INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Convective Heat Transfer

Lec- 09

Natural Convection: Uniform Heat Flux

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Hello welcome in the 9th lecture of convective transfer course in this lecture we will be disusing about national convection but this time we will be keeping the plate supplying uniform heat flux, okay in the previous lecture we have considered plate was that constant temperature here will be considering plate is at uniform heat flux okay, so let me show you that what things will be actually covering in this lecture and first we will be studying the governing equation and boundary condition.

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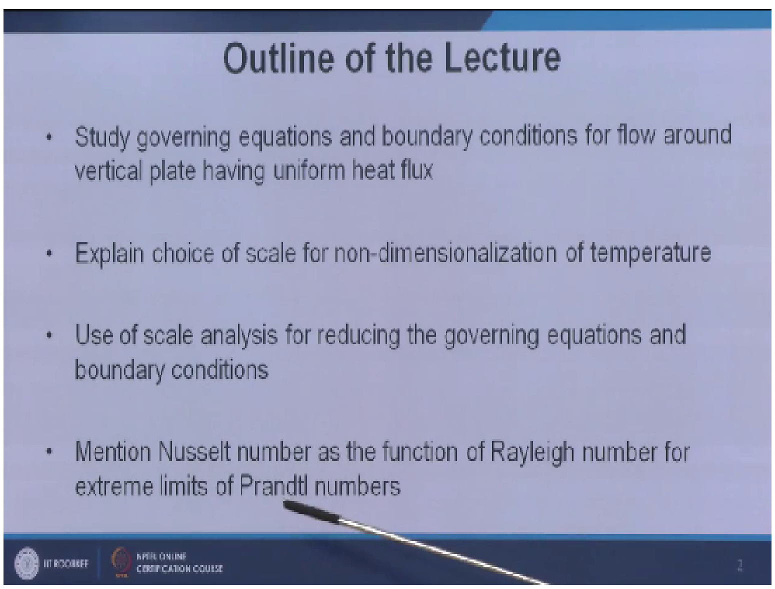

For flow around vertical plate having uniform heat flux, so uniform heat flux is very important over here okay, next we will be explaining the choice of non-dimensionalization of temperature in the last lecture the choice of velocity was important here also velocity we have to find out, but here now what will be the temperical scale that is also important okay use of scale analysis will be doing for reducing the governing equation and subsequently we will be mentioning what are the boundary conditions okay.

And towards the end will be mentioning what is the nusselt number as the function of the Reynolds Rayleigh number and Prandtl number okay, so let me then come from the very beginning already we have shown that the boundary equation for natural convection comes in this form okay.

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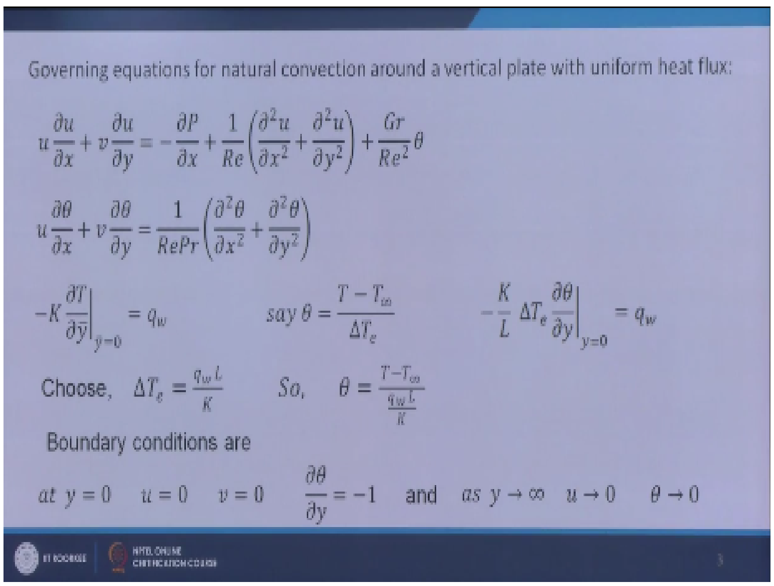

So here you can see the left hand side is having intertie then pressure gradient then viscous term and subsequently this is the by n term okay by in say term is having the coefficient grass of number by Reynolds number square okay, and in case of your energy equation left hand side is our convection and in right hand side we are having conduction equation informed of the conduction we are having the coefficient 1 by decline number or 1/RePr okay, now let me show you for uniform heat flux how boundary condition is being changed.

Now as it is uniform heat flux so obviously plate temperature is not define, but from the plate what is the heat flux that is constant, so the heat flux I am taking sQw okay, so this heat will be actually release from the solid plate solid vertical plate in the form of a conduction so I am writing the conduction as – k ∂T/∂y at y = 0 whenever these y are actually dimensional y, y bar okay now we have to construct the temperature scale so that we can write down θ as T - T∞ / ΔT_e so but ΔT_e over here is unknown.

In the previous case of constant temperature we have written this ΔTe as Tw - T∞ but here Tw is not known, so what we have to do let us try to find out that can be construct the temperature scale from this boundary conditions if you go for the scale analysis or this boundary condition we can write down K into ΔTe / L, so Te is being non – dimensonlize to θ as a result ΔTe is coming out, and small y bar is being non- dimensionalize to small y so as a result L is coming out over here, now this equation looks like little bit difficulty in nature.

So if we choose ΔTe in such a fashion that this K/L and Kw gets cancelled out then this will be very easier looking equation okay, so let us take then the value of ΔTe as qw/ l qw x l / K okay. So once we have determined what is the temperature scale we can easily write down θ is nothing but T - T∞/ Kw l /K okay so subsequently let us see that how the boundary conditions are now be coming simplify so boundary conditions, so for first the velocity as we have shown in the previous lecture the boundary conditions will be very simple.

Mostly no penetration at the wall at y = 0 u = 0 and v = 0 and juts now we have constructed simplified one, so that this Δθ/Δy cab become a simplified 1 – 1 so ΔTe we have chosen in a such a fashion that this gives writes Δθ/Δy is equals to – 1 at the wall okay. And away from the wall in the first whenever y takes to ∞ obviously you will be taking towards 0 because there is no flow away from the plate and θ will be tending towards 0, because T will be tending towards ∞ away from the wall okay so these are the boundary conditions for uniform heat flux we have constructed right.

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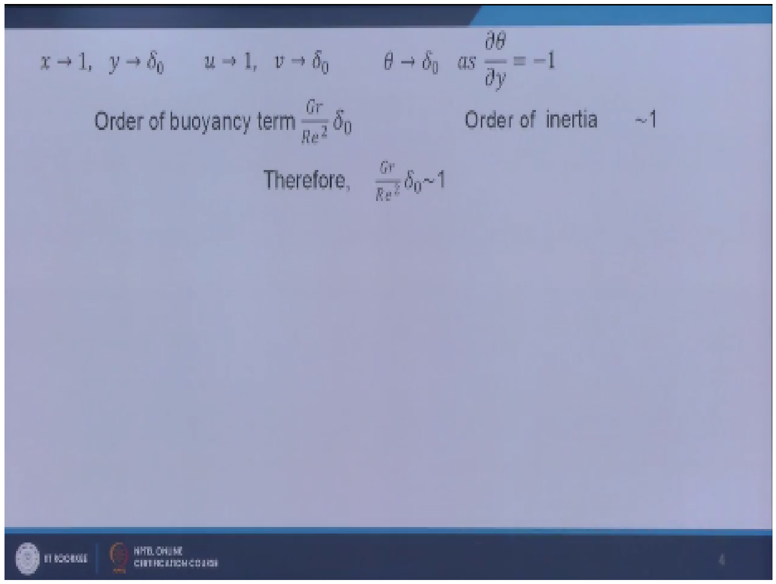

Next let us try to understand that what will be the lane scales for different equations first we start from the continuity equation, so in the continuity equation first we will be constructing the lane scale for the x and y direction for x direction we can write down integers of order one, and subsequently direction we are having a some boundary layer thickness we are calling that one has δ_0 so y direction is here actually having δ_0 thickness okay, so as we have decided x and y so from continuity equation definitely u and v will be sided okay.

So δu δx will gives me that u will be definitely of order of 1 and δv δy gives you that v will be definitely of order δ_0 okay, for θ obviously it will be of order δ_0 because ∂T/∂y is of or is actually equals to – 1, if we take y is of order δ_0 then definitely θ has to be of order δ_0 okay so this is a new thing coming from the previous lecture okay. So if you try to find out what is the order of buoyancy then we will be finding out the buoyancy term is actually Gr/Re^2 into δ_0 okay so if you see the buoyancy term so Gr/Re^2 and θ at the same time we are having order of inertia which is of order of 1 okay.

In last lecture also we have shown that inertia term becomes of the order of one now if we equate the buoyancy inertia orders then we can find out what is the value of this δ_0 in y direction okay so here you can see Gr/Re^2 into δ_0 is of the order of one okay.

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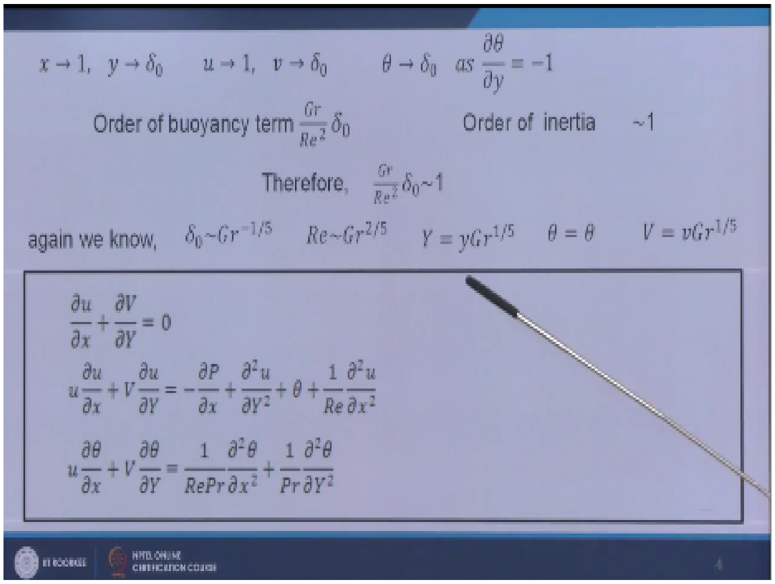

So from here we can get that δ_0 is actually your $Gr^{2\ 1/5}$ okay so this is coming from your velocity boundary layer thickness okay, so if we find out the δ^0 is $Gr^{1/5}$ from the velocity boundary layer thickness and if we took this over here we can get the information for Re and Gr number so Re number will be determine in the order of Gr number to be part 2/5 okay , so as we have already seen that y is actually of order of √Re so we can write down y bar capital Y = y x $Gr^{1/5}$ so √Re will be giving you actually $Gr^{1/5}$ okay so we are writing Y = y into $Gr^{1/5}$ so this is just touching we are doing for y variable.

For θ we need not to do anything okay because already we have considered the all scale over here and for V we are doing the ordering like this V = v into $Gr^{1/5}$ okay, now if we put all this stretched variables in my equation previous forms of equations, so these equations.

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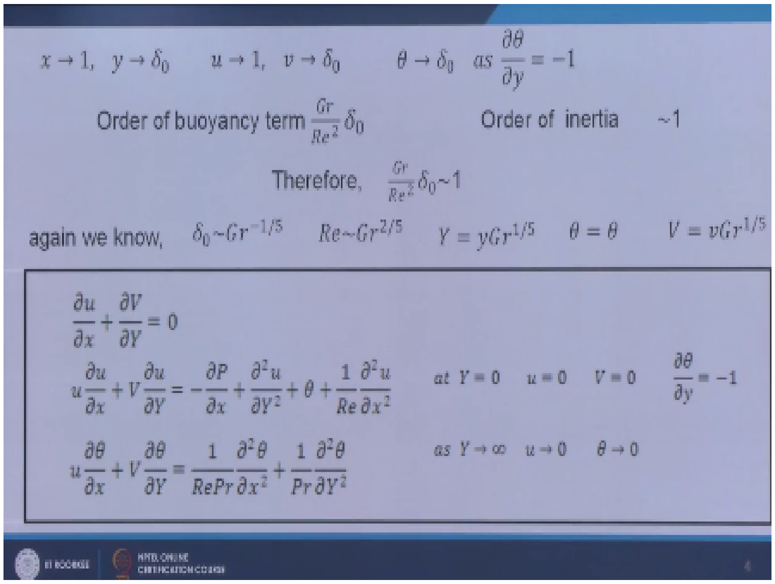

So new states of equations comes out like this, ∂u / ∂x + ∂V/∂Y = 0 and momentum equation becomes this is my inertia term, u ∂u ∂x + v capital V ∂u ∂Y = pressure gradient term - ∂P ∂x + your viscous term + δ^2 small u ∂Y^2 + θ which is your buoyancy term and this one comes from your viscous term industry noise direction vertical direction + 1/Re ∂^2u ∂x^2 right. And from your energy equation you can see this is the convection term u ∂θ/∂x + v capital V ∂θ/∂Y and in the conduction side we are having 1/RePr okay so, stream wise conduction we are having no coefficient coming out so it is becoming 1/P number into ∂^2 θ ∂x^2 and in the cross term wise deduction.

Conduction becomes 1/Pr ∂^2 θ ∂Y^2 okay side by side let us see the boundary conditions these already we have mentioned that y= 0 u and capital V become 0 and ∂θ/∂Y = - 1 from here we have seen okay and away from the plate we are having y tends to ∞ means it will be having more velocity u tells to 0 and obviously θ tends to 0 as T tends to T∞ okay, now let us take the boundary where approximation from this equation if you take the boundary where approximation this term and this term can be cancelled out and P = constant if you take and this term can also be cancelled down. Now so my

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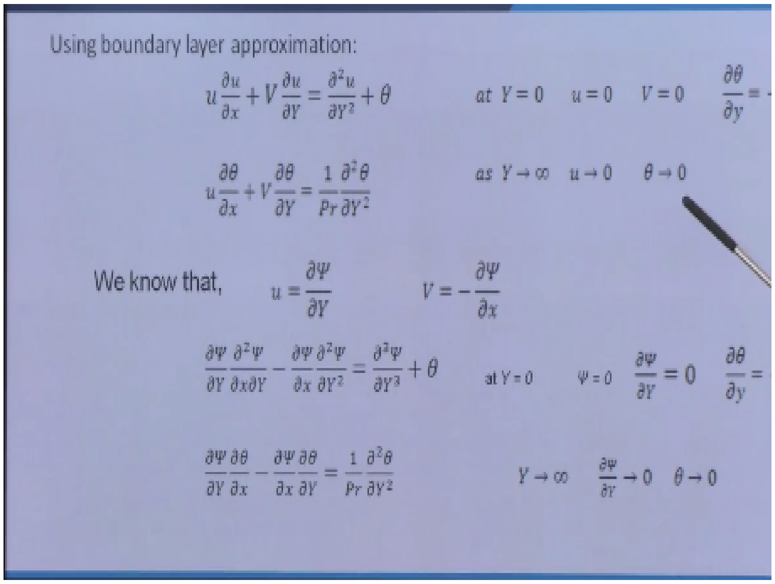

Momentum equation becomes u ∂u/x + V ∂u/∂Y= ∂^2u/∂Y^2 + θ okay so this is the ny equation and my left hand side becomes convection and right hand side is conduction 1/Pl is the coefficient in the before the conduction term. Hello with the boundary condition, boundary condition are same as I have shown you in the previous slide. Let us go for then the concepts u is nothing but ∂ψ/∂Y V is nothing but - ∂ψ/∂x we have shown in the previous lecture. So after incorporation this steam function concept this boundary layer approximation and energy equations.

We are putting the value of u over here and this is ∂u ∂x okay, so once if you do the derivative of u respective to x you will be getting this term and here this is V so V comes over here in this case. Here we are having ψ∂/Y, in the right hand side we are having ∂^2ψ/∂Y^2 + θ and the corresponding energy equitation will become ∂ψ/∂x- this is V so –V -∂ψ/∂y ∂θ/∂Y = to the right hand side 1/pr ∂^{θ} /∂Y^2 now if you see the boundary conditions at Y = 0 we are having u = 0, which means ∂ψ/∂Y, and away from the plate you see Y ∞ we are having ψ 0 which is nothing but ∂ψ/∂Y θ= 0.

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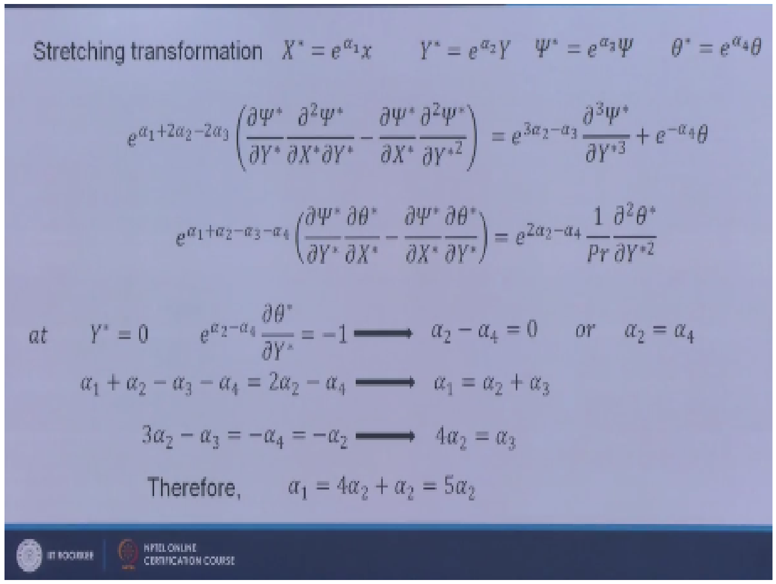

So now if you tends to apply the stretching variable to find out the similarity parameter done in the previous lecture so $X^*=e^{\alpha 1}x$, $Y^* = e^{\alpha 2}Y$ similarly $\psi^*= e^{\alpha 2}\psi$ and $\theta^*= e^{\alpha 4}\theta$. So if you consider that then your momentum energy equation will be having some coefficient to the powers okay, as we have seen in the previous lecture also. So these powers are coming in terms of α1,α2,α3 and α4 okay. These α1, α2, α3 and α4 relationship will be actually will help me to find out the similarity parameter. So let us see what is the coefficient in the actual term so $e^{\alpha 1+2a2-2a3}$.
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In before slide we discuss $e^{3\alpha 2-\alpha 3}$ before the e-α4θ in front of conviction we are having the $e^{\alpha 1+\alpha 2-\alpha 3-\alpha 4}$ and before the conduction we are having $e^{2\alpha 2-\alpha 4}$. Now let us try to find out what is the simplest form of boundary condition, so in the boundary condition we have already seen that if we are having the Y* = 0 then we are having $e^{\alpha 2-\alpha 4}$ ∂θ*/∂Y*= -1 okay, from here to make it simplify we can do $e^{\alpha 2-\alpha 4} = 0$. So α2-α4=0 or α2 = α4 okay.

Now from here your energy equation you can easily write down that α1+α2-α3-α4 = $e^{2\alpha 2-\alpha 4}$, so if you write down like this then you can get α1= α2 + α3 after cancellation okay. On the other hand if you equate this along with –α4 = θ, we can find out $4\alpha_2$ = α3. So finally α1 is only left using this one we can get this one which is nothing but $4\alpha_2 + \alpha_2 = 5\alpha_2$. So we have got α1, α2, α4 in terms of α2.

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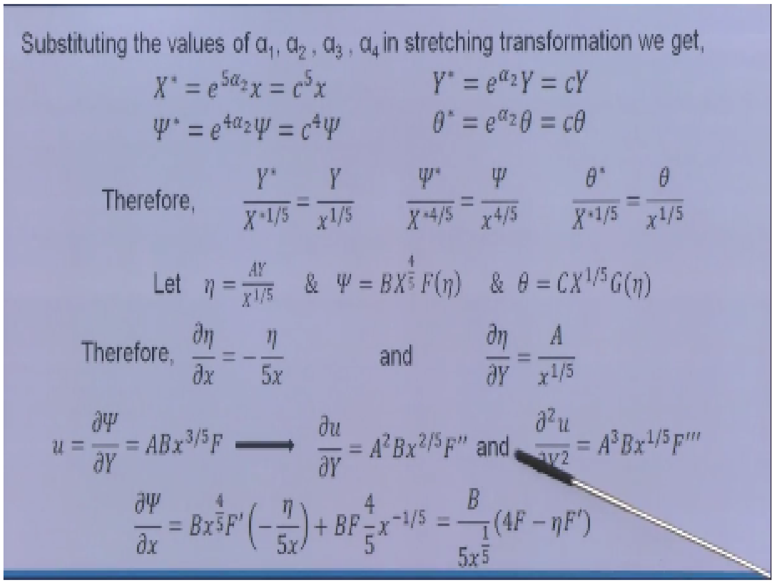

So let us write down the X* $e^{5\alpha2}$x and Y* = $e^{\alpha2}$ Y, ψ*= $e^{4\alpha2\psi}$ so from here we can easily constant the Y*/X*$^{1/5}$ = Y/ x$^{1/5}$ so from this equation and this equation we can get this okay, so by canceling the c you will be getting ψ*/x-$^{4/5}$ = ψ*/x$^{4/5}$ okay in the same fashion θ you will be cancelling θ*/X-$^{1/5}$ = θ*/x$^{1/5}$. So we have already got the significant idea so let us constant the similar idea now η=Y/ x$^{1/5}$ but in this case I do not know what is the was so I am keeping it has A okay.

In the similar fashion I can write down ψ as x$^{4/5}$ x F but I do not know the constant so let us keep that as B and similar fashion θ = x$^{1/5}$ x G so this all so another constant that we are keeping it has C. So let us make the derivatives of ∂ in last lecture also we have shown the single options can be obtained. Next let us try to obtain the derivatives for ψ okay so ψ derivative is giving V that I will show you later on, but once again if you the derivative of ψ A^{2}B$^{2/5}$ here.

And third times you do the derivative of A^{3}Bx$^{21/5p}$ this will come in conviction term okay. If you do the derivative to x we will be having X$^{4/5}$ and so the differentiation we have done for the, so X$^{4/5}$ we have kept constant and we have done the derivative for it and then X$^{4/5\ -1/5}$ and the simplification it gives this form of the equation.

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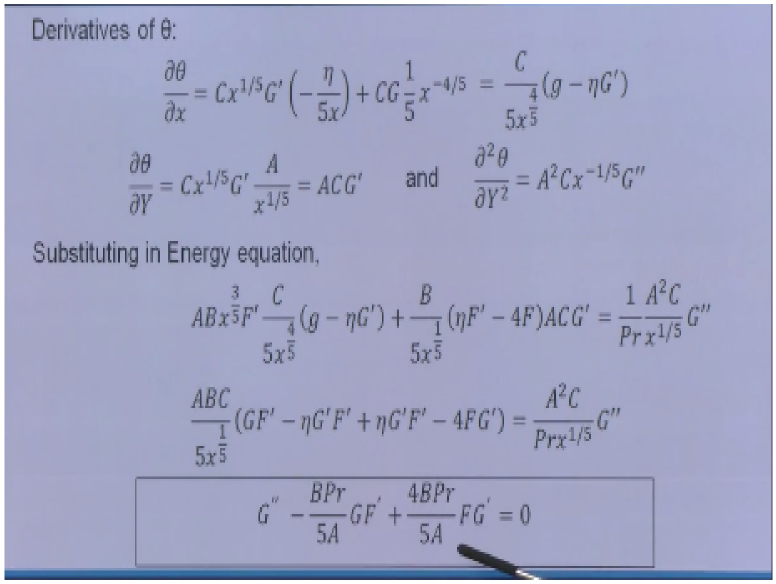

So we have got the value of V is nothing but - ∂ψ/∂x we have already found out, so this becomes ηF-4F okay, now you can out all these term in my momentum equation, no before that we have shown that what is the ∂u/∂x okay, so once again we have to do the derivative equation to get the x of this term okay, so if you do that we can find out differentiation of 2 multiplication. So first we have kept $x^{3/5}$ constant over here we have done the derivative of f over here and then f derivative of $x^{3/5}$ okay. Then from the momentum equation if you want to solve this we have to get this form of equation, so if you see this is my V and then finally this is my θ.

This will be signification will be giving you this form here these two term can be cancelled and finally you will get this form involving A and B, C. Still it requires the value of A, Band C we will found out. Let me also show you that what happens for energy equation and then we will be requiring the derivatives of θ, so let me show you ∂ θ /∂x, so here also first we have to go for x $^{1/5}$ and then you have to go G. So first x $^{1/5}$ has been kept constant G has been kept constant so this comes as δQ/δX ,in the similar fraction δQ/δY ,so its too be 1/5 this is kept as constant Gis kept done over here and the second time the Qis kept why you keeping this one .

So on so put all this in the energy equation .the simplified form of the energy equation comes out like this here you and we little stronger here previous slides okay. So if you simplify this one ,then we will get equation in this form okay, here also these two terms can be cancelled okay , so finally this becomes the energy equation already I have shown what is the.

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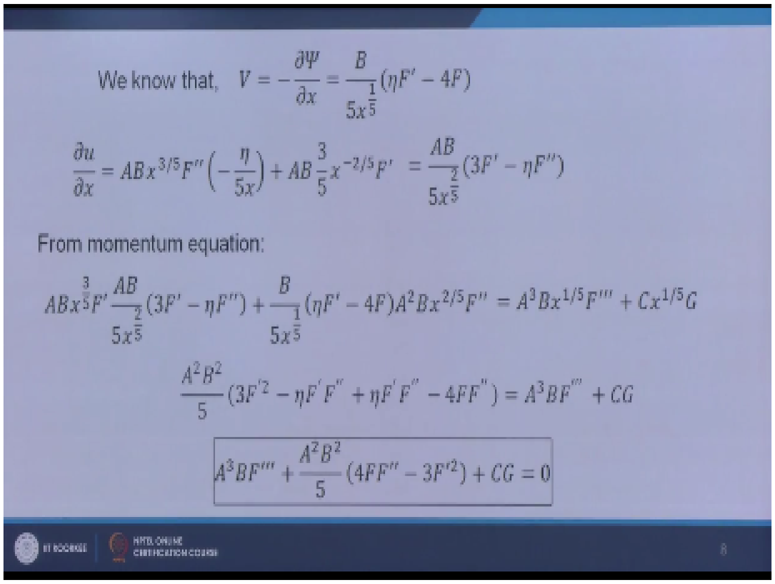

Form of my momentem equation over here but you can find out from here both this equation are coupled here also we having F Gokay,so the dependents of the G and here also we are having dependents of F G okay ,but before that we have to reduce this by chooosing the good values if A B C.

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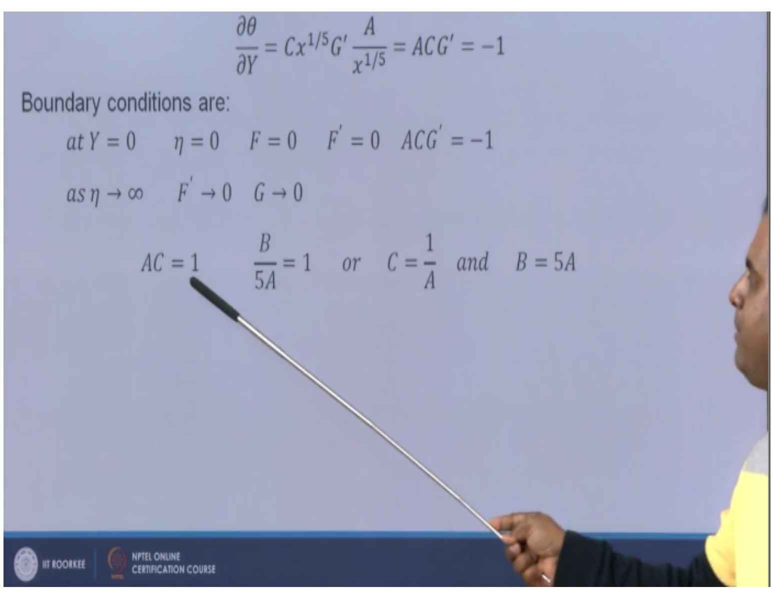

So let us do that okay before that we have to go to boundry condition also, and V=o F=0 from this we get ACG ′ so one QY g so this is the boundry condition and far we have the Qtends to infinity the f.,x=0 .

So if we see now that what can be the my A B C value the 1/1 firsst time what will get andA and Cwill mahe one so that will give G′=to _1c ,let me make the Ac=to 1 okay,in the same fasion the equation you make V/5 A=TO 1 .so we have the equation we can have c=to 1/Aand C=1/a .so if we ise this then we can see the other coefficient for exaple here ,some example we have the equation the Qa2 Qb^2 soo it will be reducing to the a2 b2/5 with c ,the c was the coefficient of the q over here in the m omentum equation okay..

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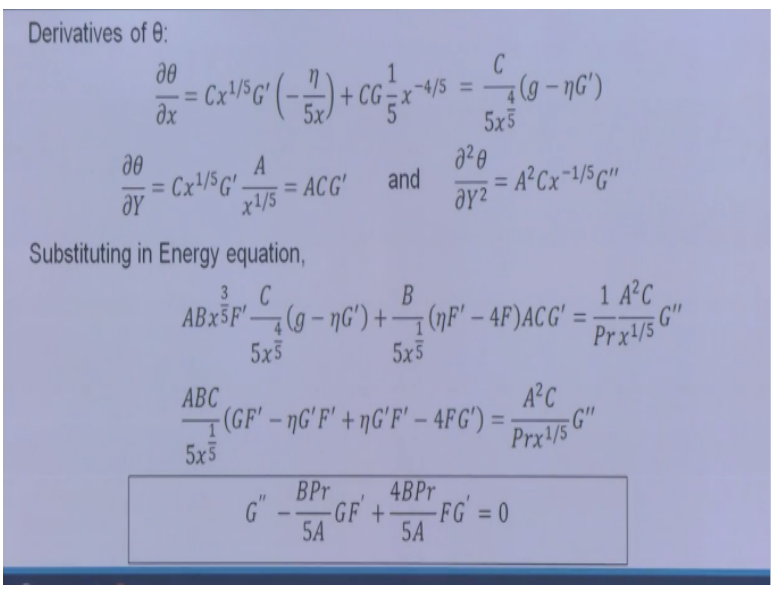

This we have equated with this one so if we equated to this one then we will finding out the relationship between the A B Cand once aghain .so here you see ABC here found oiut her ,already we showedC=to 1/A and c=to 5Aaand look in the c in the terms of the here and we get one equation here okay7.c

So here we casn reduce Ait is actually1/5to me power 1/5 okay,so if we find out A,the subsequenceB Cfinding out is ver easy and we also constructive variance and in the Q=to and Vis whish is now %/4 and this one okay. So 5 can be depend as 5 to the Xto the $and 5 in to F.so and the subsequence Cusing this one can be written as %/1/5 so now Qwill become 5 and $X^{1/5}$
In to g okay.so finally the equation under the reduction and taken over Cthis is the simplified formefd if the simple for the double _in to the f _g +o and the g double _for the f p __pr f dash e goes to thr 0 and in the momentum equation.

It can be comed energ yequation for the boundry condition we are shown up are written over here fdash and f over here and f dash o is o and fis equal is 0 and g dash o _1 stronger than the condition of the temperature and the far away the f dash is o is o ang g +to o means the t infinity is =to o in finity can be zero okay this is tge correspondinnow these equation axtally needs ni the popular and the equation from the compression speed dyanmics in the4 onwar5ds we use so we find out the corresponding value o f the f an g is and the corresponding consult numbers so let me show you what is the assent number after solving these thing can we foun d out large pantal number and small pantal number is the large pantal number is we can find out the asset number I becoming well

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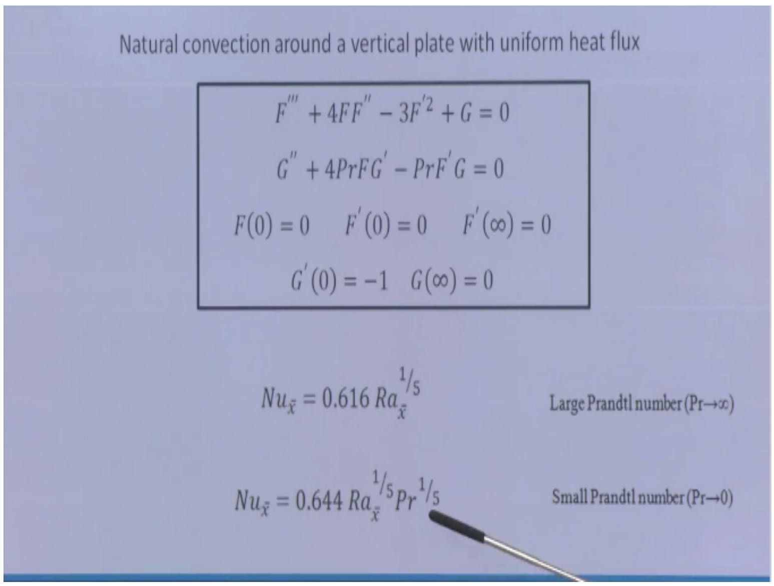

And 1/5 and for the small content it will become0.664 and then Q and /5 and 1/5 here the dependency of the vandal number comes in the fashion okay.

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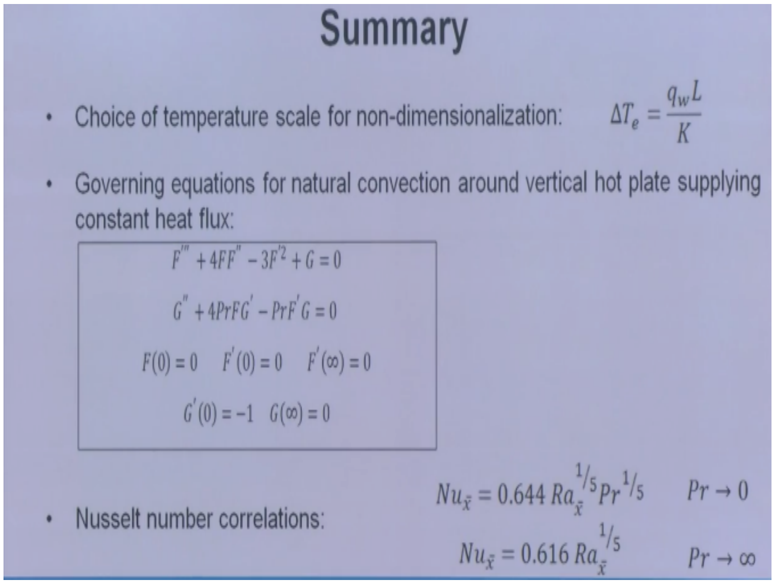

If the first lecture we have shown that is what is the choice of temperature scale and the non dimensional and we have shown that the ΔTE can be found out as keep in the L/K because here constant hit for the approximately for the temperature of the wall was the not known at E is Q W N/k powering equation what we have in the vertical around in the supplied hit for the flux shown in these two form okay simplified forms this is now momentum equation and this is for energy equation and the subsequence for A G is written over here finally we have mentioned number for the number and high mandrel number like this okay.

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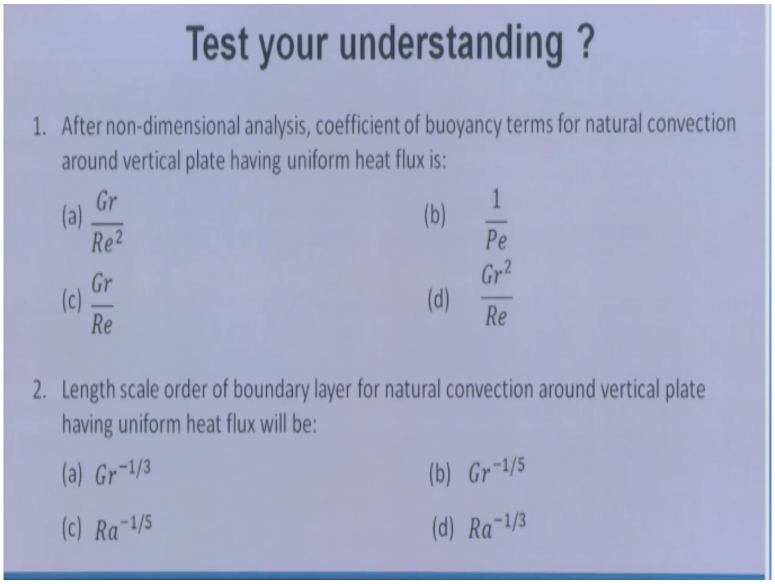

Now I taste how far you understood in this lecture and understanding having two question about here ,so first one after non immersion analysis in the co efficient of Q terms for the natural convection around vertical plate uniform deluxe is here we are having four options and in the term of buoyancy terms and we having four options $G r /Re^2$ 1/P e G r/Re $G r^2/R$ and the first one is grass of number and it denotes number square okay the correct answer is we have disused in lecture .

Correct answer will be gross of number $G r/Re^2$ .and the second question is like this the name scale order of the balancing the boundary lay for the natural convection around the vertical plate have been uniform flux will be here I am asking line scale for the boundary layer will be having .so the option over here gross of number will be one third grass of number is to fifth to the _1 fifth and two the one third okay in this lecture I have show you the correct one will be the $Gr^{-1/5}$ okay this will be right also .so I think that is the correct answer so these two cases ,so with I have thanking you and ending this lecture in the next lecture we a discussing about some tutorials on convection heat transfer of discussing it so keep on posting you in the discussion thank you.

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