Introduction to Mechanical Vibration
Prof. Anil Kumar
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Lecture - 14
Excitation of the support

Welcome to the lecture on the excitation due to support. So here in this case we have a system that is on a support and support is vibrating. So, frequent example is the earthquake. If there is some earthquake and there is a building, so the due to the earthquake motions what will be the motion on some particular floor, particular story building. So here also there are some cases like if we have some vibration on the ground and you have kept some machine.

So what is the vibration due to the ground, how much it is going to have the vibration on the top of the machine or some instrument you are keeping on certain table and the table is vibrating then your instrument, what is the vibration amplitude of the instrument? So there are these kind of examples. So here we will take a single degree of freedom system and that is on the support.

So when we say support means we say that it has some damping and some stiffness. So it could be the some like something kept on some elastomer, okay. So what is the vibration of this kind of system? So let us take this example.

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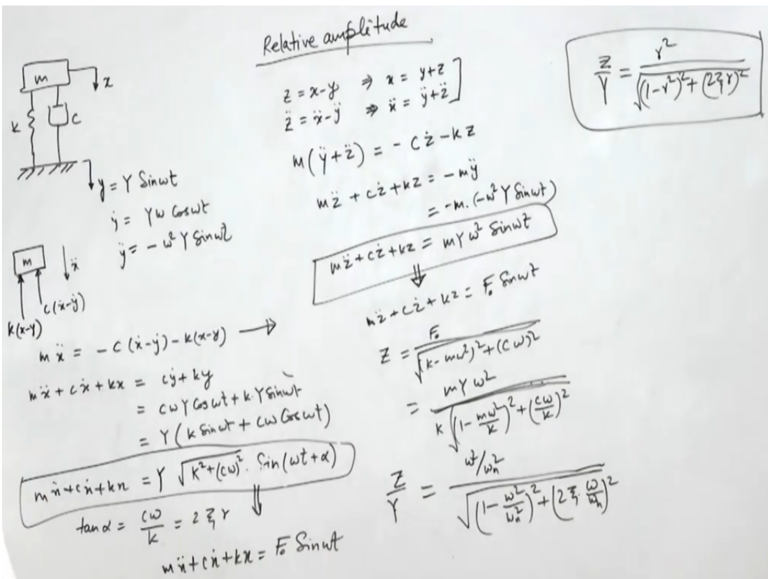

So we have the system consisting the mass element, stiffness element and damping element. Now if there is the motion of the support so this has some motion Y, harmonic excitation of the support. So Y = Y sine omega T. So due to the excitation of the support what be the response of this mass. Let us say this is X. So what will be the X and so we will right the equation of motion.

So first we will make the free body diagram of this mass. So let us say we have this mass. Because this mass is let us say the direction of the escalation of the mass and this mass is compressing, this is spring, so this spring we will apply the force but the net compression of this spring will be the difference between the X and Y, okay. So, the force that it will apply will be K X - Y.

Because this is moving X this is moving Y, so the net compression of the spring or its stretch of this spring is X - Y. Similarly, the damper the net velocity X dot - Y dot. Now, we can write the equations of motion applying the Newton's law. So sigma F = MX double dot. So here MX double dot = - ZX dot - Y dot - K X - Y. So here we can write MX double dot + ZX dot + KX = ZY dot + KY, okay.

So here we have - ZX dot we bring it to the left side so it is ZX dot and this is KX and + ZY dot and + KY. Now we have Y = Y sine omega T and Y dot = Y omega multiplied by cos omega T. So here we can write Z omega into Y cos omega T + K multiplied by Y sin omega T. So we can write Y into K sin omega T + Z omega cos omega T. Now we can write these 2 in the addition of the 2 harmonics of the same frequency.

So the amplitude will be under root K square + C omega square and this will be sine omega T + some phase let say alpha where tan alpha = Z omega by K = 2 into zeta into R, okay. Where R = omega by omega N. So what we see that this equation is converted into equation of forced vibration where the amplitude of force is this and this is the harmonic motion.

Now the response X will be = X sin omega T + alpha - phi. Where again the phi is the phase lag. It is the phase lag between this and the X. So what will be the X? So again we will compare this equation with the equation MX double dot + CX dot + KX = F0 sine omega T. So here what we see that in that case X = F0 upon K - M omega square whole square + C omega whole square under root. Here F0 is like Y root case square + C omega S square.

So we can write here Y root K square + C omega square upon root K - M omega square whole square + C omega whole square. So here we can write X by Y =, so here we can write, take K outside, so under root here is K 1 + C omega by K whole square upon again here we can take K outside. So it is under root 1 - M omega square by K whole square + C omega by K whole square. So this will cancel out. So we can write C omega by K as 2 zeta.

So here we can write under root 1 + 2 zeta R whole square upon under root 1 - R square whole square + 2 zeta R whole square. So here we get X, so here X is, X = the absolute amplitude of the mass. So, because X = X sin omega T + alpha - phi it represents the absolute motion of the mass. Now here the phase lag, phase lag = phi - alpha. So that is equal to and here phi, here tan phi is 2 zeta R upon 1 - R square.

So here phi = tan inwards 2 zeta R upon 1 - R square - alpha and alpha is here tan inverse 2 zeta R. So this is the phase lag. Now we take the case when R is much less than 1. So when R is much less than 1, X by Y is tending to = 1. So X by Y = 1. When R = 0 so when we put R = 0 here we have X by Y = 1. So when R = 0 we have X by Y = 1. So for the lower values of R XY is approximately 1 irrespective of any damping value.

So far all the curves it starts with X by Y = 1 and when R is much greater than 1, so X by Y is tending to 0.

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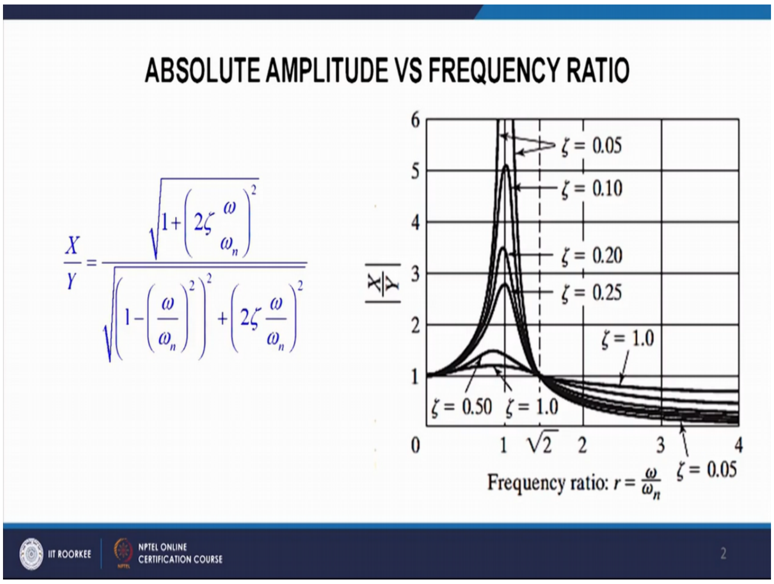

We can see from this frequency response curve and we can see that here all the curves they start from 1 at R = 0 it starts from 1 at near the resonance they get the larger magnitude then they are again coming down and passing through all the curves again pass through R = under root 2 with the unity X by Y = 1 and then when R is very large they all the curves are tending to 0 amplitude, okay.

For higher value of we can see that from for higher value of R the lower damping values give they are reaching to X by = 0 faster than higher damping. So what we have derived X that is for the absolute amplitude of mass. Now we can take another case of relative amplitude. So relative amplitude means the motion of mass with respect to the support because the mass is on the support.

So what is the relative motion of mass? So, we can here take some case, so let us assume. So, the relative amplitude, so let us assume that Z = X - Y. So it means that Z double dot = X double dot - Y double dot and from here we can write X = Y + Z and X double dot = Y double dot + Z double dot. So we put these values in this equation, the equation of motion.

So we can have MX double dot. So X double dot we can write Y double dot + Z double dot = - Z. X dot - so here X dot - Y dot we can write CZ dot - KX - Y we can write Z. So here we can write MZ double dot + CZ dot + KZ = - MY double dot. Now what is Y double dot. So we can do 1 more difference is in here of Y dot. So we will get - omega square Y into sin omega T. So we can put here - M into - omega square Y sine omega T.

So we will get MY omega square sine omega T and this is MZ double dot + CZ + KZ. So from here we can again compare it with MZ double dot + CZ dot + KZ = F0 sin omega T and here we can find the Z = F0, so upon K - M omega square whole square + Z omega whole square under root.

So here F0 is MY into omega square and here we take K outside, so we will have 1 - M omega square by K whole square + C omega by K whole square under root. We can write Z by Y =, so here omega square upon M by K so or K by M, so it is omega N square upon root 1 -. So here omega square by omega N square whole square + 2 here zeta into omega by omega N whole square.

So we can write here Z by Y =, so omega by omega N is the frequency ratio R. So we can write R square upon 1 - R square whole square + 2 zeta R whole square. Okay, so this is the, this and here the phi, the phase we can have the, we can get the similar way. So we see that we have got the, here the relative amplitude of the mass and this is Z by Y = R square by 2 to 1 - R square whole square + 2 theta whole square under root.

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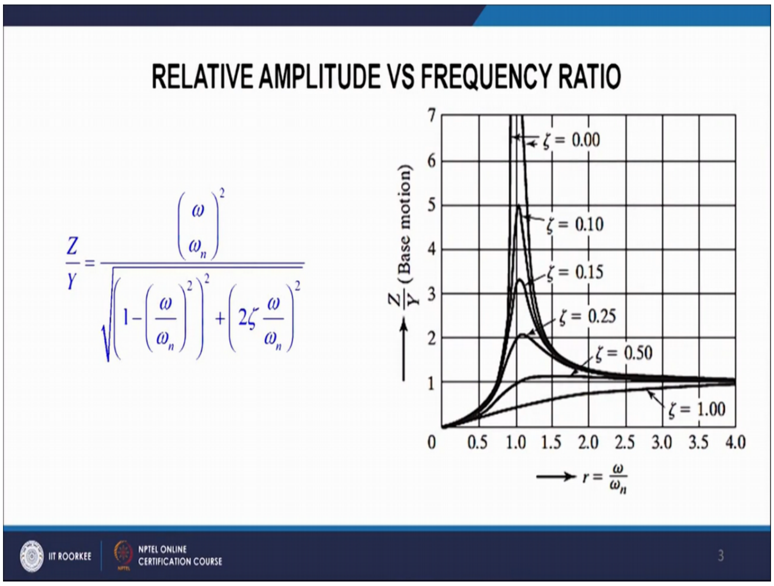

So we can see this plot of the relative amplitude was its frequency ratio and we see that this curve is start from 0, so here because when you put R = 0 it is 0. So it starts from 0 it reaches to, reaching to towards maximum values near resonance and then it is when R is increasing it is tending towards Z = 1, okay. So the relative amplitude of the mass and ratio with the support amplitude, support excitation amplitude that is tending to 1 when it is approaching to larger values of this R.

Now we will do one problem based on this concept that we discussed. So here we can see the numerical this example.

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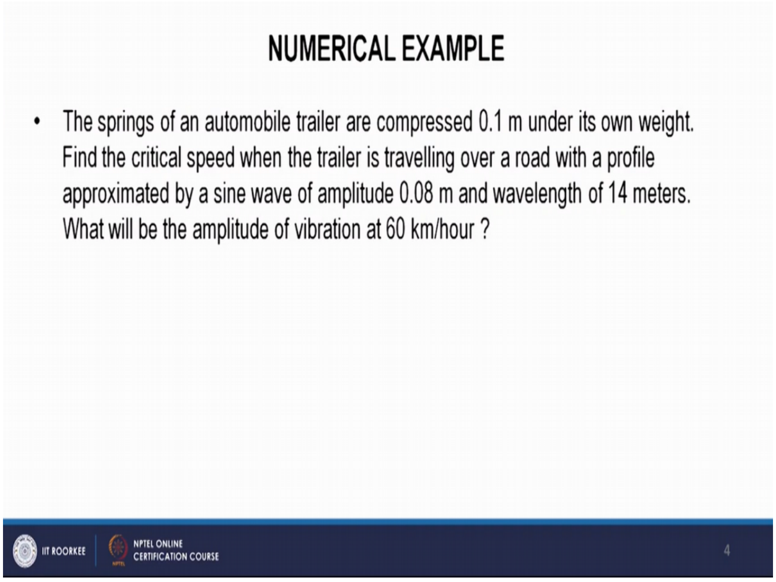

So there is springs of an automobile trailer are compressed 0 point 1 meter under its own weight. So we have automobile carrier or automobile trailer. So it is moving, it has some springs, may be suspension is springs and they are compressed point 1 meter under its own weight. Find the critical speed when the trailer is travelling over a road with the profile approximated by a sign wave of amplitude point 08 meter.

So and we have length of 14 meter. So this trailer is moving on certain road with profiles of sine curve and the amplitude is given point 08 meter of that road profile and wavelength is given. Now it is asked that what is the critical speed? Okay and then what will be the amplitude of vibration at 60 km per hour. So again it is asked at if it moves at 60 km per hour what will be the amplitude.

So now how this problem is related to the support excitation? Because this vehicle is passing on road that has some harmonic profile so when it is passing on that it will be subjected to some harmonic motions, okay, on the tyre and this will affect its vibration behavior and therefore it is the case of the support excitation. Now here it is asked find the critical speed.

So how the critical speed, what will be the critical speed how we will find the critical speed? So when say critical speed usually we, when we talk in terms of frequencies we say that when the natural frequency of the system is near to the excitation frequency. It is the critical condition. So if our road profile from the support some excitation is coming as why sine omega T?

So omega is the excitation frequency and our system has some mass and stiffness so it has the natural frequency omega N = root K by M and when the omega = omega N this is the critical condition and there is speed that we will get under this condition will be the critical speed. So let us solve this problem.

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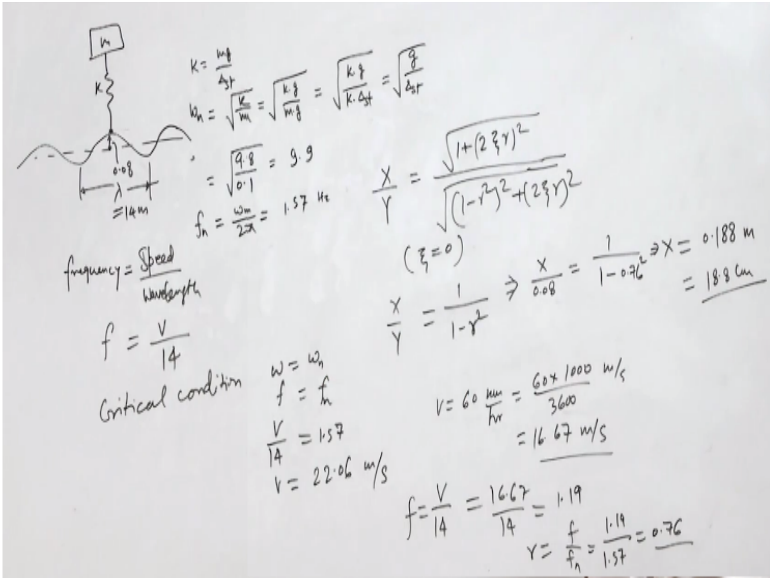

So here it is said that we have some, here is our trailer and it is said that due to its own mass that is MG it is some compression of delta HT. So you knew that its stiffness K = MG by delta HT. Because this is the force, the weight and this is the deflection due to its weight. So this will give the stiffness of this spring and then what is the natural frequency? Omega N = root K by M and we can write here K into G by M into G.

And then we can write K into G by M into G. M into G is K into delta HT. So K into delta HT. So we will get here G by delta HT, okay and here delta HT is given. Delta HT that is static, it is static deflection due to its own weight that is point 1 meter. So it is 0 point 1 meter. Now we put in this equation so we will get G = we take 9 point 8 meter per second is square the acceleration, gravitational acceleration and delta HT 0 point 1.

So we can calculate omega N, so it is 9 point . So it is coming to be 9 point 9. So it is 9 point 9 radian per second and F natural frequency FN = omega N upon 2 Pi so = 1 point 5 7 hertz. So we have got the natural frequency. Now, this trailer now it is so, this trailer is now moving on a road profile. So let us say we have some road profile that is harmonic, okay.

So this is the road profile on which this trailer is moving it has now the, we know the stiffness of this trailer, K and this road profile is like a sign curves similar to sign curve of amplitude point 08. So it means that we have, so this is 0 point 08 meters and wave length is 14 meters. So this is the wave length, so either 2 peaks or these, so this is lambda that is wave length = 14 meter. Now so what will be the excitation that is going from the support.

So we know that frequency = the speed on wave length. So let us say speed is V and wave length is 14. So this is the frequency F. Now in the critical condition omega = omega N or F = FN. So this is your excitation frequency F and you know the amplitude point 08. So here V by 14 = your FN we have already calculated 1 point 5 7. So we can get here V so that is 22 point 0 6 meter per second, okay.

Now what will be the amplitude of vibration at 60 km per hour. So we want to know the X. So we know that this we have, this equation and here we assume zeta = 0 because we have not given any value of zeta. So we assume it, so when we put theta equal 0 we will get 1 by 1 - R square. So we have to get at first the omega when our speed is 60 km per hour. So V = 60 km per hour we can convert it into meter per second.

So that is 16 point 6 7 meter per second. So at this is speed V by 14. So we have F = V by 14. So, 16 point 6, 7 by 14. We have 1 point 1 9 and we can get R, R = F upon FN or omega by omega N. So this is 1 point 19 and FN is 1 point 57, so it is 0 point 76. This we can put here, so X upon Y, Y we have point 08, 1 upon 1 - R square are 0 point 7 6 square. This implies X. So it is 0 point 1 8 8 meter, so or 18 point 8 centimeter.

So this is the absolute amplitude of the vibration of the trailer for in it, it will move at V = 60 km speed of 60 km per hour. So, we understand that how we can apply this theory of support excitation in our practical systems. And we applied here on some vehicles moving on harmonic road profile. So, I would like to thank you for attending this class and let see in the next lecture, thank you.