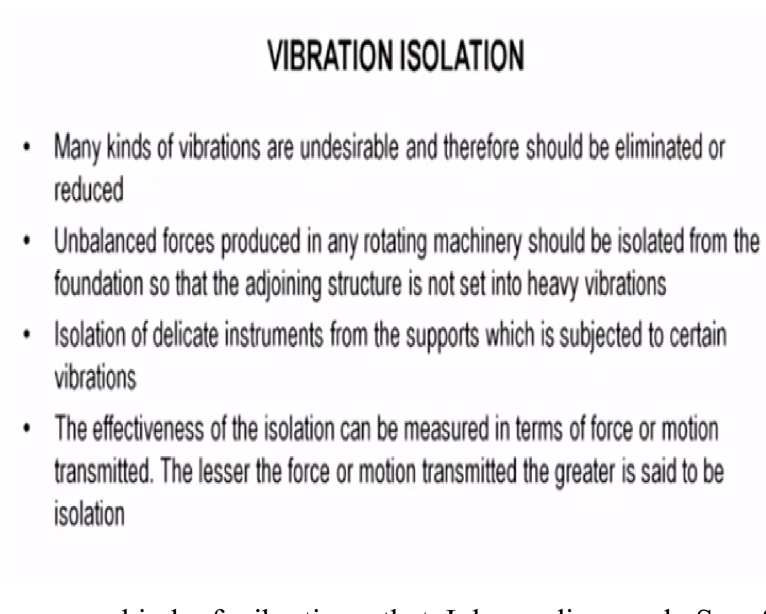

So here we have many kind of vibrations that I have discussed. So effectiveness of a vibration isolation, how do we measure, so effectiveness we measure that if a vibration after introducing the vibration isolation, the force transmitted to the foundation or the motion transmitted to from support to the instrument or some sensitive article, the motion transmitting is reduced.

So force and motion transmission is reduced then we say that this is effective vibration isolation. So here we have some materials for the, that we use for the vibration isolation. So basically, they provide the stiffness and damping okay they have some stiffness and damping of those materials and we can see here some examples.

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So the first example you can see here there is some machine and this is the isolator. So this is in the form of some spring. Another example is empty this injection moulding machines and here you can see the anti-vibration pads. So there are some pads made of rubber and they can be used as the vibration isolator. Then there are some felt or cork and some metallic springs okay. So these are some common materials, common components that are used as a vibration isolation.

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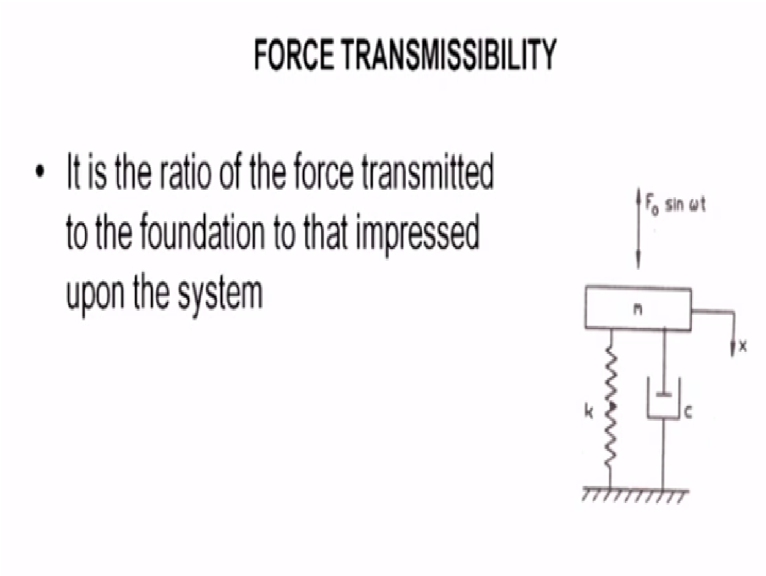

So what is transmissibility, so transmissibility of course, we are discussing the force transmissibility. So this is the ratio of force transmitted to the foundation to that impressed upon the system. So we will discuss in detail how to find the transmissibility. So we have this system.

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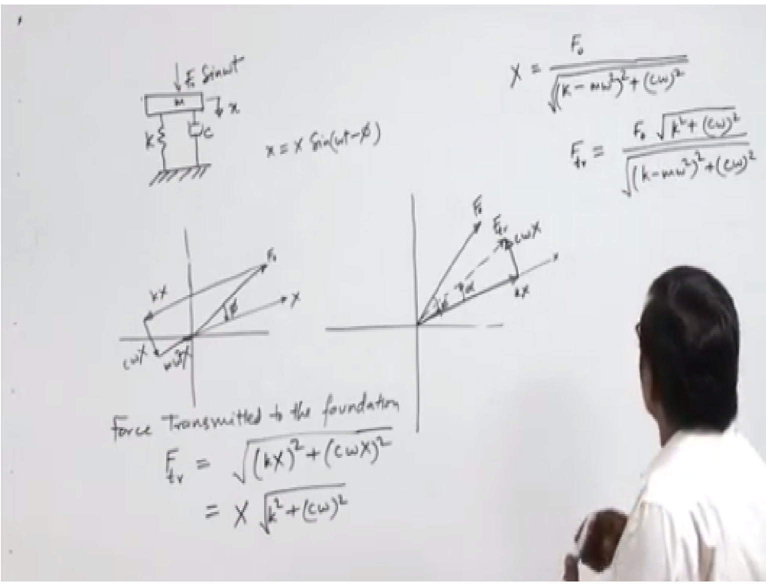

This is mass and so this part is isolator, so it has some stiffness and damping and this is the machine that is kept on this isolator and this is the foundation. We want, so this system is subjected to some force f0 sin omega t and so it is vibrating. So some forces are going to transmit to this foundation and we want to know what are those forces. So as we have already discussed this system because this is single degree of freedom forced vibration system.

And we have shown the vector diagram for the forces that are relevant to the system. So basically they are of course the impressed force f0 then the spring force, the damper force and inertia force. So here we can recall the vector diagram, so if we have, so this is the direction of x and this is f0 and this was the angle phi, so x lags so in fact x was x sin omega t - phi. So there was a phase lag.

Then there was a spring force kx then the damper force 90 degree c omega x, then there is the inertia force m omega square into x okay. So this is the vector diagram of the forces of the system and the system is in equilibrium under the effect of these forces. Now we are talking transmissibility means the force that is transmitted to the foundation, now how the force can be transmitted to the foundation.

So the force can be transmitted to the foundation through the elements that are attached to the foundation. So what are those elements, they are this spring element and damper element. So stiffness and damping element through this the force that is acting on the system is passing to the foundation. So we can make and of course the force that are passing to the foundation that is the spring force and the damper force.

But their direction will be opposite to what we have shown here because these forces are shown on the mass m and the opposite forces will be transmitted to the foundation okay. So the spring force, this is k x, so suppose this is x direction, now the spring force kx o this force in the opposite direction, so this is here kx and then damper force, the omega x but in opposite direction.

So this is the damper force c omega x and if we do the vector sum of these forces let us keep this name f t r and here we have f0 okay, and here is this phi and there is some angle here alpha, let us alpha. So f t r, so the force transmitted to the foundation f t r. So f t r is the vector sum of these two forces, spring force and damping force, so it is under root kx square + c omega x square.

And so it is k square + c omega whole square under root into x okay. So this is the force that will transmit to the foundation. Now what is x, so x we already have known from these vector diagram and that x = f0 upon k - m omega square whole square + c omega whole square under root, so this is x. So we put these value of x here, so we have f t r =, so f0 under root k square + c omega whole square upon k - m omega square whole square + c omega whole square under root.

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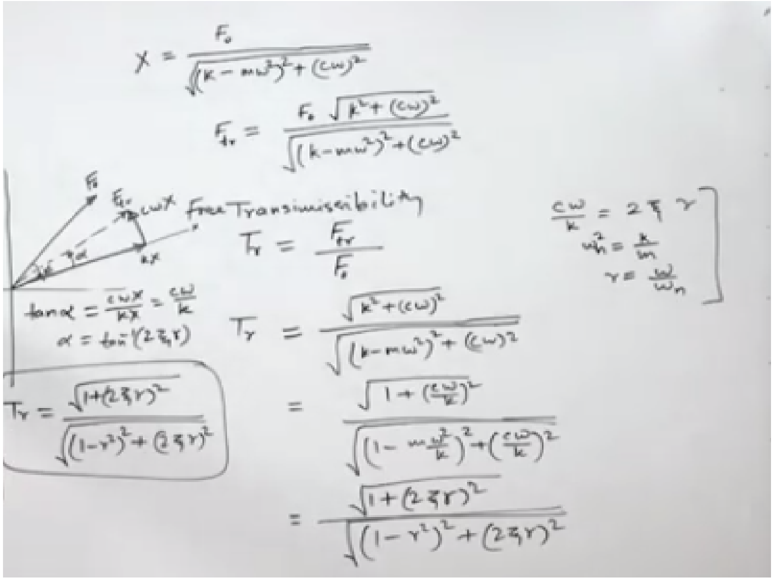

So we have replaced k with these term, x with this term. Now as we say transmissibility, of course force transmissibility that is force transmissibility. T r =, we say that transmissibility is the ratio of the force transmitted to the foundation, so this is the force that is transmitted to the foundation, that is f t r upon the force that impressed upon the system. So the impressed force is f0 sin omega t, so amplitude f0. So here we have f0.

So from this equation, we can find f t r by f0 = k square + c omega whole square upon under root k - m omega square whole square + c omega whole square okay. So this is the initial formula for the force transmissibility. So it says that it represents the ratio of the amplitude of the force transmitted to the foundation by the force impressed on the system.

Now we take k outside, we can write root one + c omega by k whole square upon one - m omega square by k whole square + c omega by k whole square under root. So I take k here outside also here, so they will cancel out. So we have this term and this term we can write like, so we can write this term because c omega by k already we know that c omega by k is = two into zeta into r and zeta is c by cc okay.

And omega n = omega n square is k by m and r = omega by omega n, these things we know. So always we try to relate to finalise our formula in this in terms of this parameter. So we have one + two zeta r whole square under root upon one - r square whole square + two zeta r whole square under root. So finally, we can define, we can write that transmissibility t r = one

+ two zeta r whole square under root upon one - r square whole square + two zeta r whole square under root.

So this is the final formula of the transmissibility. Moreover, we can write, so this is phi and this is alpha, so what will be alpha. So here tan alpha, tan alpha is c omega x by k x, so c omega x by k x, so x x cancel out, so c omega by k. So it means alpha is nothing but tan inverse c omega by k, and c omega by k is two zeta r, so we can write two zeta into r okay.

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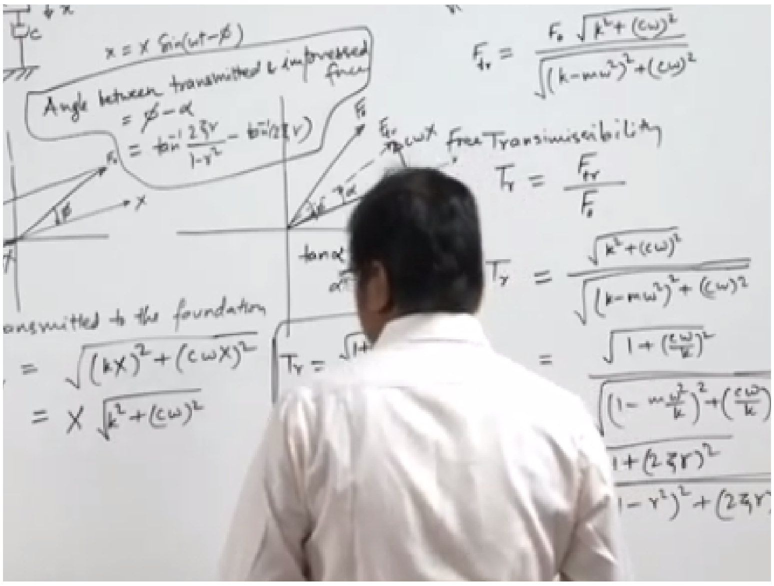

So angle between transmitted and impressed force =, so it is phi - alpha. So phi - alpha and phi already we know that phi is tan inverse two zeta r by one - r square. So it is tan inverse two zeta r by one - r square - tan inverse two zeta r okay.

So this is the angle of lag. Now we can understand as we always do, we can draw the frequency curves for the transmissibility because we see the transmissibility is a function of the frequency ratio that is, so the function of the excitation frequency upon the natural frequency and the damping factor zeta. So we can draw the frequency curves for transmissibility and we can get some conclusion, some guidelines okay, that can help us two design our system.

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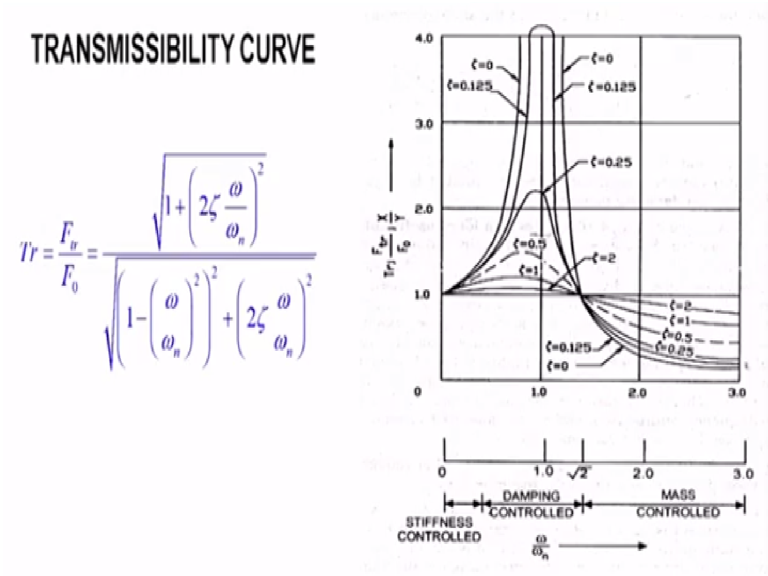

So we can next see this, so here transmissibility curve I have plotted. So this transmissibility curve is for, so here we can see that this transmissibility on the y axis that is f t r by f0 and here is the omega by omega n that is r. Now one thing we should we can one conclusion we can get from here.

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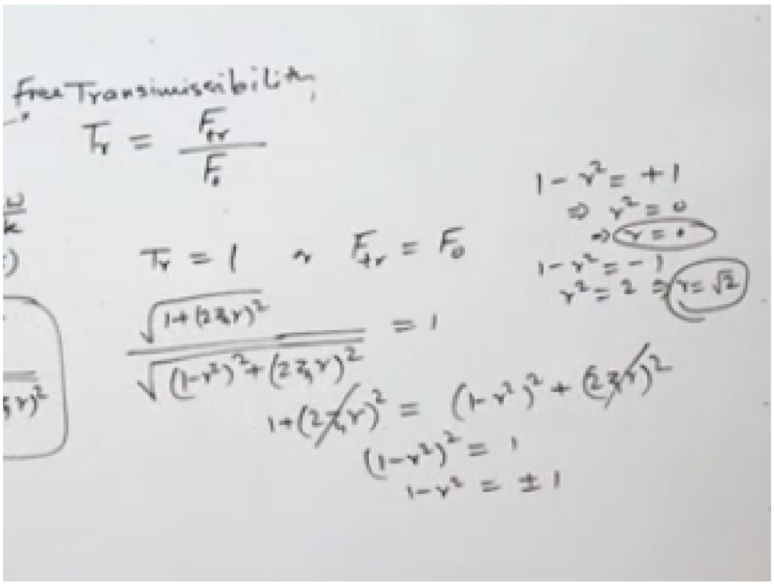

So if we do like when the transmissibility is one means we want to know when the transmissibility will be one means the force that is impressed, that same force is transmitted to the foundation. So transmissibility is one in this case or f t r = f0. So when we put t r = one, so this equation root one + two zeta r whole square root one - r square whole square + two zeta r whole square, this is = one.

Now we square these terms, so one + zeta r whole square = one - r square whole square + two zeta r whole square. So these terms will cancel out and we have one - r square whole square = one. So one - r square = + - one. So if we take one - r square = + one, so we get r square = one - one zero means r = zero.

When we take one - r square = - one, we get r square = two this means r = root two. So we are getting two values of r when the forced, the same amount of force will be transmitted to the foundation as will be impressed on the system that is f t r = f0 and that is r = zero and r = root two. So from this curve, we see this curve what we see that, at r = zero, here transmissibility is one and again this is again r = root two, this is root two here when transmissibility is one.

Moreover, when r is much less than one, for this transmissibility is tend to one and when r is much less than one, much greater than one, the transmissibility each tending to zero okay. So we see that the transmissibility curve, transmissibility curve starts from one and near the resonance condition, that is r = one, it is amplified and again it falls and passes through one at r = root two.

And then it is tending to towards zero at the higher values of omega by omega n. So now we can get some important conclusion here. So this completed a region can be identified in three different regions. Of course, here this first region stiffness controlled region because here we have r much less than one.

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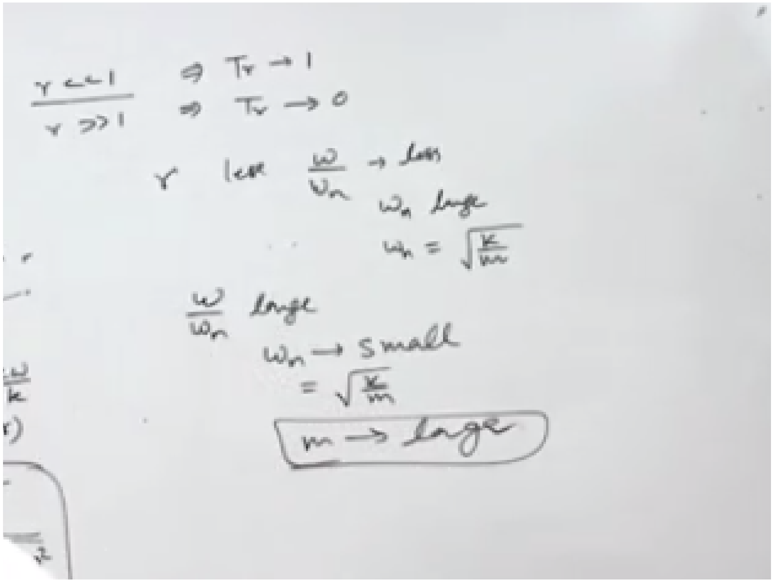

It means when r is very less, so omega by omega n is less, so it means omega n is large and so omega n = root k by m. So we can change the stiffness of the spring, we can take some large value of stiffness and we can, so we are in this region. In the region near one, here the effect of damping is quite clear that if we have low damping, the transmissibility is very high and we have higher damping to transmissibility is low.

So this region is completely dominated by the effect of damping. When we come to this region, when omega by omega n is large, so omega by omega n is large, means omega n should be small. It means, omega n is root k by m. So it means, to have a small omega and we must have m large okay. So it is large. So we see that in mass controlled region for lower by lower damping, we have the better performance means the transmissibility is less in the mass controlled region, transmissibility is less for the lower value of damping okay.

And the overall transmissibility is decreasing, so this is the region that we want to operate our system. So we always try to keep omega by omega n large, so that we are in the mass controlled region and of course, how can we change the mass of the system. So we usually to the machines, we attach some heavy concrete blocks, so we increase the mass and then we support them on some stiff, some high stiffness springs.

So due to the very heavy mass of the concrete block, we already shift to the mass controlled region and so the transmissibility of the vibrations is less to the foundation. Now we come to the phase lag. So phase lag we have already defined that the phase, the angle of lag between the force, replaced force and the transmitted force, this plot is showing.

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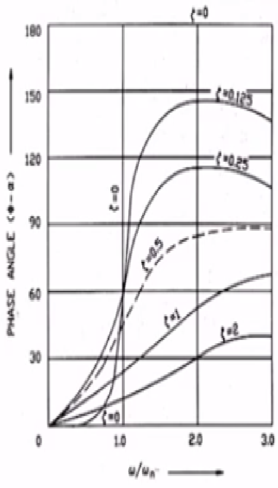
PHASE LAG

$$\phi - \alpha = \tan^{-1}\left[\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right] - \tan^{-1}[2\zeta(\omega/\omega_n)]$$



We see that for value of damping less than 0.5, the curves are going reaching to some maximum value and then falling and tending towards 90 degree while for curves greater than damping of 0.5, the curves are directly they are increasing to towards the 90 degree phase angle.

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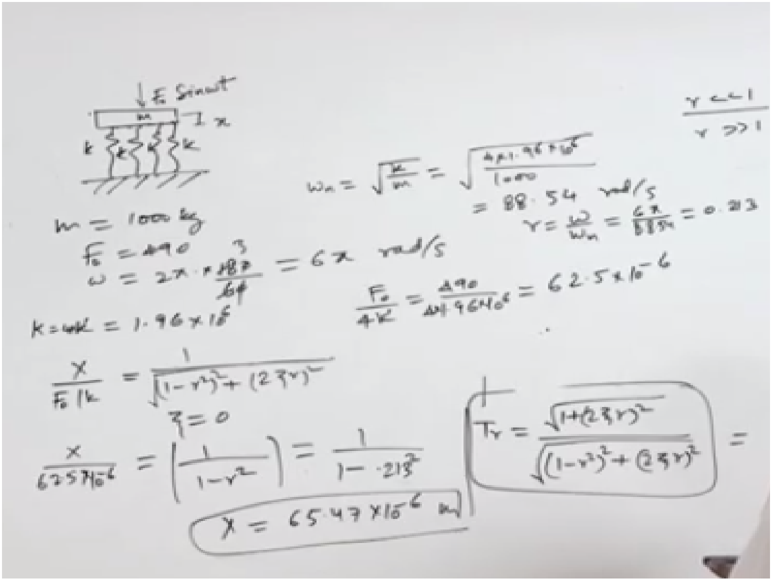

So now we take one numerical example to apply these expressions and to understand the concept more better way so let us start this okay. So we have 1000 kg machine is mounted on four identical springs. So here we have four springs, so like here is one, two, three, and four and similarly four dampers here okay. So there four springs okay. So here it says an negligible damping.

So we have no damper, so we have only four springs, so we can make these, so these are four springs okay. And each has the same spring constant k. So we have m = 1000 kg and four springs is and k, they have a spring constant k. The machine is subjected to harmonic external force of f0 490.

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NUMERICAL EXAMPLE

- A 1000 kg machine is mounted on four identical springs of total spring constant k and having negligible damping. The machine is subjected to a harmonic external force of amplitude F_0 = 490 N and frequency 180 rpm. Determine the amplitude of motion of the machine and maximum force transmitted to foundation because of the unbalanced force when k = 1.96×10^6 N/m.

So f0 is 490 and frequency is 180, so omega is, so two pi into n, so two pi into 180 by 60 in radiant per second. So we can find it is 6 pi radiant per second. Now we determine the amplitude of motion of the machine and maximum force transmitted to foundation because of the unbalanced force when k =, so k is k by n 1.96 into 10 power 6. So first we have to find the amplitude of motion.

So for this we have this formula x upon f0 by k = one upon one - r square whole square + two zeta r whole square under root. So now if zeta is zero because we have negligible damping, we have it is one by one - r square. So x upon f0 by k, f0 is 490. So f0 by k we calculate that is 490 by 1.96 into 10 power 6 but here we have four springs, so the total stiffness is four times k.

So it is four times k, so this is into four. So we can compute this, so four into 1.96 =, so it is 62.5 into 10 power - 6. So here in fact this we say k = four times, so let us say this is k dash okay. So this is 62.5 into 10 power - 6 and one upon one - r square. So now omega n = root k by m and it is four into 1.96 into 10 power 6 upon m, m is 1000, so we can get, so it is 88.54 radium per second.

And so we can get r = omega by omega n and omega is six into pi. So six pi by 88.54. So it is 0.213. So we have 0.213 square so we can find x, so we have 65.47 into 10 power - 6 meter. So this is the x and that is very less x. Now we have to find out forced transmitted. So forced transmitted we can find =, so we have zeta = zero, so we have same as one - r square. Sorry, this is transmissibility, we can find the force transmitted.

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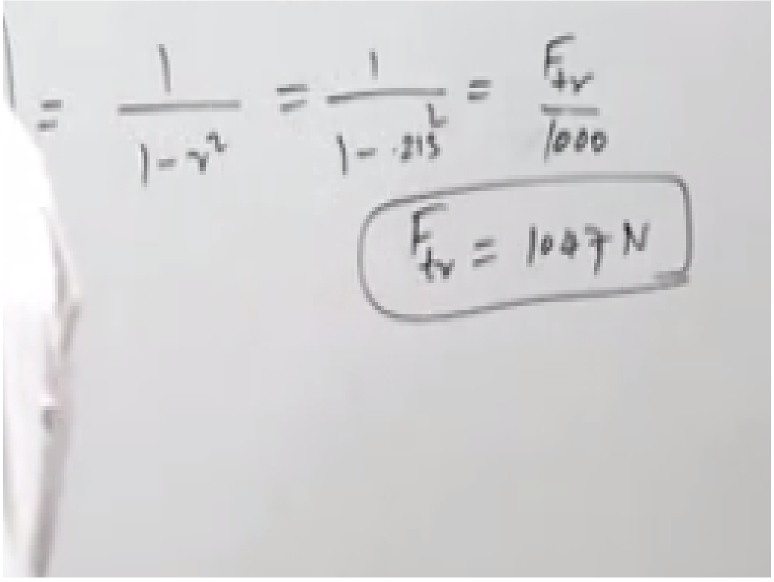

So first we find one by one - r square, so one by one - 0.213 square and that is f t r by f0. F0 we know it is 1000, so from here we can find, so here f t r = 1047 newton post transmitted to the system. So here we had four spring in parallel, so we have to combine, took the combine stiffness of these spring and then we can use apply these formula when data is zero, we can simplify.

And similarly, we can find the transmissibility and that is f t r by f0 as we know. So we can find the force transmission. So here we would stop this session, so thank you for attending this lecture and see you in the next lecture.