Introduction to Mechanical Vibration
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Lecture – 12
Magnification Factor and Frequency Response Curve

Welcome to the lecture on force vibration of single degree of freedom system. Today we will discuss the magnification factor and frequency response curves. So we were discussing about the single degree of freedoms system under harmonic excitations and we found that there are the complete solution is of the 2 parts; the transient part and the steady state part. The transient part decays with time and the steady state part is remains.

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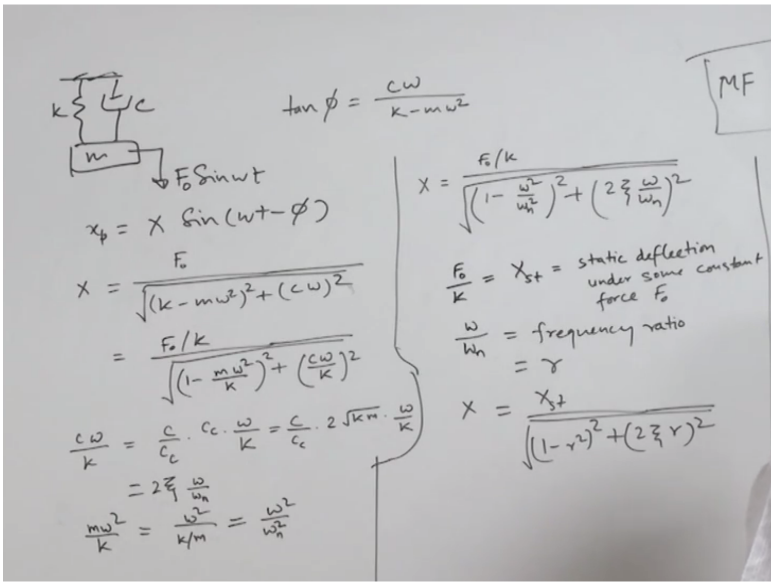

So here we will study some characteristics of the steady state part. So we write the equation, so we have the equation; so this was our system, is spring mass damper system subjected to some force F0 sin omega t and here we found that the xp, that is the steady state part that is; x sin omega t minus phi and here x was given as F0 upon k minus m omega square, so this was x and we had the phi that is tan 5, that was = c omega upon k minus m omega square.

So we have rearranged this terms; these terms, so we can have F0 by k, so we take k outside, of this under root. So we have 1 minus m omega square, okay by k. So we are taking k outside here, we have c omega by k here and this k will go on in the numerator. Now we can write here c omega by k = c by cc into cc into omega by k. So we multiply and divide by cc; cc is the critical damping and we have already defined this term.

So we can write here c by cc into cc is 2 root k into m into omega by k. So c by cc is nothing but damping factor zeta and here 2; so let us take 2 here and here k will be like omega and root k; so here root m by k, so it is omega n = root k / m, so it will be like this. Similarly, this term m omega square by k can be written as omega square by k by m and k by m is nothing but omega n square.

So we can write here omega square by omega n square. So here x; we can write x = F0 by k upon 1 minus omega square by omega n square; whole square, plus; c omega by k is 2 zeta omega by omega n; 2 omega by omega n whole square and under root. Now what is F0 by k so this is Xst, we defined Xst that is the static deflection under some constant force F under some constant force F0. So if we apply some constant force F0.

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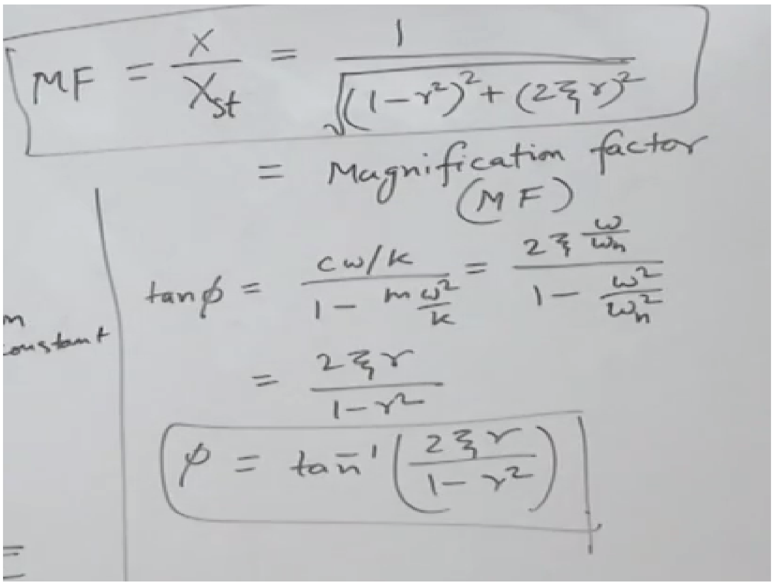

The system will deflect with these amount, so that is why it is called a static deflection or 0 or 0 frequency deflection. So we can write and omega by omega n let us defined as the frequency ratio = r, so we can write this equation X = Xst by 1 minus r square, whole square; plus 2 zeta r whole square under root or X by Xst or here we can write X by Xst = 1 by 1 minus r square, whole square, plus 2 zeta r whole square under root.

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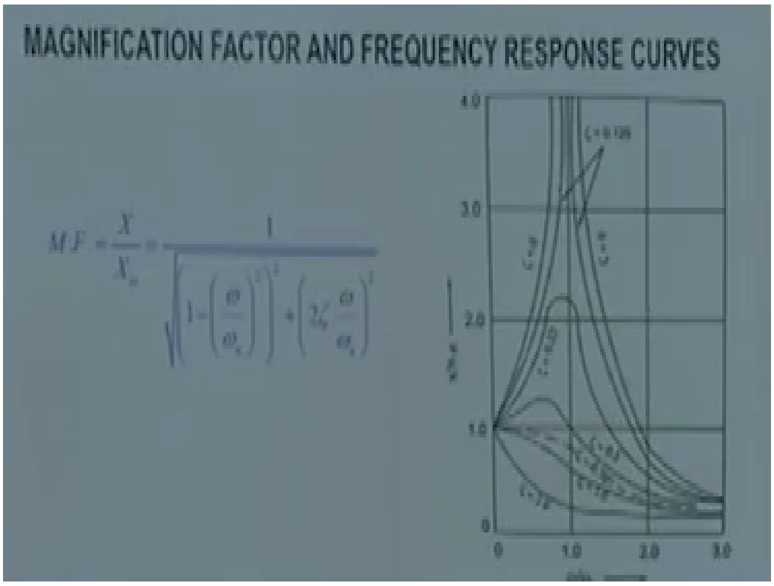

So this quantity X by Xst is nothing but it is called the magnification factor, so this is called as equal to magnification factor mf, let us say it is go mf, we represent it, so this is = mf. Now what about this phase, so tan phi = c omega by k upon 1 minus m omega square by k, so here we can write = so c omega by k is 2 zeta omega by omega n upon 1 minus m omega square by k is omega square by omega n square, so omega square by omega n a square.

So this is equal to 2 zeta r upon 1 minus r square. So phi = tan inverse 2 zeta r upon 1 minus r square. So this is the phase angle of this equation. So this; the response of the system that is xp = x sin omega t minus phi, so it is the steady state response and what is this x; x is the amplitude of the steady state response and Xst is the 0 frequency deflection or static deflection under some constant force F0.

So this is magnification factor is the ratio of the steady state amplitude of the steady state response upon the 0 frequency deflection. So this magnification factor is a function of r and zeta, so zeta is the damping factor and r is the frequency ratio. It means that for any given damping, if we change the r means, we change the ratio of the force frequency upon the natural frequency of the system.

We will get a curve, okay for different r for one damping factor. Similarly, if we take several damping factors, we will get a group of similar curves.

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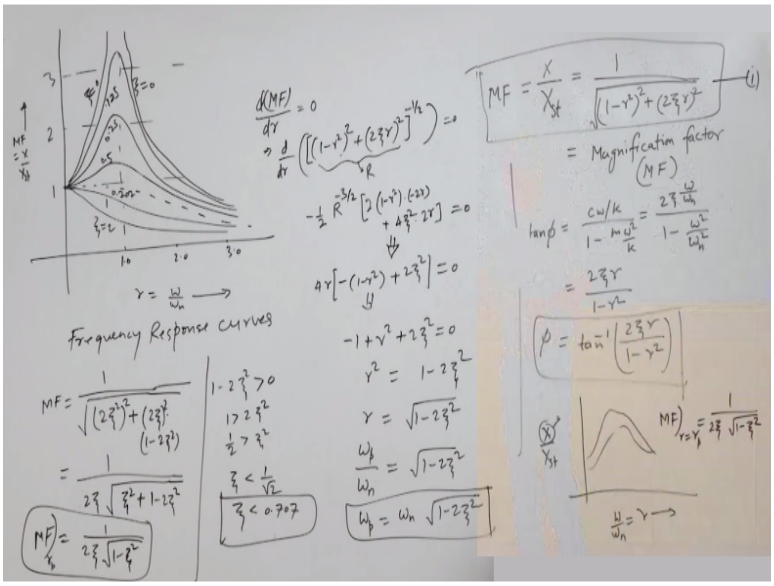

So these curves, because they tell; so for example if we take here like X by Xst and here r that is omega and for some damping, we are getting some curve. Then for other damping, we are getting some curve, so we are getting these r is nothing but omega by omega n so frequency ratio. So this curve tells us the response, a steady state response amplitude at some force frequency, some forcing frequency.

So that is why these kind of curves are called the frequency response curve. So here frequency response curves, we can plot for a single degree of freedom system, okay. So r = omega by omega n and here it is mf or X by Xst, so this is in this curve. So we can plot this frequency, so this is the frequency response curve that we are going to plot frequency response curves.

Because they are for certain frequencies, so we can plot these, so here we have, so this is zeta=2, 1, 0.7, 0.5, 0.25, 0.125 and then zeta =0, zeta = 0. So this is the frequency response curve. Now this curve tells some important characteristics and it helps us to design a system that of our choice based on the limits of the vibration response or the amplitude of the vibration.

Because here we can select our interest of frequency range that in which my system is subjected to some force so we have some frequency range and in that frequency range, what response we are expecting? we can select that damping value, so that our system response is staying in our choice, so this frequency response curve we are seeing that; please note this curve.

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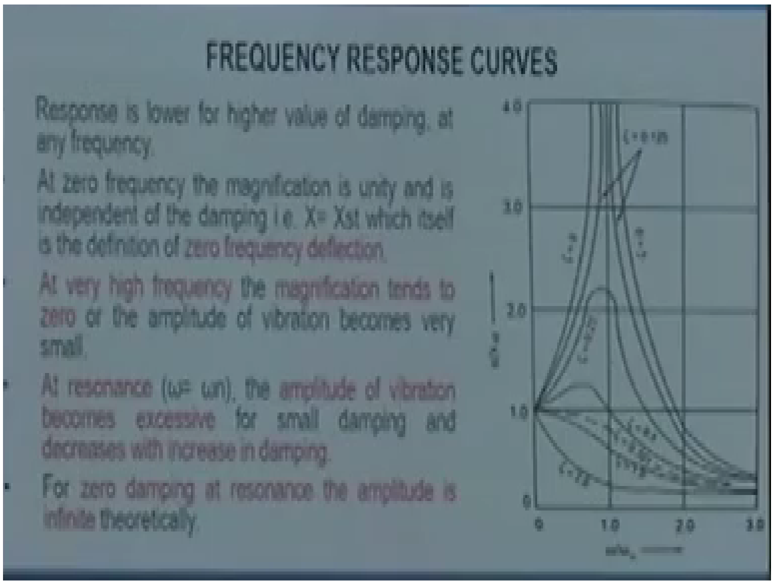

We can see that if we are encouraging the damping, the response of the system is decreasing, means the amplitude x; a steady state amplitude x is decreasing, if you increase the value of damping zeta and that is obvious because we; damping is the parameter that controls the vibration response. Moreover, we can see that when we are near the r=1 or omega by omega n=1 or our forcing frequency is equal to the natural frequency of the system.

Then the response is very high, response is very high. Although if we increase the damping, the response is decreasing but in compare to the other regions, for example, r less than 1 or r much greater than 1, the response is quiet high. When r=0, the response is X by Xst =1 means X= Xst. So that is the definition of the 0 frequency deflection, so when r =0 means, we have some force F0, a constant force is acting, so the response is equal to X Xst.

And that so X/Xst =1, so all the curves are we starting at X/Xst =1and they are moreover we can see that there are some peaks occurring, we can see some peaks; here are some peak point 5, some 0. 25, 0.25 and so we see that the response is not taking place maximum at r= 1 means, at resonance, the maximum is not taking place but maximum is taking place little earlier than the resonance point, okay.

And we see that when we are decreasing the damping, we are more going close to the; peak is more moving towards the resonance condition that is r=1. If we have the 0 damping, so zeta = 0; I mean no damping, the response is infinitely large, okay. So now we are; we want to know that what is the peak frequency or at what value of r the peak of this curves occur? So if you want to get the peak, we have to; you can get it by differentiating this equation with respect to r and equate it to 0.

So d MF by dr=0, then we can get the maximas, so we have; this implies that d by dr and this quantity we can write; 1 minus r square, whole square plus 2 zeta r whole square and this is minus half, okay so this equal to 0. So now we differentiate it and let us say this quantity is like we say R, so minus half into R minus 3/2, okay this is R power minus half, so minus half R, minus half minus 1; so minus 3/2, now we differentiate inside.

So here we have 2, 1 minus r square into minus 2r, so x power n, so nx n-1, so 2, 1-r square into inside, so 0 minus 2r plus this quantity, so this is 4 zeta square into RA square, so RA square again is 2r, okay and this is equal to 0. Okay. So let us take this term mainly so here we have, we can take 2r outside, so we have, in fact we can take 4r, so we take 4r, and so it is minus 1 minus r square and plus 2 zeta square =0.

So here r=0, where we get this unity or we make this equal to 0, so minus 1 plus r square plus 2 zeta square equal to 0, minus 1 plus r square plus 2 zeta square, so it means r square = 1 minus 2 zeta square and so r equal to under root 1 minus 2 zeta square and so omega p by omega n equal to root 1 minus 2 zeta square plus omega p equal to omega n root 1 minus 2 zeta square.

So what we are getting that our maxima occurring at r equal to root one minus 2 zeta square and to occur a maxima, these quantity should not be negative, it means, 1 minus 2 zeta square is greater than 0, so it means that; so we take the case of positive quantity so here, zeta is less than 1 by root 2 or 0.707. So if we have the value of damping, so we will get the peak only if our damping is less than 0.707.

And this we can see from the curve, we see that when this is the dotted line is the 0.707, here we are getting the peak at unity but after this we are encouraging the damping because it is 1,2 we are not getting any peak, so and for zeta, less than 0.707 so it is 0.525, we are getting clearly the peaks, so a peak only will occur if zeta is less than 0.707, then what will be the value of magnification factor when r at the peak?

So what is the value of magnification factor for the peak, so we can put this in the equation here, so we will get mf=1/ under root, so 1 minus r square is; we can write here, so it is 1 minus r square here, from here we can write 1 minus r square = 2 zeta square, so here we can write 2 zeta square, whole square plus 2 zeta into r; so r here we get, so 2 zeta r square, so 2 zeta square into r square; r square is nothing but 1 minus 2 zeta square.

So here we can solve it, so 2 zeta whole square, we can take outside; under root 2 zeta whole square, so we can take 2 zeta outside here and this is zeta, so zeta square plus 1 minus 2 zeta square. So here we can write 2 zeta into zeta, so 2 zeta whole square that comes and here zeta square remains. So with this we can write 1/2 zeta under root 1 minus zeta square, okay so this is mf at rp, so peak**;** so this is at; so here we can write that MF at r equal to rp.

So peak equal to 1 by 2 zeta, 1 minus 2 zeta square, so this is only the function of damping, here this peaks we can find okay and we see that if we are decreasing the damping okay, we are tending to 1/2 zeta, so we are going to tend towards this line, okay. So here it is clear that why we have this 0.707 that glides the peaks and the peak response and absence of the peaks and from here we can understand the effect of damping.

Because if you we are encouraging the; if we are near the resonance condition and we do not have the damping then the amplitude is infinite but if we are encouraging the damping, the amplitude is deducing, so in this near r = 1 or omega=omega n and we have only method to control the vibration is the; we can induce some damping. Now come to the phase angle, phase angle we have already written here, phase angle.

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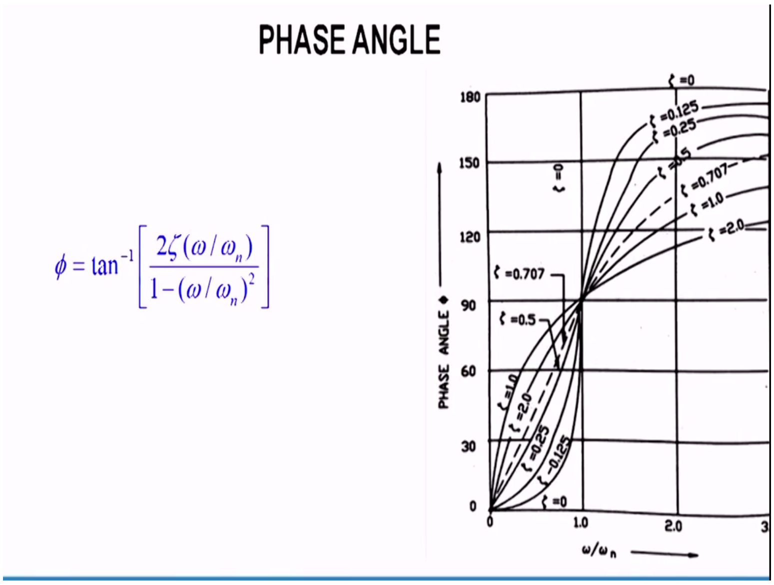

So phi = tan inverse 2 zeta r upon 1 minus r square and we see here that we can plot the phase angle similar to the frequency response curve, as zeta is also function of zeta and r so far, r and we have plotted phase angle so what we can see that phase angle is; it is 0 about 0 at low frequency, at low r, okay.

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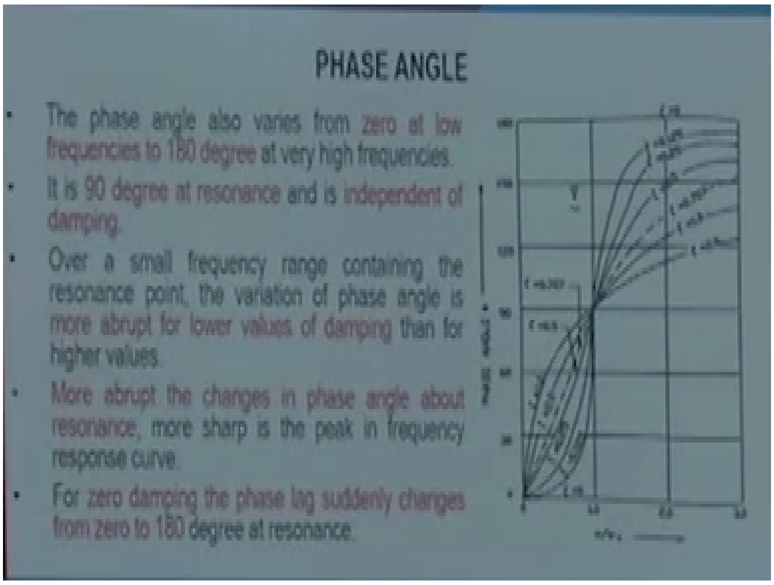

So it is start from 0, when r=0, it is start from 0 and it is increasing and going towards 180 degree, when we are changing the r. Moreover, we see that all the curves are passing, all the curves are passing through phi = 90 degree at r=1, so at resonance condition for any value of damping, we have the 90 degree phase angle that is fixed 90 degree phase angle and this is independent of damping.

For any damping curve, for any value of damping, all the curves pass through this phi = 90 degree. So if we have lower value of damping, in the phase angle plot, okay, there is more abrupt changes. So here we see that if we have zeta = 0.125, okay so or zeta=0, you see zeta =0 is this line so this line at 1, it is going to 90 and then it is going to 180 degree, so very abrupt change of the phase angle.

Similarly, here zeta =0.125, 0 to 90 and then 180 degree; towards 180 degree. So at higher value of r they are tending to 180 degree and lower values of r, they are near 0 degree and more abrupt the changes in phase angle about resonance, the most sharp is the peak in the frequency response curve. For example, zeta = 0.125, we have very abrupt change, we are 0 to changing to 180 degree and here we can see zeta = very abrupt peak.

So here in the frequency response curve, sharp peak and for 0 damping, the phase suddenly changes from 0 to 180 degree at resonance. So we discussed the magnification factor and frequency response curve as frequency response curve are the function of damping and for different damping values, they give the response a steady state response, amplitude response at the; in the different frequency force frequency.

So they gave us some information that can help us to design our system I mean; we can select that what is our frequency range and based on that what the response we want? we can select the damping value, okay. So these frequency response curve and phase angle plot they give us some very important information and that is useful for about the single degree of freedom systems. So thank you for your attention and see you in the next lecture.