Introduction to Mechanical Vibration

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Lecture – 17
Structural Damping and Equivalent Viscous Damping

So welcome to the lecture on structural damping and equivalent viscous damping. We have already discussed the forced vibration response of a system, single degree of freedom system having viscous damping and then we discussed in the previous lecture the column damping and we found that to deal with column damping, we calculated the equivalent viscous damping and then we used the same equations that we derived in case of viscous damping.

So as I said that the energy dissipated in one cycle can be equated for different type of damping and equivalent viscous damping can be obtained. And that can be used in the differential equation and the amplitude response that we already have obtained in some lectures. So today we are dealing with the structural damping, so what is structural damping, so we can recall that a structural damping when we load some material.

And then unload that material, so the hysteretic diagram that is different in loading and unloading. So there is some loop that we can see here. There is a loop formed when we load it and when unload it, so means in loading we give the energy, but during the unloading we do not get the same amount of energy. So where does the difference of energy goes, so this difference of energy is expanded in the molecular friction of the material.

So this concept of energy dissipation is used as damping and that is called structural damping or solid damping or material damping or hysteresis damping because here the loop that is formed is called the hysteresis loop. So from the experiment it is found that the energy dissipated per cycle in hysteresis damping is proportional to the square of the amplitude. While we saw that in case of the viscous damping, the energy dissipated per cycle is pi c omega x square.

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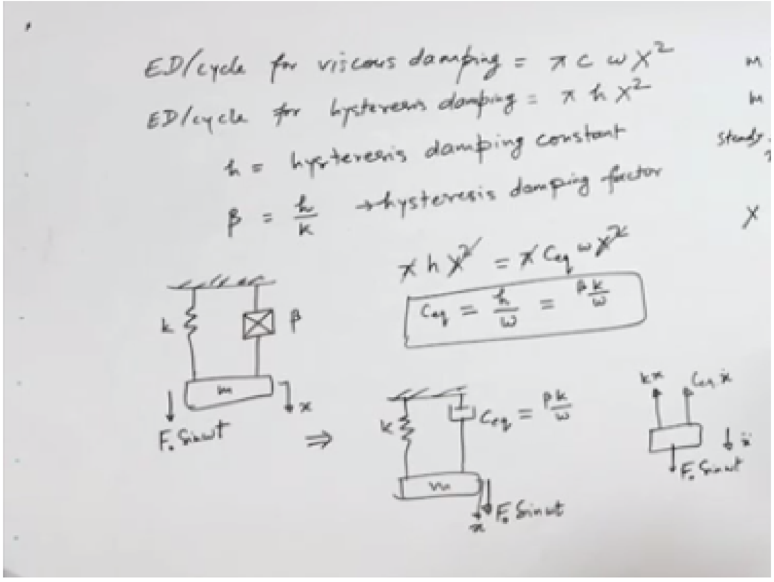

So we can write, so we see that, the energy dissipated per cycle for viscous damping is pi c - x square. However, energy dissipated per cycle for hysteresis damping or solid damping, it does not depend on the frequency, but so it is like pi h x square, x is the amplitude of the response. So here h h hysteresis damping constant and we can define beta and beta is h by k is hysteresis damping factor where k is stiffness.

So if we have a system, that is so we show the hysteresis damping in this way. So this is m k and this is let say beta. So we represent h or beta and this system is doing some motion here and it is subject to some force, f0 sin omega t. This we can see here that is the same system.

Now as I said that we have to convert this system to an equivalent viscous damping system. So that we can use the derived equations and the formula of the response, so here we can equate the energy dissipated per cycle. So energy dissipated per cycle pi h x square. That is the energy dissipated per cycle in the hysteresis due to hysteresis damping and we assume that we have to know the equivalent viscous damping.

So in a viscous damping, the energy dissipated per cycle is pi c omega x square. So we want to know what is the c equivalent. So c equivalent is the viscous damping that is equivalent to the hysteresis damping. So we can find c equivalent here. So here we will cancel out some term and we will find c equivalent is h by omega r because h is k times beta. So we can write beta k by omega.

So this is the equivalent viscous damping corresponding to the hysteresis damping. Now this system, we can represent to an equivalent viscous damping system. So here we can write it is like k and viscous damping. We can show a viscous damper, this is mass k and here it is c equivalent and this is equal to beta into k by omega or h by omega.

And this system is subjected to f0 sin omega t. And this is the response of the system here okay. So as we can make the free body diagram on the system, this is here it is f0 sin omega t. Here are the spring force k into x, here c equivalent into x dot and this is x double dot.

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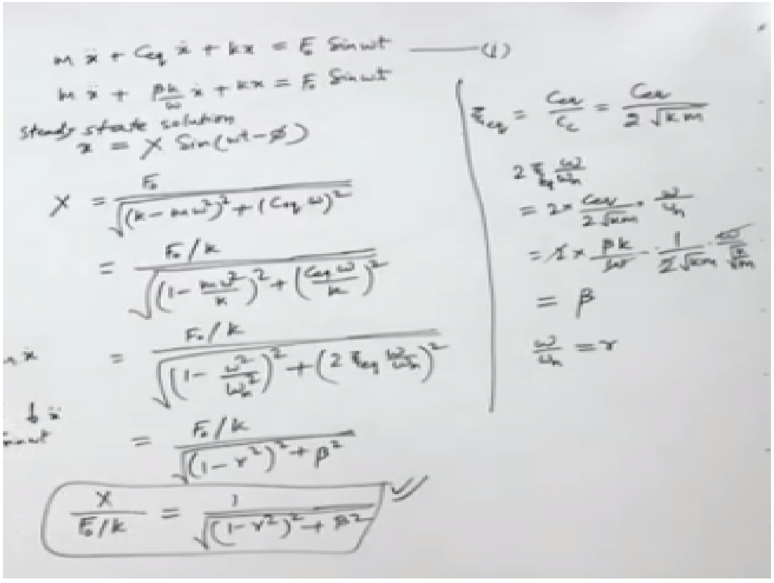

So we have already, we can write the equation on most in that will be mx double dot + c equivalent x dot + kx = f0 sin omega t. So we can write mx double dot + c equivalent is beta k by omega x dot where kx = f0 sin omega t. So for such a system with m c and k, we already have find the response of the system.

So the solution of this equation, let say equation one will be steady state solution, here we say steady state solution, x = x sin omega t - 5. So here as if we recall x = f0 by k - m omega square whole square +, now here c is c equivalent. So c equivalent into omega whole square under root. So because these formula we have already derived when we discussed the forced vibration of viscous damp system.

So there were stiffness and viscous damping and mass elements of a single degree of freedom system. So these equations are as well discussed how we achieve this equation, so this term we can write f0 by k because we can take here k outside and this is one - m omega square by k whole square +, so c equivalent into omega by k whole square. So let us write this like f0 by k upon one - omega square and k by m is omega n square.

So we can write omega n square upon whole square +, this we can write two zeta equivalent into omega y omega n whole square. So it is zeta equivalent, so zeta equivalent is nothing but c equivalent y, c c, c c is critical damping and we know that cc = two root k into m that we have already derived. Now c equivalent is beta k by omega, so we here write, now if we come to this term, two zeta omega by omega n so this zeta equivalent.

So two into c equivalent by two root k m into omega by omega n. Now two into c equivalent is beta k by omega. So c equivalent is beta k by omega, so we write beta k by omega into one by two root k n into omega by omega n and omega n we can write root k by n. So it is root k by n. So here is two is cancel out, omega is cancel out. Here m m cancel out and root k, root k it is k and k k cancel out. So it has come to equal to beta.

So we can write = f0 by k one -. So let us say omega by omega n = r as we have already used this type of representation. So one - r square whole square +, + here two zeta equivalent omega by omega n = beta. So we can write beta square and then under root. So here x by f0 by k = one by one - r square whole square + beta a square.

So this is the response amplitude of the system having the hysteresis damping. Now we have to find the angle phi.

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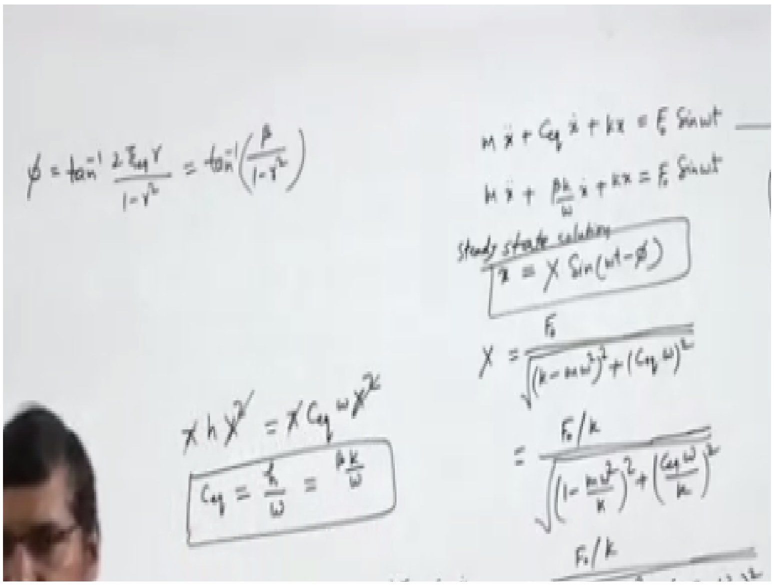

So angle phi as we know that phi = tan inverse two zeta r one - r square. So it is equal to tan inverse two zeta r is beta, so beta upon one - r square. So this is phi, so from here we can find this response, we know x, we know phi, so this response is obtained. Now we can see because this response is function of this beta n and r and r is nothing but the ratio of the excitation frequency and the natural frequency.

So we can have this plot of amplitude ratio and phase angle.

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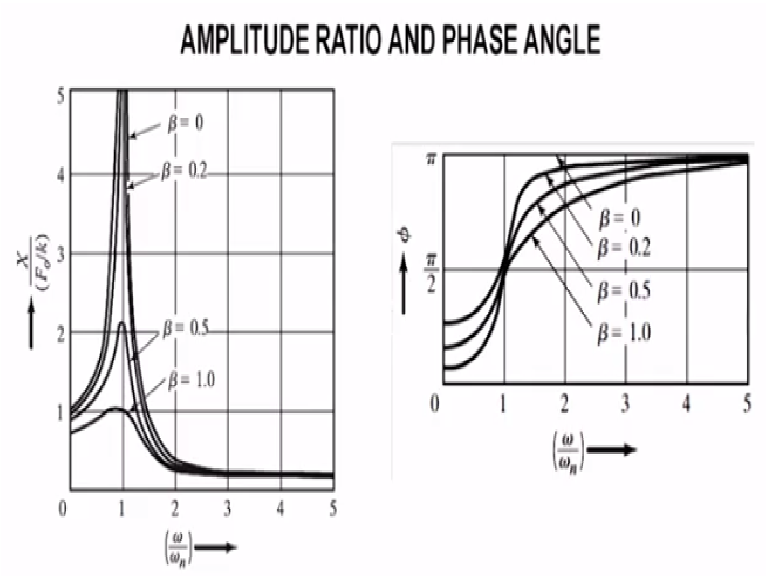

So we have plotted this x by of0 k on the y axis and r that is omega by omega n on the x axis and for different value of beta means different value of hysteresis damping factor, we have plotted these graphs and we can see that these graphs, so if we put here, r = one this is maximum and that is one by beta okay. So here we see that when r = one, we are getting the maximum amplitude for any value of damping, any value of beta.

So the response is increasing when omega by omega n is less than one, so it is increasing till one omega means till the resonant frequency and then it becomes maximum at resonance and then it is decreasing and tending towards zero when we are increasing the omega by omega n value okay. Now when we see the phase angle curve, so what we see that, in this phase angle curve, we see that when r = zero, it is stramineous beta okay.

So means that this curve starts with some phi > zero okay. And so the force and the response x, they are never in phase. And they all passing through the phi = 90 degree for any value of damping and finally when we increase the value of omega by omega n there are tending towards the phase of 180 degree. So now we will discuss one problem that it based on this theory. So let us take one numerical example okay.

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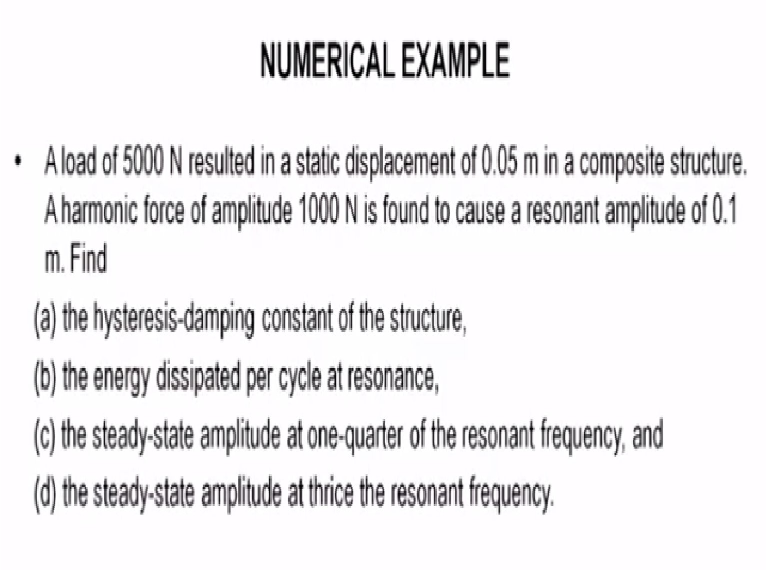

So we can see the problem here, numerical example, so a load of 5000 newton resulted in a static displacement of 0.05 m in a composite structure. So there is some structure, some composite structure and when we apply a load of 5000 newton static load, so we find a static displacement of that structure 0.05 m. Then a harmonic force of amplitude 1000 newton is found to cause a resonant amplitude of 0.1 m. So here it is said that, so let us write.

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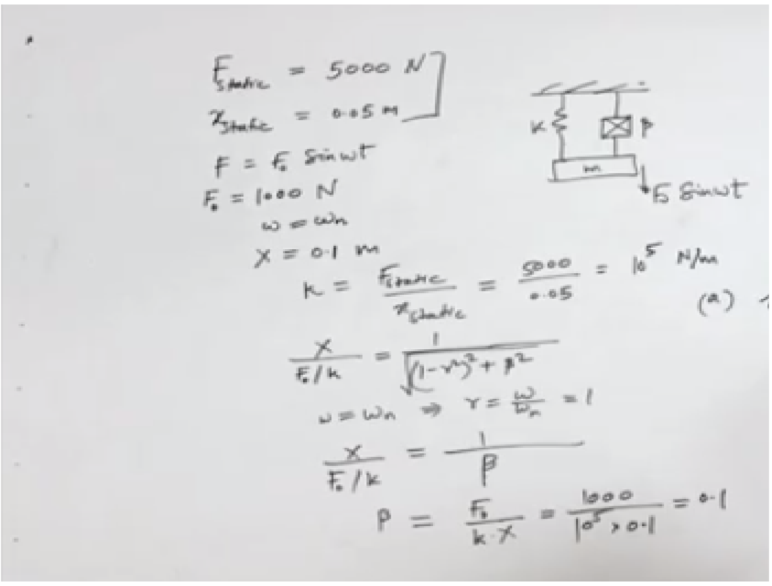

So here f static = 5000 newton and x static = 0.05 m. Now it is said that a harmonic force of amplitude. So let us say that harmonic force f = f0 sin omega t. This is our harmonic force and the amplitude of the harmonic force. So f0 = 1000 newton. So this is 1000 newton. Found to cause a resonant amplitude of 0.1 m. So here it is said the resonant amplitude. So what is resonant amplitude.

So resonant amplitude means when the excitation frequency matches with the natural frequency means excitation frequency is equal to the natural frequency of the system. So means, here omega = omega n and at this frequency of forcing, the x that is the amplitude of vibration is 0.5 m, so x = 0.5 m. Now what we have to find, we have to find the hysteresis damping constant of the structure.

So hysteresis damping constant, we have to find and we have already defined what is hysteresis damping constant then we have to find the energy dissipated per cycle at resonant, so what is energy dissipated per cycle, the resonant condition. Then the steady state amplitude at one quarter of the resonant frequency, so if the excitation frequency is one fourth of the omega n okay.

So we are in the region of less than omega by omega n less than one and the steady state amplitude at thrice the resonant frequency, it means omega = three times omega n. So we are in the region when omega by omega n > 1 okay. So we have to solve this problem. So now the first thing is that if we have supposed we have this system, so this is m k and this is let say beta and this is f0 sin omega t okay.

And we have to, so here we have to find the stiffness k, how we can find k from this information as it is said that if we apply 5000 newton, the displacement, the static displacement at 0.05, so k = f static upon x static because the stiffness is force upon displacement. So we have 5000 upon 0.05. So we can find, we can solve it and we can calculate it 5000 by 0.05 =, so it is 10 power 5 newton per newton. So this is the stiffness k.

Now we have to use because we are given x at omega = omega, we have to use this formula x by f0 by k. So x by f0 by k = one upon one - r square whole square + beta square under root. Now when omega = omega n that is, this means that r = omega by omega n = one. So if we put r = one here, we will find x by f0 by k = one upon.

So one - one is zero, so we will have beta square and under root beta square, so we will get one by beta okay. So from here, we will get beta = f0 by k into x. So we get beta = one by x and this is f0 y k. Now what is f0, so f0 is 1000 newton, so we have 1000 newton, k is 10 power 5, so this is 10 power 5 into x, x is 0.1 m. So we will get, this is 10 power 3, 10 power 5, so it is 100, 100 into 0.1. So we have 10, so it is 0.1. So we get 0.1 beta value.

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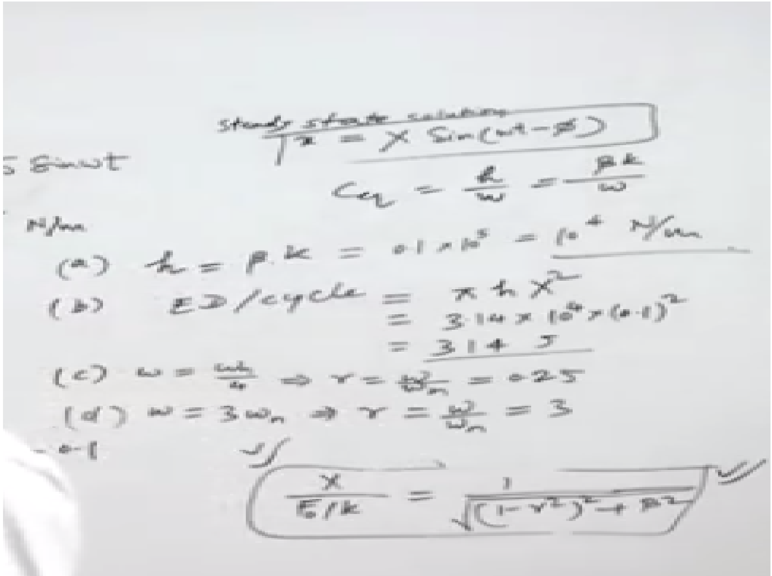

Now we have to find the A part the hysteresis damping constant. So what is hysteresis damping constant A. So hysteresis damping constant is h and h is beta into k, so beta is 0.1 into k. So k is 10 power of 5. So we get it is 10 power 4 newton per meter okay. So we have got the part A.

Now we have find the energy dissipated per cycle at resonant. So energy dissipated per cycle, we have already found this formula of ed per cycle. So ed per cycle is pi h x square. So pi is 3.14 let us and h = 10 power of 4 and x at resonant is point one, so into zero point one square. So from here, we can find, so it is 0.01, so we can find 314. So it is joule okay. So this is the energy dissipated per cycle.

To find the steady state amplitude at one quarter of the resonant frequency, so c part we have to put, so here omega = omega n by 4, this means that r = omega by omega n = 0.25. Now if the value of r, we put here and the value of beta what we obtain here 0.1, we put here, we know f0 the magnitude and k we have already know. So we can get x from this formula.

Similarly, in case d, we have omega = thrice the omega n, so three times omega n, this means that r = omega by omega n = 3. So similarly, in similar to case c, we put r = 3 here, beta = 0.1, f0 = 1000 and k = 10 power 5, so we can also get the steady state amplitude x. So we complete this part. So thank you for attending this lecture and see you in the next lectures.