**Introduction to Mechanical Vibration**
**Prof. Anil Kumar**
**Department of Mechanical and Industrial Engineering**
**Indian Institute of Technology – Roorkee**

**Lecture - 13**
**Rotating Unbalance**

Welcome to the lecture on forced vibration of a system having rotating unbalance. So we have already studied the forced vibration of a single degree of freedom system that is subjected to harmonic force having amplitude F0 and frequency omega. And we have already discussed the frequency response curves and phase angle curves. Now, we will take case of rotating unbalance.

Usually the rotating machineries that rotates about certain access like turbines or motors or some shaft. So they have some unbalanced mass. It could be due to some problem or some weakness in design or due to some manufacturing error. So when there is some unbalance mass, this unbalanced mass when the system will rotate about certain access will apply certain force and due to that force in addition to the rotation the machinery will also have some vibration in the transverse direction.

So, therefore we will study today the force vibration of a system that has some rotating unbalance. So let us show a system and then right the equations of motion and then the solutions and then we discuss the response and then we will carry out some numerical examples.

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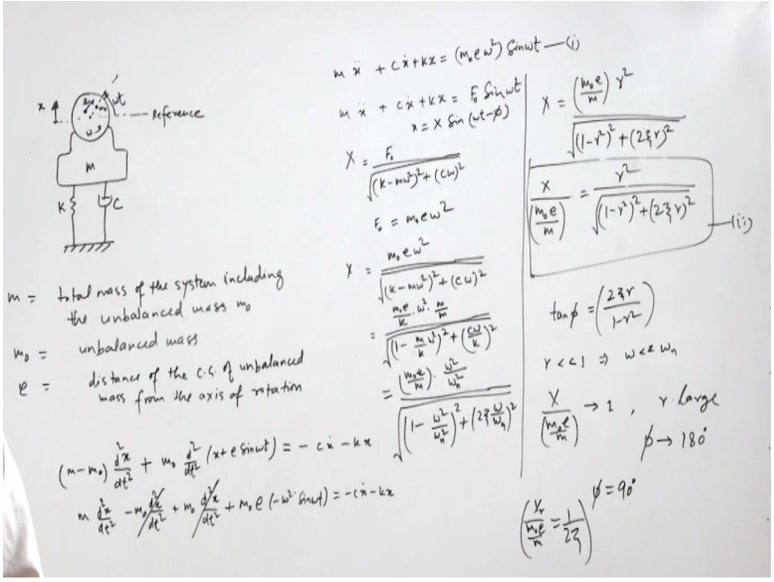


So here you can see a system. So here is the element that is rotating and this is the complete system that so it can be a machine that is resting on some elastic pad. So that has some stiffness and damping. Now this machine is rotating, so this is rotating with certain frequency omega. So this is rotating with omega and here is the unbalanced mass M0. Here M is the total mass.

Total mass means it includes, so it total mass of the system including the unbalanced. Unbalanced mass M0 and M0 is the unbalanced mass. So this is the access of rotation about which our machine is rotating and from this access our rotation there is this distance E the center of gravity of this unbalanced mass. So E is, it is we call it eccentricity also. So it is the distance between the axis of rotation, distance of the CG or center of gravity of unbalanced mass from the axis of rotation.

Okay, so let see, I mean this system you rotating and there is a situation this is the difference axis, from here we assume that the system is vibrating in x direction and at particular instant of time here this is omega T because we measure time from here and this is omega T and my mass is at this portion. Of course when it will, the time it will rotate it will go here and so on. Now we have to write the equations of motion.

So here we have mass, M - M0 that is balanced mass. So balanced mass is the total mass - unbalanced mass and these mass center is here. So we can directly right the equations of motion. So that is D square XY DT square. Okay so, MX double dot. Now for the unbalanced mass M0 we have this distance X + E and this is omega T so this is sine omega T. So M0 D square by DT square X + E sin omega T. And this is equal to, so these are the forces and the system due to whispering and damper.

So they are - CX dot - KX. So now we can see here, we can write MD square X by DT square - M0 D square X by DT square + M0 D square X by DT square + M0 E and D square by DT square into sin omega T. So it is - omega S square if it differentiates twice, so it is cos omega T then again - sine omega T = - CX dot - KX.

So these will cancel out so our equation will be MD square X by DT square +, so here we can write M D Square X by DT square we can write MX double dot + CX dot + KX =, so this term we take to the right hand side, so it is M0 E omega S square into sin omega T. So this is the equation motion of the system. Now, we have already perform the derivation of the amplitude response of a system subjected to harmonic force.

That is we have already if our system is MX double dot + CX dot + KX = F0 sine omega T. So for this system we have the solution X = X sin Omega T - 5 and for which X = F0 upon K - M omega square whole square + C omega whole square. So this already we have derived. Now here if you see here F0 we can put F0 = M0 U omega squared in this equation and we can find the amplitude response of this equation number 1.

So from this equation we can see that due to this, on this unbalanced mass there is some centrifugal force that is proportional to omega square is working acting on this unbalanced mass. So it is M0 E omega square acting the centrifugal force and M0 E omega square and its component in the X direction is M0 E omega square sine omega T. So this same force here is represented.

So if we write X = M0 E omega square by K - M omega square whole square + C omega whole square under root. Now, here we can write, we can take K outside from this so we can write 1 - M by K omega S square whole square + C omega by K whole square under root and K will be here so M0 E by K and 2 omega S square. Now we here multiply and divide by M so, we will get M0 E by M into omega E square and K by M. So K by M is nothing but natural frequency square.

So omega N square upon and here 1 - omega square again K by M omega N square whole square + C omega by K we have already derived that is 2 theta omega by omega N Square that is an under root. So from here we can find X = M0 E by M multiplied R square upon 1 - R square whole E square + 2 zeta are whole square under root or we can write X upon M0 E by M = R square upon 1 - R square whole square + 2 zeta R whole square under root.

So this equation let say equation 2, this gives the amplitude of the vibration of the system due to the unbalanced mass. Now the phase angle 5, so here we can write 10, 5 that is 2 zeta R upon 1 - R square. Now again we can plot the curves for X versus the frequency ratio R and we can see that what are the properties of this kind of vibrations, vibrating system at different frequency and damping.

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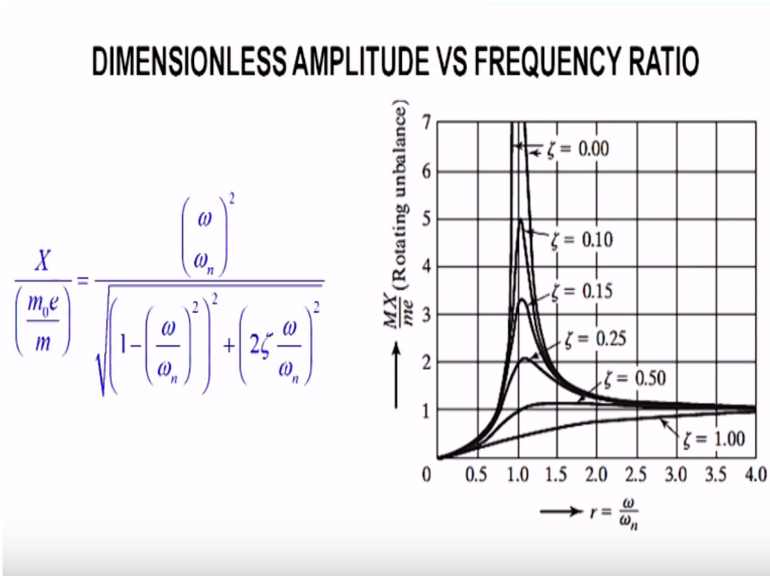


So here we have plotted, so this is the dimension less amplitude because E has the same minute as the X and M0 and M has the same. So this is dimension less and this curve can be plotted here and we see that when R is very less than 1, it means that omega is much less than omega N. So we have this X by MO E by M = about R square that is M0. So here we have the X by M0 E by M very less.

Here when we have R very less it starts from 0. Okay when omega is 0 this is 0 because here omega by omega and omega is 0. So it starts from 0 this response curve and as soon as we approach to R = 1 that is the resonance condition. When omega = omega N, the amplitude is increasing. Moreover, if we increased the damping even at the resonance the amplitude is lower.

And therefore if our operating speed is we should divide the operating speed at as soon as however, if we have somehow to pass through the resonance speed which would increase the damping. so that the amplitude is lower. However, when we increase the R the response amplitude is tending to 1. So X by M0 E by M is tending to 1 as R 10 to R is larger. So when R is large this X by M0 E by M is tending to 1.

However, when at resonance and here if we see the phase angle. So, in the next slide we can see the phase angle.

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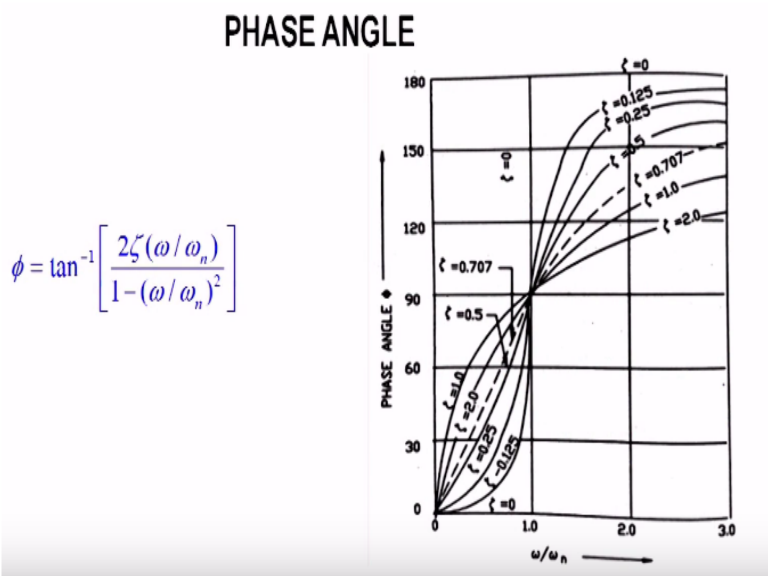


So phase angle when the R is very large it is trending to 180 degree. So if phi is tending to 180 degree. It means that when R is large the unbalanced and balanced mass vibrating out of phase. So if M0 - M - M0 that is balanced mass is vibrating in upward the unbalanced mass will be in the downwards. So, they are out of phase.

At the resonance condition phi is 90 degree and we can see the phase is 90 degree when R = 1 irrespective of the value of the damping and at this situation the X by M0 E by M. If we put R = 1 here. So this is 0 and here it is 2 theta and here R = 1. So it is 1 by 2 zeta. Okay so this is the amplitude at resonance. So, now we will take 1 example to numerical example to understand this concept in a better way and to apply on some practical problem.

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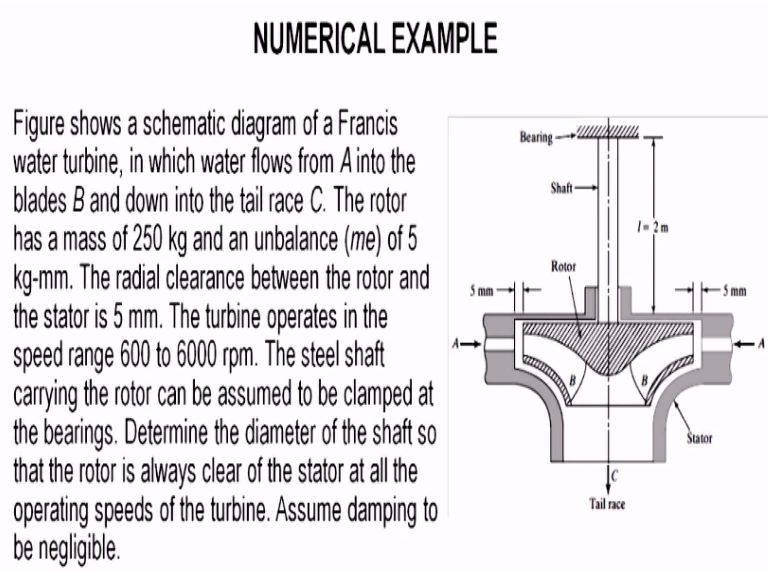


So here we can see the numerical example. It says that there is a figure of turbine that is Francis turbine and here is the bearing and there is shaft and on the shaft this turbine is installed. So here is this rotor and blades so from the A site here the water is flowing it passes through the blades and it gives its energy to the blades, I mean they rotate and so they generate the power.

Now we can say, so this is the stator that is the static part and this is rotor, so it is rotating part. You can see the length of the shaft is 2 meter and this is the terrace, so the water falls down from this in this opening. So it says that the rotor has the mass of 255 kg and an unbalance of 5 kg mm. So it means the rotor, so rotor has the mass. So this is the total mass of the system. So it is M, M = 250Kg and an unbalanced ME.

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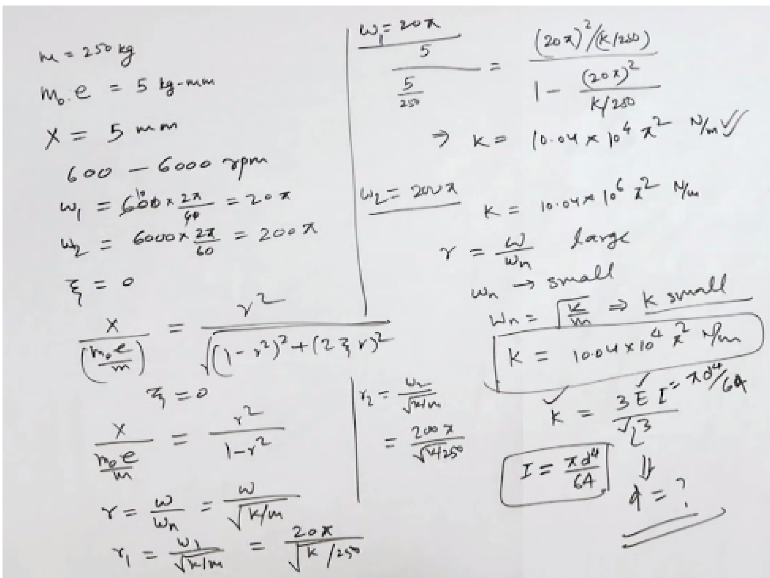

So here we took M0 as the unbalanced mass and E the distance from the axis of rotation. So M0 E = 5, so M0 into E = 5 kg mm. The radial clearance between the rotor and the stator is 5 mm. So there is the 5 mm gap between the router and the stator. So one thing we should understand that there is some unbalance in this router system. So due to this unbalance the system will vibrate. So if I have this system and here is some unbalance, so this is my rotor and here is some unbalance let say and this is the center.

So it will vibrate in the transverse direction and due to this vibration it is possible that the rotor may heat the stator and the gap is 5 mm. So it means that our X is 5 mm. So X is the amplitude of the vibration and the turbine operates in the speed range if 600 to 6,000. So it is operating in 600 to 6,000 rpm range. So it means let say omega 1 = 600. So it is RPM, so we can convert it into 18 per second 2 Pi by 60 and omega 2 = 6,000 into 2 Pi by 60.

The still shaft carrying the rotor can be assumed to be clamped at the bearing. So it is the shaft is clamped and so when it is vibrating we can assume it as a cantilever beam and I have already derived the how to find the stiffness how to find the natural frequency of cantilever beam. So we can refer that. So now we have to find the diameter of the shaft so that the rotor is always clear of the stator at all the operating speed of the turbine.

So what is required is the diameter, we have to find the diameter so that the rotor is always clear of the stator. So it does not touch the, does not heat the stator, so it means the maximum X is 5 mm or less. Maximum x is 5 mm so it could be less also. So assume damping to be negligible. So here we have to assume zeta = 0. And so let us this is 20 Pi and this is 200 Pi.

So now we have this formula XY M0 E by M = RS square 1- R square whole S square + 2 theta R whole square under root. Now zeta = 0, so we can reduce this equation R square 1 1 - R square. Now for, so what is the, so we have R = omega by omega N and omega by omega N is root K by M. So here, we have 2 cases of omega, so we have some lower range 20 Pi and higher range 200 Pi.

So we can have let say R1 = omega 1 by root K by M. So that is equal to 20 Pi point root K let say K is K1 here, K1 K by M. So K by M, so M is 250 and here R2 = omega 2 by root K by M. So it 200 Pi by root K by M. Now we put in this equation. So X is 5 mm and M0 E is 5 kg mm, point M is 250 = RS square. So let say first we are solving for the R1. So R1 is square, so that is 20 Pi square by 250 upon 1 - 20 Pi square upon K by 250.

So from here we can get K and if we calculate K, so we get 10 point 04 into 10 power in 2 Pi square in MK 78. Now we take the R2 and here MH 250 and this for omega equal 20 Pi that is omega 1 and here omega 2 = 200 Pi. So we get K if you solve this equation for putting R2 with K get K = 10.04 into 10 power 6 Pi square in MK's unit. So here it is newton per meter. Now we have 2 stiffness's which one we should select? We can select only one stiffness.

So here the stiffness because according to the frequency response curve R should be larger. So R = omega by omega N is large, so it may omega and should be smaller and omega N = root K by M. This implies that K is smaller. So K should be small. To make omega any small and so the R larger, so we have to select these smaller ones. So we select this so our K = 10.04 into 10.04 Pi square newton per meter. Now we have to find the diameter and we know that I said that this is like a cantilever beam.

So for a cantilever we have K = so K = P by delta and delta is PL cube point 3 E I. So 3 E I by LQ. And from here we know that I = Pi D for by 64 for a circular shaft and E is the elasticity of the material and L is the length given here = 2 meter. So we know E we can take if it is steel it is 200 giga Pascal, we can take we know L and I we know Pi D4 by 64, so we can replace I = Pi D4 by 64. From here we can get the diameter.

So, we see that how the theory that is based the frequency curve that we already understood helps us to take the decision of the design and to select the correct stiffness and finally we can get the diameter of this shaft that is used in this turbine. So we understand today, we study the forced vibration of system that has the unbalanced mass, rotating unbalanced mass and we found out the amplitude and phase and we understood a problem based on this concept.

So thank you for attending the lecture and see you in the next lecture, thank you.