Introduction of Mechanical Vibration
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Lecture - 15
Energy input and dissipation by viscous damping

Welcome to the lecture on input energy and dissipation of the energy due to viscous damping in a single degree of freedom post vibration system. So we know that in a spring mass damper system, we have these three elements. When we apply force on this system, we give the energy input in the system. If we do not apply the force and we disturb the system due to the damping, the vibration will be damped out it will, the system will come to rest.

It means that to maintain its motion we need to apply, provide the energy through application of the external force, because in each cycle there is the dissipation of the energy in the damper, because damper is the element that dissipates the energy. So today we have to discuss that how much energy is given to a single degree of freedom system, there is some harmonic force applied to the system like f = f0 sin omega t.

So if this force is applied on the system, how much energy per cycle is going into the system, and then we will try to find out that how much energy is being dissipated in the system, mainly due to the damper element. So let us discuss this

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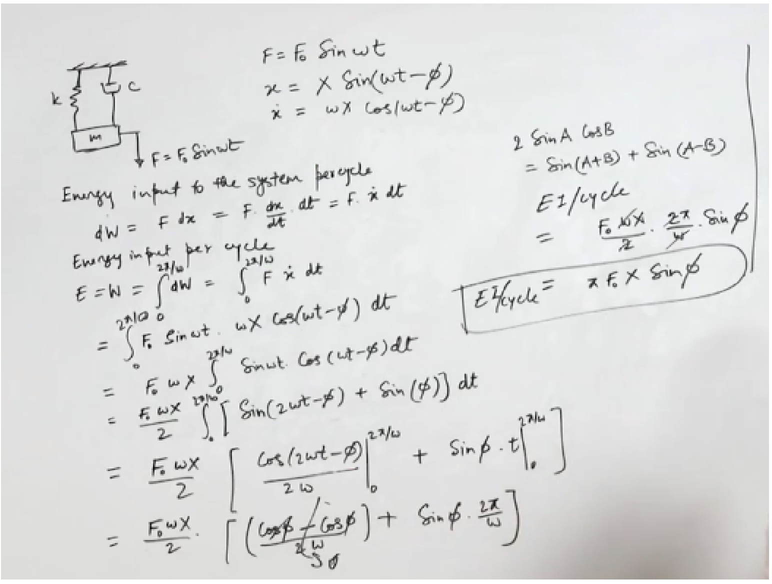

So we take a single degree of freedom system we have this system and this system is subjected to some force F0 sin omega t. So it is harmonic force the system is subjected, so we have F = F0 sin omega t. Now we know that due to the effect of this force, the response of the system will be also harmonic, but with some phase lag phi. We are talking here about the STD state response so here x = X sin omega t - phi, so this is x.

Now if you want to know the energy input to the system per cycle. So what is the energy? Energy is nothing but the work that this force will do to move certain displacement of the system. So if we say let the elementary energy dw = it is F into dx. So if you want to know the energy input per cycle, here we can write F into dx by dt into dt. So that is nothing but F into x dot into dt, here if you want to know energy.

That is the work done for one cycle we say zero to 2 Pi upon omega, because zero to time period and time period is 2 Pi by omega and dw, so = zero to 2 Pi by omega F into x dot into dt. So here we can write, F = F0 sin omega t and x dot, x dot we can find from here, x dot it is omega into x into cos omega t - phi we differentiate x with respect to time t. So x dot = omega x cos omega t - phi.

We can write it omega x cos omega t - phi into dt and this is zero to 2 Pi by omega. So here we can write F0 into omega into x zero to 2 Phi by omega sin omega t into cos omega t - phi dt. So now we have to solve this integral and we know this formula that 2 sin a cos b = sin a + b + sin a - b. So this is the formula that we can apply here, so we can write F0 omega x by 2.

When we divide 2 multiply 2, this 2 sin a cos b, we can write sin a + b, it is two omega t, omega t + omega t - phi, 2 omega t - phi + sin a - b, so omega t - omega t + phi, so it is sin phi into dt. So this we can do F0 omega x by 2, here sin theta, we have cos 2 omega t - phi and we are integrating, so we will have upon 2 omega and this limits zero to 2 Pi by omega + sin phi.

So this will have sin phi into t and this will have the limit zero to 2 Pi by omega. So F0 omega x by 3 into, here we have, when we will have 2 Pi by omega, it will be 4 Pi cos 4 Pi - phi. So cos four Pi - phi will be cos phi -, when we have t = zero it is cos - phi, so cos - phi is again cos phi upon 2 omega + here sin phi into 2 Pi by omega square - zero. Now this is, this term will be zero, this is zero.

So we will have energy input per cycle = F0 omega x by two into 2 Pi by omega into sin phi. So we will have omega cancel out 2 here cancel out, we will get Pi F0 x sin phi. So this is the energy input per cycle in a system, single degree of freedom system that is subjected to some harmonic force.

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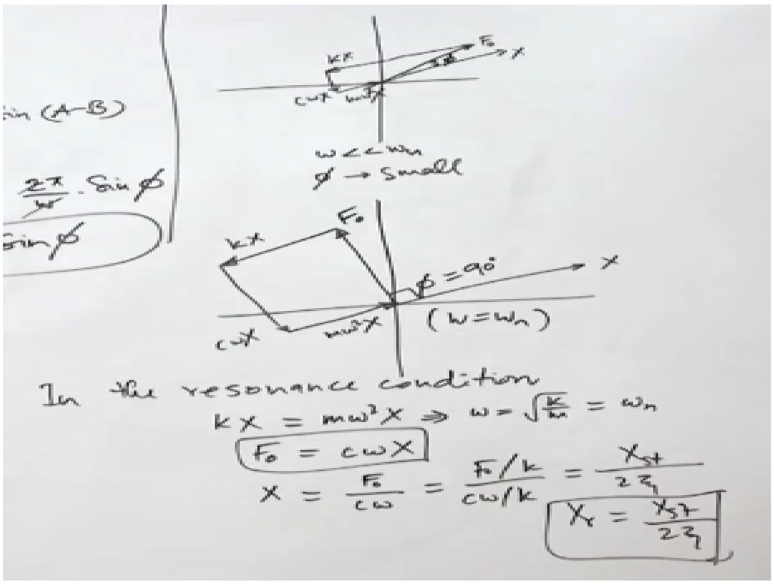

Now we take some cases here, if we remember the vector diagram that we made for the forced harmonic vibration of a single degree of freedom system. So we have this vector diagram, here this is x and this was F0, then this is kx, this is c omega x and this is m omega square x and this is phi. So if we see while omega is much less than omega n, the phi is very small and in this case, so omega is very less.

The energy of force m omega square x is the energy of force. That will be negligible or very, very small even this force will be small. So mainly the applied force is mainly balancing the spring force, however when we come to the resonance condition in the resonance condition when omega = omega n. So let us take the resonance condition, so this is x, now in case of resonance, as we remember the phase angle is 90 degree means the angle phi that is between x and F is 90 degree.

So this is x and here this is F0 and this angle phi = 90 degree. Then this is spring force kx, this is damping force c omega x and this is the energy of force, that is m omega square x. So here phi = 90, so in the resonance condition we see that spring force kx = m omega square x. So this makes that omega = root k by m and that is nothing but omega n. This is what is the resonance condition, that is omega = omega n, that is satisfied.

Again the F0 = c omega x means the applied force is balanced by the damping force. And so the x = F0 upon c omega, and we can write F0 by k c omega by k and that is nothing but F0 by k is the static zero frequency deflection and c omega by k it is 2 zeta into r, r is omega by omega n and that is 1 for the omega = omega n. So the resonance amplitude is inversely proportional to the damping.

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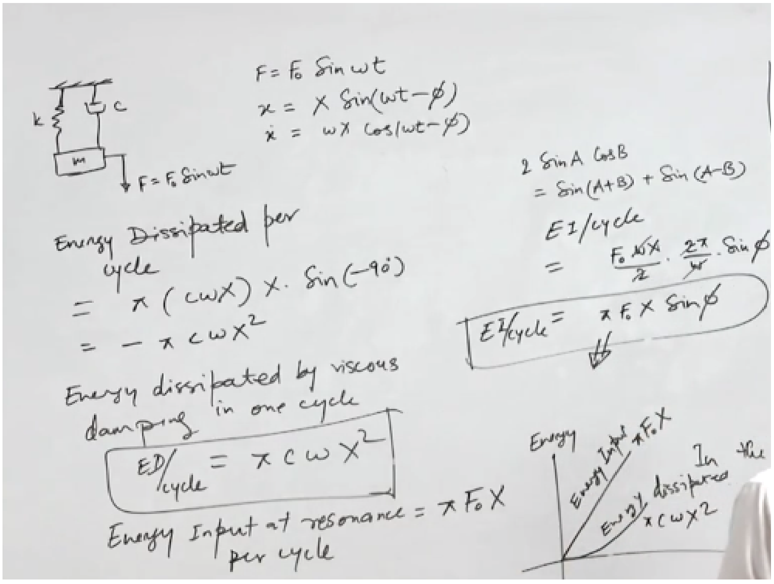

So the energy dissipated per cycle, so of course this is energy dissipated by the damping force, c omega x. So if in this formula we put, here in spite of F0 I put c omega x, so I can put Pi and c omega x, because this is the damping force, I want to know the energy dissipated by the damping force. So in spite of F0 I put c omega x into x into sin phi. So sin phi is - 90 degree, because c omega x makes - 90 degree angle from the x, so it is - 90 degree.

So this is - Pi c omega x square, because sin 90 is one and - is here. So it only shows that energy is dissipated. So energy dissipated by viscous damping in one cycle = Pi c omega x square. Now energy input at resonance =, if we put phi = 90, so it will be Pi F0 x. Now if we make a plot of the energy or work done and here, so if we have x here, so one line like this is energy input, that is Pi F0 x so it is a straight line.

While the energy dissipated, here it is per cycle, that is parabolic, so it is like this, so it is, this is Pi c omega x square, so energy dissipated, this is energy dissipated. So here we see that, of course energy input is greater than the energy dissipated, because not all the energy is getting dissipated some energy is used by the system for its vibration and there is the conversion of the kinetic energy and potential energy interchange, inter-conversion between potential energy of the spring and the kinetic energy of the mass.

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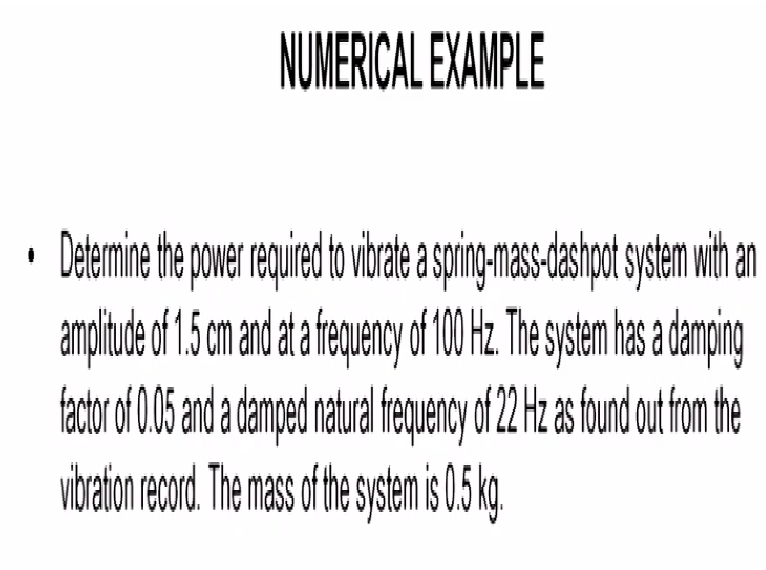

Now we can take one numerical example, so here we have to determine the power required to vibrate spring mass dashpot system with an amplitude of 1.5 centimetre and at a frequency of 100 Hertz. The system has a damping factor of 0.05 and a damped natural frequency of 22 hertz as found out from the vibration record. The mass of the system is 0.5 Kg.

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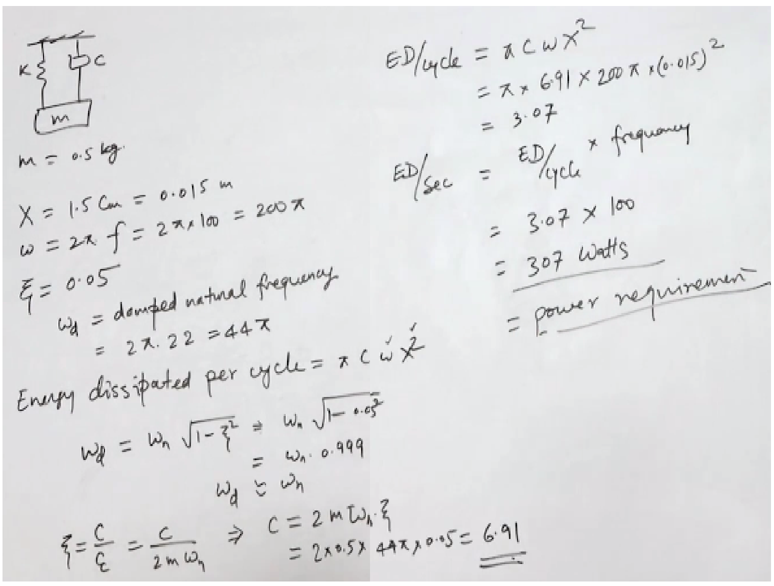

So we have a system, this is our spring dashpot mass system, so this is our system. So here mass is given 0.5 Kg. We need to determine the power required, so power means the energy per unit time that is required to the system. So the system vibrates with the amplitude of 1.5 centimetre, so here x is 1.5 centimetre that is 0.015 metre, and at a frequency of 100 hertz, so your omega = 2 Pi into F, so here 2 Pi into F is 100 , so it is 200 Pi radian per second, so here.

Now the system has a damping power of 0.05, so here the zeta is 0.05 and damped natural frequency of 22 hertz. So omega d, that is the damped natural frequency that is = 2 Pi into 22 hertz, so 22, so that is equal to 44 Pi radian per second, because we are converting this hertz into radian per second. So we are giving these data and we have to find out that what is the power required.

Because the power required per cycle will be the power dissipated and to get the power dissipated, first we have to find the energy per cycle, energy dissipated per cycle. So energy dissipated per cycle, what is the energy dissipated per cycle? Because this is viscous damping, so we just derived the expression, energy dissipated per cycle = Pi c omega x square.

Here we know omega, we know x, you have to find the c, c is the damping of this system. So to get the damping, here we have the omega d, so omega d is = omega n root one - zeta square. So damped natural frequency is root 1 - zeta square into omega n. So this implies that, so omega n root 1 - zeta is 0.05 square. So = omega n square into, if we solve this, this is about 0.999.

So we can assume that omega d = omega n, because this quantity is 0.05 is very small and this is almost = 1.99. So we assume omega d = omega n, and omega d we know, and omega n so now if we want to know the damping c. So c, we know that zeta = c by cc, and cc = 2 m into omega n or 2 root k n. So from here we can find c = 2 m omega n into zeta.

So 2 into m is 0.5, and omega n = omega d, that is 44 Pi, into zeta, zeta is 0.05. So we can get c, it is 6.91. So now we get c, now we can get the energy dissipated per cycle. So ED per cycle = Pi c omega x square, so = Pi into c that is 6.91 into omega, omega is 200 Pi into x, x is 0.015 square. That is we can calculate, that is 3.07. So now this is the energy dissipated per cycle. So we need the energy dissipated per unit time, or per second.

So energy dissipated per second is energy dissipated per cycle into the frequency, because frequency is number of cycles per second. So we multiply, 3.07 into, here frequency is 100 hertz, so we multiply with 100, and we get 307 watts, so this is in watts. And this will be = the power requirement, so this is the power required for this system, okay. So we stop here and thank you for attending this lecture and let us meet in the next lecture. Thank you.