Introduction to Mechanical Vibration
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Lecture - 16
Coulomb damping and equivalent viscous damping

Welcome to the lecture on forced vibration under Coulomb damping. So far, we have discussed the case of the viscous damping. Now when we have Coulomb damping in the system, it means that there is some mass that is moving on some surface, some dry surface and there is friction between the two surfaces. So, there will be some energy dissipation and if we have this type of damping, what will be the response of the system.

So there is some easier ways to study this kind of systems. One is that we convert this kind of damping, like Coulomb damping to an equivalent viscous damping. How and why it can be convertible. So first, because the basic function of damping is the energy dissipation, so in a viscous damping case, there is some damping in the slow movement of some piston in the fluid, this kind of damping is regarded as viscous and there is energy dissipation.

But in friction also, there is energy dissipation, so if we can get that, how much energy is dissipated in each cycle of Coulomb damping system and we equate this energy to the energy dissipated per cycle of a viscous damping system. From there, we can get the equivalent viscous damping coefficient and if we get the equivalence, we can use all the expressions and all the equations that we derived for viscous damping.

So therefore, we are going to do this study of the Coulomb damping with converting it into an equivalent viscous damping system. So let us take a system.

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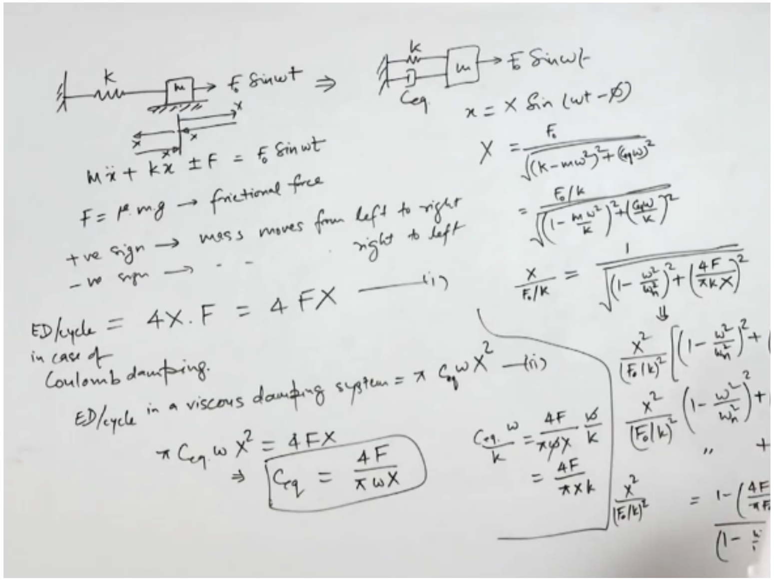

So we have a spring mass system that is moving on a dry surface and there is some forced applied that is F0 sine omega t, the harmoning force. So if we can write the equation of motion for this kind of system, so equation of motion, mx double dot + Kx and here it is +/- f = F0 sine omega t. So Fx mu times m into G that is the frictional force. So there is +/- sign and because the friction force depends on the direction of the relative velocity.

It can be positive. So here the + sign, positive sign is taken when the mass moves from left to right and take negative sign when mass moves from right to left. Now we want to solve this system and as I said that we will use the equivalent viscous damping concept. So first, we have to know that how much energy in case of Coulomb damping is dissipated in every cycle. So we know that this mass is moving against some frictional force F.

And this mass first it is moving here the maximum displacement, let us say x, then again coming back to its equilibrium position, it is moving here x, then again going here the maximum that is x and again coming to this equilibrium, so moving x. So it means that this mass is moving in one cycle. So one cycle is complete when it starts from here and reach at the same point, but in this one cycle it is moving by 4x, so four times x.

Because in quarter cycle, it is moving the maximum amplitude x and during it movement, there is constant friction force acting on the system. So the work done by this friction force or the energy dissipated per cycle is equal to 4x into F that is 4Fx, in case of Coulomb damping. Now we know that in case of viscous damping system. The energy dissipated per cycle is pi C omega x square.

So energy dissipated per cycle in viscous damping system = pi C omega x square. So it means we are going to convert the system into an equivalent viscous damping system. So this is mK and here is C equivalent and this is F0 since omega t. So we are going to covert the system into this system and here we say C equivalent because we are going to covert equivalent viscous damping for the Coulomb damping case.

So now we equate this equation #1 and 2. Because we are equating the two energies per cycle, so pi C equivalent omega into x square = 4 times Fx, this means C equivalent = 4 F upon pi omega into x. so this is the equivalent viscous damping. Now if we have the equivalent viscous damping here, we can use all the expressions that we have derived for the viscous damping case. So the basic thing is we want to know the x that is the amplitude.

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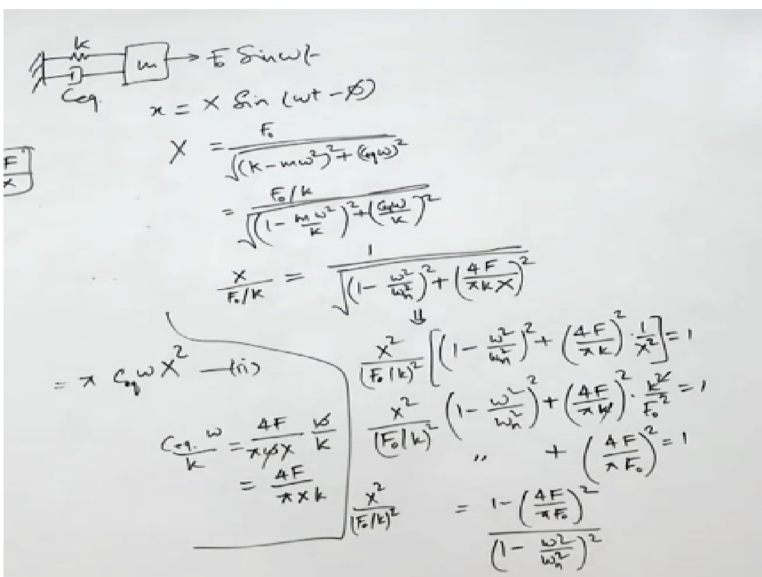

And in this case, we have our equation of the response of the system x = x sine omega t - phi. So here x = F0 by K, so F0 upon root K - m omega square whole square + C omega whole square under root or we can write F0 by K 1 - m omega square by K whole square + C omega by K whole square under root. Now it is C equivalent. So here we can put C equivalent. Here also we put C equivalent because now our C is C equivalent.

And now we can write here x by F0 by K = 1 upon 1 - omega square by omega n square whole square +. We can put here C equivalent value here. So C equivalent omega by K = 4 F by pi omega x into omega by K. So omega will cancel out. We will get 4F by pi x K. So here we will get 4 F by pi K x square under root. Now this equation we have to solve. Because x is here and x is also here, so we have to take out this x in one side.

So we square these both sides, so we will get x square by F0 by K square into, this is under root, so it will be just like 1 - omega square by omega n square whole square + 4F by pi K square into 1 by x square = 1. So we will have x square by F0 by K whole square 1 - omega square whole square +. So x square here will be canceled out and here will be 1 by F0 by K square, so it is K square by F0 square and that is equal to 1.

Here, this is also K square, so this K square will be canceled out. So we will have this term as such + 4 F + pi F0 whole square = 1. Now we take this other side, so it is 1 - 4F by pi F0 square and that is x square by F0 by K square upon 1 - omega square by omega n square whole square.

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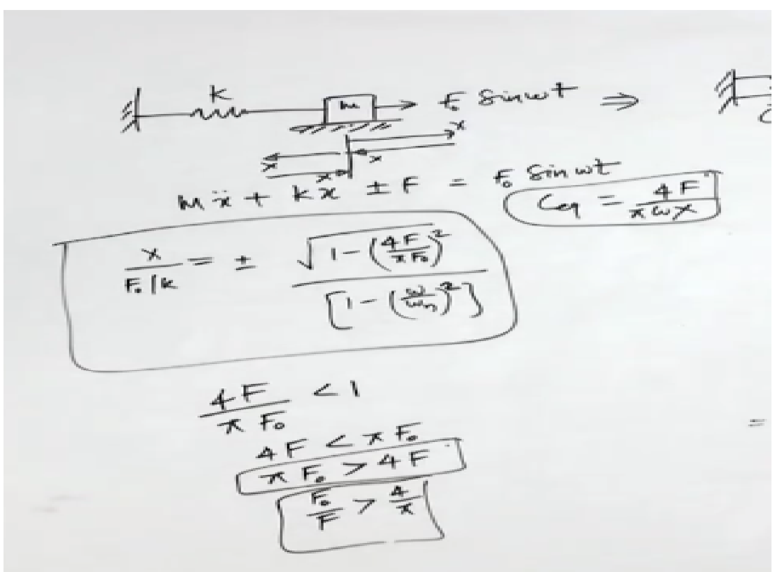

So here we got C equivalent = 4 F by pi omega into x. So we can find x upon F0 by K = maybe +/- under root 1 - 4 F by pi F0 whole square upon under root 1 - omega by omega n whole square. So this will be under root, here we will have, because we have a square, so we will have only this term. So this the x that we get the amplitude of response of Coulomb damping system.

Now we see that to have a real solution, that is a solution that is not complex. We must have this under root quantity must be greater than 0. It means that 4 F by pi F0 should be less than 1, but if it is less than 1, then only this 1 - this quantity will be positive. So it means that 4 F is less than pi F0 or pi F0 is greater than 4F or F0 by F is greater than 4 by pi. So these are the conditions we can see that. F0 should be greater because F is the frictional force.

And F0 is the amplitude of the external harmonic force and it demands that to have some harmonic motion, this external force amplitude must be greater than 4F by pi. Now this condition must be satisfied to have the real values of x. Now what is pi?

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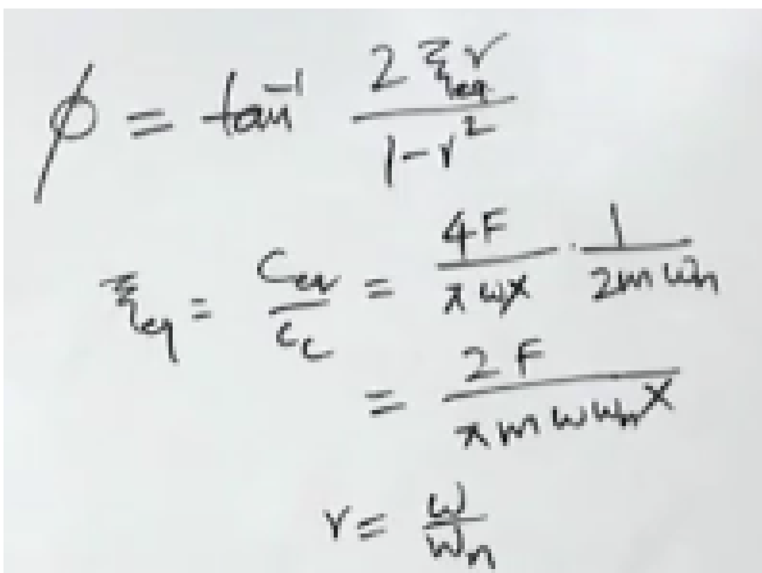

We know that pi is an inverse to zeta upon 1 - r square and here zeta/zeta equivalent and zeta equivalent = C equivalent by CC and C equivalent we already know 4F by pi omega x into 1 by CC 2m omega n, so this will be 2F by pi m omega, omega n into x and r omega by omega n. so we can get the F. now if we want to know what is the energy dissipated.

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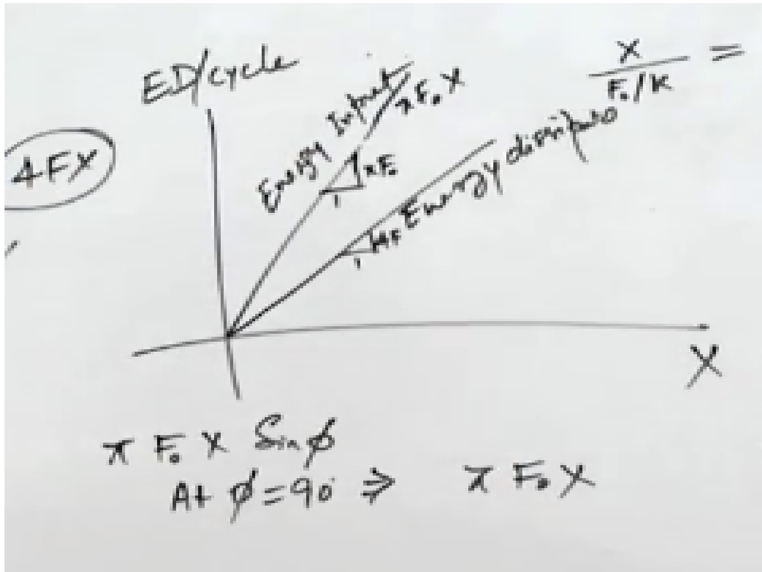

So we can plot. So this is the energy dissipated per cycle and this is I think x. So energy input in the system is as we have already derived pi F0 x sin phi. So at phi = 90, the resonance condition, we have it as phi F0 x and if you plot this energy input, so this is the energy input at resonance and this pi F0 x. This is x. So here this is the slope. This is pi F0. Now energy dissipated in Coulomb damping is 4Fx.

So it is here. This is energy dissipated in case of Coulomb damping and this is the slope for F. So we see that the slope pi of 0 should be greater than 4F and that is what conditions we are getting from here, pi F0 is greater than 4F. It means the energy input will be greater than the energy dissipated. So let us see one numerical example.

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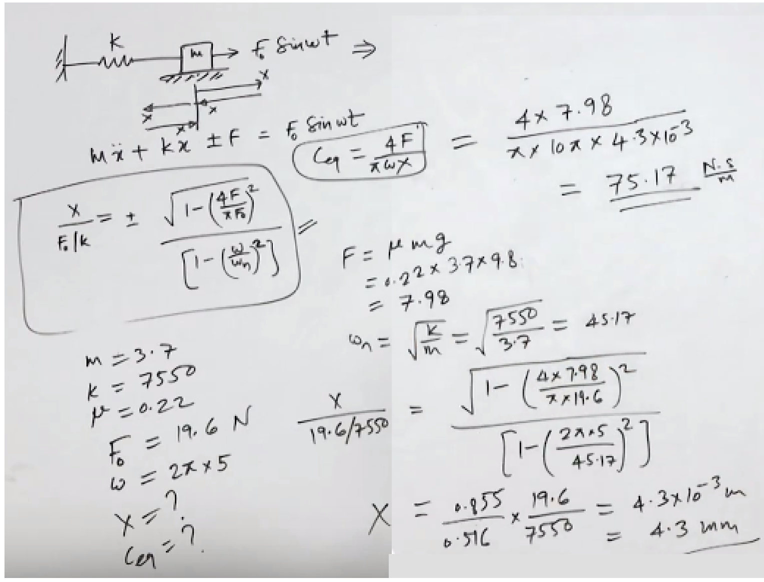

So it is said that there is a horizontal spring mass system, so we have made the system here. That is subjected to dry friction damping and has the following physical data. So mass is 3.7 Kg. So here we have m = 3.7 Kg spring constant K = 7550 Newton per meter, coefficient of friction between the mass and the horizontal plane is 0.22. The mass is subjected to a sinusoidal forcing function of amplitude 19.6 Newton.

So amplitude of the force F0 = 19.6 Newton. So F0 and the frequency is 5 Hertz, so here omega is 2 pi into 5. Now find the amplitude of vibration of the mass, also calculate the equivalent viscous damping. So first we have to find the amplitude of vibration of the mass. So we can use this formula to find the amplitude of vibration because we have to find the x, first x = constant, then C equivalent. The equivalent viscous damping is required.

So here what we have to find? We have to find first in this expression, we have to find F first. F = mu times m into g, m 0.22 into m is 3.7 and g is 9.8, so we get 7.98, this frictional force. Now we have to get the omega n. So omega n is root K by m so that is equal to under root 7550 by 3.7, so that is 45.17 radiant per second. Now we put these values. So x by F0 is 19.6 upon K. K is 7550 = under root 1 - 4 F, F is 7.98 upon pi into F0, F0 is 19.6 square.

1 -, omega by omega n, so omega 2 pi into 5 by omega n is 45.17 square. So we can calculate, so this is coming 0.516 and the top one is 0.855. so x =, this will be multiplied 19.6 by 7550. So it is

about 4.3 into 10 power - 3 meter, so it is equal to 4.3 mm. So we get the amplitude of the vibration of the mass that is under the Coulomb damping. Now, we have to find the equivalent viscous damping.

So equivalent viscous damping, we know C equivalent = 4F by pi omega x. so 4 into F, so F is 7.98 upon pi into omega. So omega is here 10 pi, x we have already calculated 4.3 into 10 power - 3. So we can find, so it is coming to be 75.17 Newton sec/meter. So thus we understand that to solve the case of the Coulomb damping, we can convert it into the equivalent viscous damping.

And then all the expression that we have derived for the viscous damping can be used here and we can find the amplitude of vibration of the system. So thank you for the attention to this lecture and see you in the next lecture.