Introduction to Mechanical Vibration
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Lecture - 10
Coulomb Damping

Welcome to the lecture on logarithmic decrement and free vibration under coulomb damping. So we have already discussed the viscous damping case and we discussed the over damping, critical damping and under damping.

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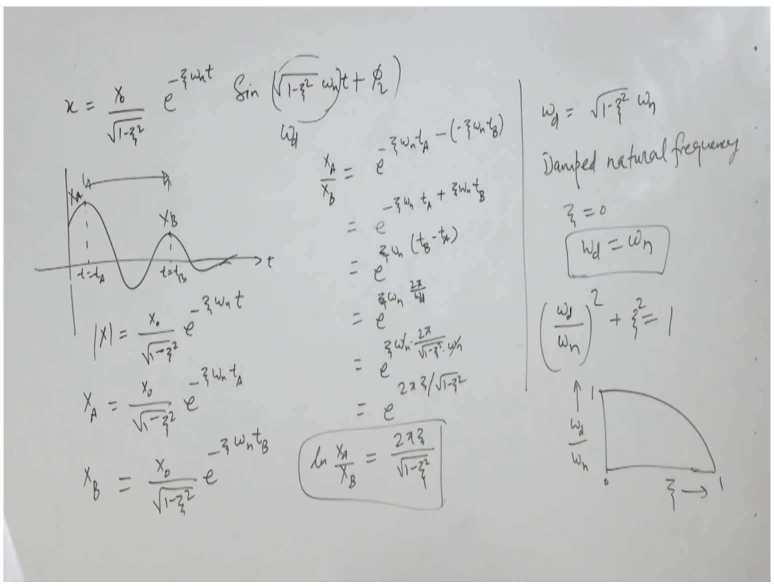

And in case of under damping we find the response that is x = X0 by root 1 - zeta square exponential e power - zeta omega n t sin root 1 - zeta square omega n t + here pi 2 - pi 2 is the phase, so and this - this all the three responses we can compare on in the figure shown here so we can see here we have this over damped system zeta greater than 1.

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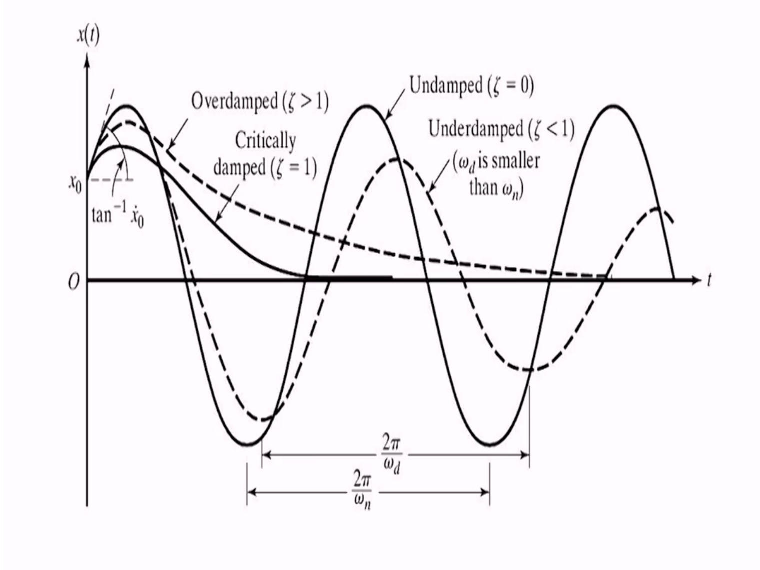

And we can see that okay through all the initial conditions they start and it is reaching to the equilibrium or 0 for this amount of time, then the critical damping zeta = 1 and this is reaching to 0 here and we can see both are aperiodic motion they do not do any oscillations and we see that the critical damping is faster than the over damping and we said that when we increase damping the system becomes the motion becomes slower.

And critical damping is the least amount of damping that will make the system to reach to 0 or equilibrium. now comes to the under damping system so here is the under damping system this is undamped system zeta = 0 means there is no damping so system is following the harmonic motion and we have the under damped zeta < 1 and we see that this system is the amplitude is going to decay with time.

Now comes to we come to discuss logarithmic decrement that is relevant to the under damped system and logarithmic decrement is the, it shows the rate of decay in an under damped system because underdamped system has oscillation - oscillations that decay with time in the exponential manner so logarithmic decrement it is the ratio of the two successive amplitudes on natural log so we say delta = ln X1 by X2, where X1 is the first amplitude and X2 is the next amplitude.

So we can find out let us say this is our system so let us say this is A this is B, X A X B the amplitude here, so because our amplitude - amplitude is given as X0 by root 1 - zeta square in to e power - zeta omega n t, so our amplitude is given like this. now if we say X A that is a time t = - t = t A and this is time t = t B this is time, so X0 by root 1 - zeta square e power - zeta omega n t A.

And similarly X B = X0 by root 1 - zeta square e power - zeta omega n t B, so they are X A and X B they are two consecutive amplitudes of the under damped system motion, so we got X A and X B. now we put X A by X B - X A by X B = so this will be cancelled out we will have exponential - zeta omega n t A -, - zeta omega n t B so this is equal to e power zeta omega n t A + zeta omega n t B.

So here we can write e power zeta omega n t B - t A, e power zeta omega n t B - t A, this is t B - t A and this is nothing but the time - the time period because this is the time between two period and what is the time period we can see the time period is coming from this because this is the omega we call it omega d, that is omega d = root 1 - zeta square omega n, so this is called damped natural frequency - damped natural frequency.

When zeta = 0 omega d = omega n, so the system will vibrate with this frequency when we have a damping zeta while the undamped system was vibrating with its own natural frequency that was omega n, so omega d is root here we can write also omega d by omega n square + zeta square = 1, so this is showing some ellipse and we can plot this omega d by omega n here and this is zeta.

So this is showing an ellipse so here 1 so omega d, now here we can write t B - t A that is one time period, so it is 2 pi upon omega d and = e power zeta omega n into 2 pi upon omega d is root 1 - zeta square into omega n, so omega and cancelled out we find e power 2 pi zeta upon root 1 - zeta square, now we take the natural log ln X A by X B = 2 pi zeta upon root 1 - zeta square. So here we see that this logarithmic decrement is depending nothing.

But one - only on the damping factor zeta, when we have a zeta much less than 1, so we can write this as delta = ln X A by X B = 2 pi zeta, so this is an approximate relation for the logarithmic decrement, okay.

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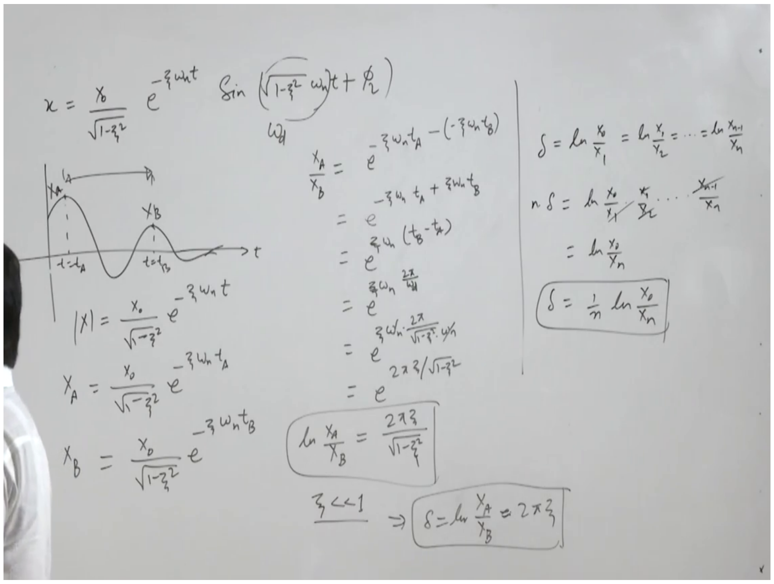

Now we can also write delta = ln X0 by X1 or ln X1 by X2 or ln Xn - 1 by Xn, because delta is the log of ratio of any two successive amplitudes, so it could be between X0 and X1 or X1 and X2 or Xn - 1, now we - we do like we add this terms so we add so this is n terms so it is n into delta and when we add they will come into multiplication so X0 by X1 into X1 X2 into Xn - 1 by Xn. So all these intermediate terms simply cancelled out and we will get ln X0 by Xn,

So here delta = 1 by n ln X0 by Xn, so this is another expression for the logarithmic decrement. So now here we complete the viscous damping free vibration under viscous damping. Now we start the another type of damping and free vibration of a system that as the coulomb damping. So we already discussed the coulomb damping, a damping due to friction between the two surfaces and there is the dry friction and coefficient of friction mu is there.

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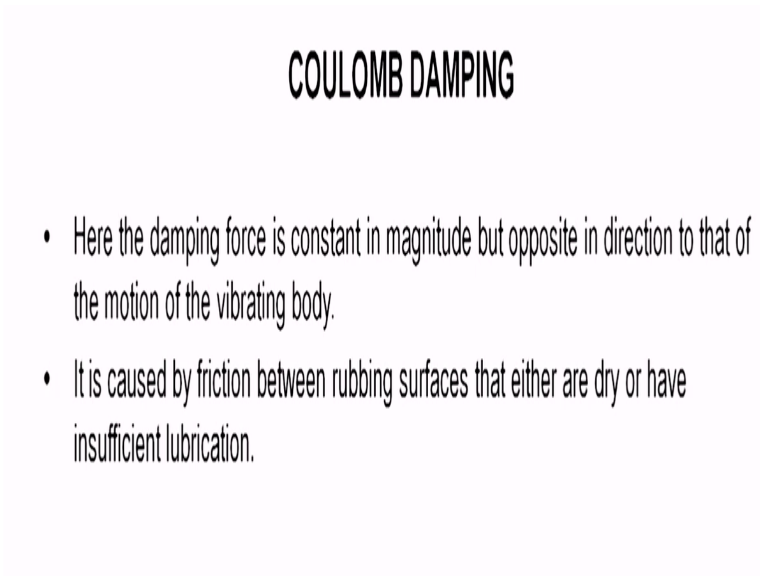

And due to this coefficient of friction and due to the moment relative moment between the two bodies there is this kind of damping. So we have now this system so we represent the system.

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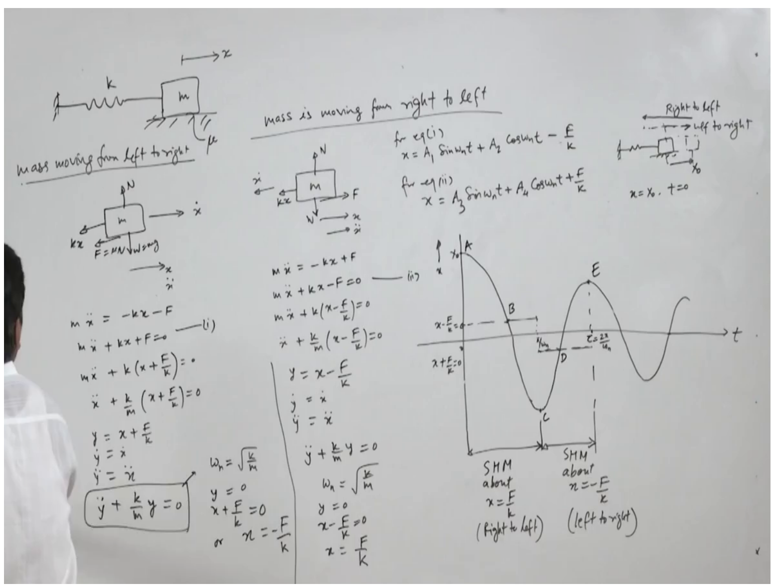

So we have a system, let us say this is our spring and this is our mass, this mass is moving on a surface that has a coefficient of friction mu, so this is m and k and here it is moving in this direction let us assume and there is the friction coefficient mu between the two surfaces on the it is moving on horizontal surface, so there is a coefficient of friction mu between the horizontal surface and the block m.

Now because the direction of the frictional force depends on the relative velocity, so in half cycle the mass will move from left to right and another half cycle it will move from right to left so we will have two cases of this moment, so let us say this is the mass, so mass moving from left to right, so we have this mass and it is moving from left to right, so it is moving this - this is the direction of x dot and this is x and x double dot.

So the in this direction we assume x and x double dot as positive, so when it is moving we will have the spring force k x and because it is moving this side so the friction force will be working in this direction, so we will have a friction force here F that is equal to mu times N and N is the reaction and here W is the weight and N is the reaction here W = m times g, similarly so we can write a equation of motion so this is a free body diagram.

Now we write m x double dot = the forces so x double dot direction is this and force - k x and - F, so we have m x double dot + k x + F = 0, now we can have this we can readjust these terms so x double dot + so m x double dot + k x + F by k = 0, so we can write x double dot + k by m x + F by k = 0. Now we assume another axis y = x + F by k, so y dot = x dot and y double dot = x double dot.

Now we put these terms here and we change the co-ordinate so y double dot we can write x double dot = y double dot + k by m and here x + F by k is y so here it is y = 0, so this is nothing but showing differential equation showing similar to the undamped free vibration and so from here we see that the system is vibrating with the natural frequency omega n = root k by m, okay.

So in this case the mass will oscillate with the natural frequency that is omega n = root k by m but it will oscillate not about x = 0, but it will oscillate about y = 0, so y = 0 means x + F by k = 0 or x = - F by k, so it will oscillate about this line rather than x = 0. Now we take the another case, the second case which will mass is moving from right to left, so here mass is moving from right to left.

So we can make the free body so this is mass and it is moving in this direction so this is x dot, however x is this and x double dot we assume in this direction positive and so spring force will be working here, because mass is still stretch to this direction to this side but moving towards the left side, so k x is the spring force this time the friction force F will work in this direction and here will be W and here the N.

Now again we write the equation of motion so here m x double dot = - k x + F, so here m x double dot + k x - F = 0. so this is the equation of motion of this system for this cycle, because in the half cycle the system, when the system is moving from left to right it will follow this equation, when it is moving from right to left it will follow this equation. Now again we do like m x double dot + k x - F by k = 0 and x double dot + k by m x - F by k = 0.

So again we assume y = x - F by k here and y dot = x dot and y double dot = x double dot, okay. So d square y by dt square = d square x by dt square. Now we will put these values here so we will have y co-ordinate y double dot + k by m into y = 0, from here we find that the natural frequency omega n is root k by m, but the system is vibrating about y = 0 that is x - F by k = 0 or x = + F by k. now what is this is the system, now we can plot and see the response of the system although if we want to see the response of this differential equation.

So here the response of the differential equation this one will be x = A 1 sign omega n t + A 2 cos omega n t + and - F by k, while for equation this is for equation for equation one, for equation two x = A 3 sin omega n t + A 4 cos omega n t + F by k. so now if we want to see the response of the system, so we can plot it with for this we, let us assume some initial conditions, so x = X0 at t = 0.

So for such initial conditions we will start, so it means that we have this mass say equilibrium position and we have moved pushed it this mass here, so we pushed it with X0 and then we release it, so it will move start moving from right to left, so it is moving from right - right to left and then in the next cycle it will move left to - left to right, okay. So when we have pulled it with X0 and leaving it is moving right to left.

So here this is X0 so and this is x, so it is moving, so the system here at X0 when t = 0 this is time and till here, because here is the one cycle completed and so it is tau - tau - tau = 2 pi upon omega n and this is pi upon omega n, so this is half cycle - half cycle this system is moving so this system is in this cycle doing SHM about because this in this part it is moving right to left. So here we have the equation right to left and it is moving about x = F by k or x - F by k = 0.

So this is x - F by k = 0 line, so this is the line for this cycle, so SHM about x = F by k, so in this half cycle it will vibrate about this axis. In the next half cycle when it is moving so here it is moving right to left - right to left, now in this cycle till here it will do SHM about x = - F by k. Because here it is moving left to right - left to right, so left to right when it is moving it will oscillate about x = - F by k or x + F by k = 0.

So this is the line here, so this is the line x + F by k = 0, so this is this line and so in this it will vibrate about this axis, so this response we can of this system having coulomb damping we can see also from this curve.

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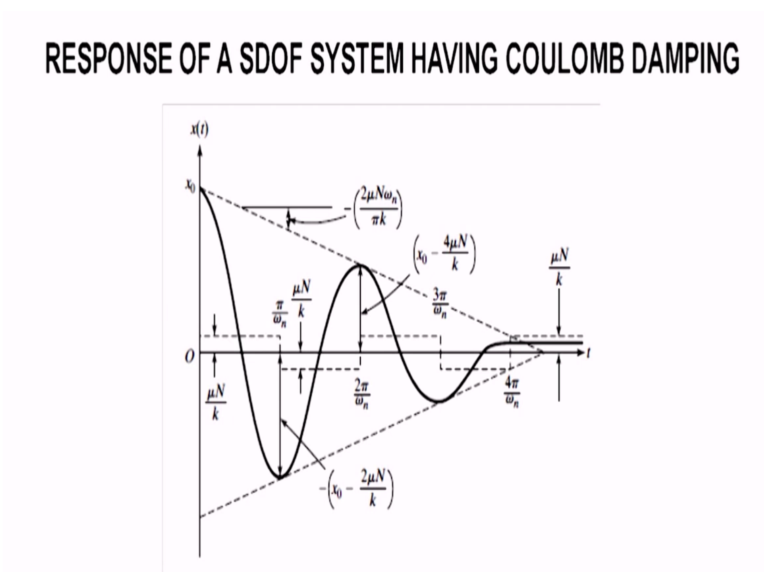

Now we want to know that what is the - what is the oscillations so the rate of decay, because we want to know that what is the kind of decay here because although the amplitude is going to decrease but how much so this - this information we want to know, so for this let us we say this is a point A, this is point B, and this is point C, point D and point E, so it starts its motion at A and passing through B, C, D and E, so this is the half cycle.

So we can - we are interested to know that what is the decay in one half decay of the amplitude in one cycle this is what we want to know and how we can know this, because we have the response we can see here the response, now we can see that when we have this system k and m x and this system is vibrating, there is a constant friction force F.

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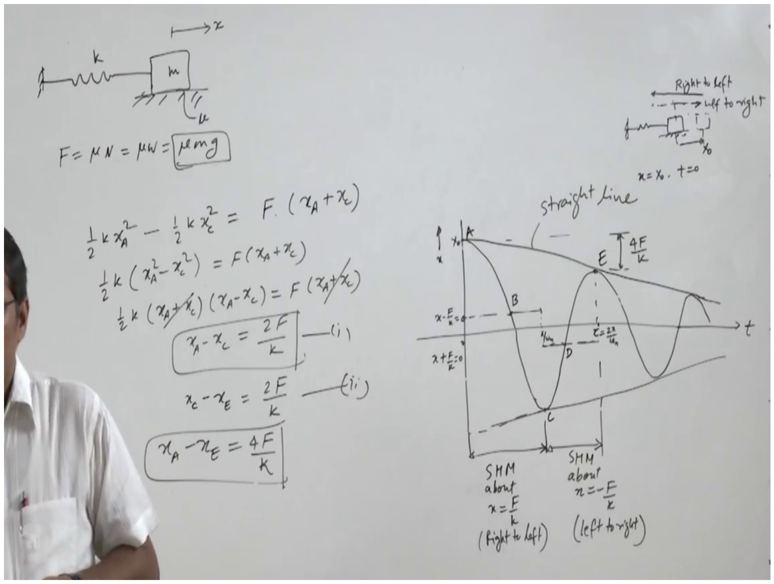

That is F = mu times n and N = W so mu times m g, here all these things are constants, because mu is constant m is constant g, so our force is constant so our mass is moving against a constant force, so it is doing some work against this constant force F and in doing this force the systems energy is dissipated and systems energy in terms of potential energy in the spring or kinetic. So this potential energy of the system.

So we have at A we have a potential energy half k xA square, when the system is reaching to point C it has potential energy half k xC square, now the change in potential energy will be used to do the work against the friction force, so change in potential energy is this half k xA square - half k xC square and this is friction force is F, and the net moment of this system is so this is xA and this is xC, so xA + xC.

So total this is the moment of the system, now we can write half k, so xA square - xC square = F xA + xC, so here we can write half k xA + xC and xA - xC = F xA + xC, so xA + xC this will cancelled out and so we will get xA - xC = 2F by k, so we see that the amplitude there is the decay in the amplitude from A to C and that decay xA - xC = constant 2F by k. Because the F is constant and k is the stiffness of the spring that is constant so it is a constant quantity.

So this is what we calculated for half cycle because from A to C it is half cycle, similarly for this half cycle we will find that xC - xE = 2F by k, so let us say this is an equation one and this is equation two, now we add this so we will have xA - xE = 4F by k. So here we see that from A to E there is a reduction in the amplitude 4F by k a constant quantity, so here if we have this and this is this so we have this quantity that is 4F by k, and so this is constant.

So this relation is linear, similarly here this relation will be coming as linear and same here, so we will have a linear relation, so this is a straight line. So we see that in case of coulomb damping the reduction in the amplitude the two consecutive amplitude is for 4F by k that is a constant quantity and the envelope that is passing through the maxima of this amplitudes maxima is a straight line, if we compare with respect to the viscous under damped viscous damping case where the envelope was exponential curve.

So the reduction envelope was the amplitude was reducing in a - in an exponential way and the rate of decay we defined in terms of logarithmic decrement and logarithmic decrement we defined that was the ratio of the two consecutive amplitudes and taking the natural log of that, but here we can directly define the difference between the two amplitude xA - xC that is equal to 4F by k.

And another important point is that in case of viscous damping the vibration was occurring about the x = 0, but here in the half cycle it is occurring about x - F by k = 0, in the next half cycle it is occurring about x + F by k = 0. Here the frequency of vibration is the same as the undamped vibration, undamped natural frequency. So here the system is vibrating with the omega n that is the natural frequency of the system omega n = root k by m.

In case of viscous damping the system was vibrating with not omega n but a changed the frequency that was omega d, the damped natural frequency we defined omega d that was equal to omega n root one - zeta square. So there are the differences between the two the physics of the two damping's and we understand basically the two things we were interested the frequency of vibration and the rate of decay and both things.

We have discussed for viscous damping as well as the coulomb damping. So thank you for your attention for this lecture and see you in the next lecture. Thank you.