INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

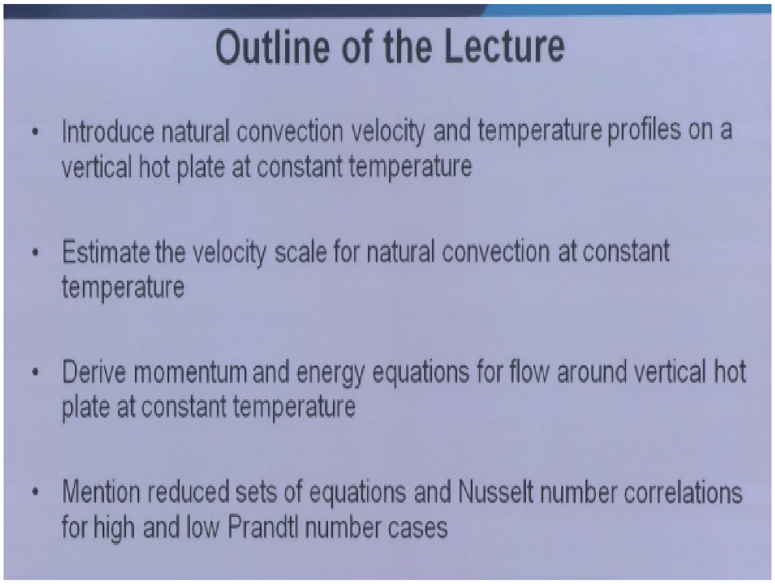
Convective Heat Transfer
Lec -08
Natural Convection: Uniform Wall Temperature

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee

Hello, let us welcome you all in our eight lecture of convective heat transfer course, in this lecture we will be discussing about natural conviction, in earlier versions of this course what we have seen that there will be a flow over the flat plate but in this case we will be seeing there is no flow over a hot plate rather the flow is being created by virtue of the temperature gradient between the plate and the first team.

So this type of conviction is actually called natural conviction and in this natural conviction we will be actually concentrating in this lecture on uniform wall temperature, so today's lecture is based on natural conviction, uniform wall temperature, okay. So let me tell you that what things will be covering in this lecture at first I will be introducing you what is natural conviction, how its velocity and temperature profile looks like, okay.

(Refer Slide Time: 01:18)

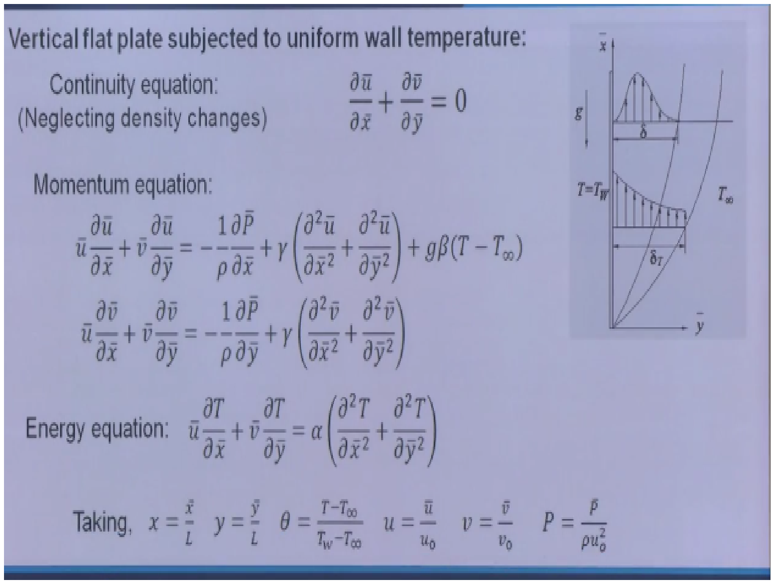


Whenever you are having a hot plate in vertical orientation, remember for natural conviction vertical orientation is utmost important in the previous lectures you have found out the plate was kept horizontal and there was a flow which is actually force conviction, in this case at the plate will be vertical and natural conviction will be occurring so I will be introducing you what is the velocity and temperature profile.

That too will be considering the plate is being kept at constant temperature, okay. As I have told you that there is no concept of forced velocity so what will be the velocity scale determination of that velocity scale is very important, so I will be introducing you how velocity scale for natural conviction can be considered especially for constant temperature case, constant temperature of the wall case, okay. We will be deriving the momentum and energy equation for flew around a vertical hot plate which is kept at constant temperature, okay.

And at the end you will be mentioning sets of equation and Nusselt number code relations for both high and prandtl number flow around the vertical hot plate.

(Refer Slide Time: 02:35)



So let me first introducing with the situation what happens in case of natural conviction as I have told you that in case of natural conviction there will be no predominate flow, flow will be rather generated by virtue of conviction so let me show you this is the plate whatever I am considering over here is kept in the vertical orientation so G is actually perpendicularly G is actually coming from top to bottom.

So in the downward direction G is acting over here gravity is acting over here and as we are interested in uniform wall temperature case, so we are keeping that the wall is having at constant temperature TW, okay and let us consider that the fluid which is there around is at in temperature T infinity and there is no flow predominant flow in the fluid so a fluid is at rest away from the plate, okay.

So freezing velocity is 0, okay. Now as we are having high temperature TW over here in composite T infinity which is a freezing temperature will be finding out there will be some temperature profile because at wall you are having very high temperature and away from the wall you are having low temperature, so the temperature profile you will be reducing in such in this fashion.

So here I have shown in the temperature profile, and what we can consider wherever the temperature has reached to 99% of the freezing temperature we can cal that point as thickness of the boundary layer, okay like this if we consider along the plate axial detection that means x – bar so we can find out locus of points okay which actually resembles 99% of the freezing temperature.

So if we join those we will be having the thermal boundary layers so this actually is the thermal boundary layer, okay. On the other hand to give this temperature profile we will be finding out there is some flow around the plate, okay though away from the plate we are having 0 velocity okay and at adjacent to the plate due to your no seen boundary condition we will be having o velocity.

But around the plate you can find out that to enhance the conviction from wall to the freeze team you will be having some velocity so we are considering that at wall we are having 0 velocity and far away from the wall once again we have regained the 0 velocity and in-between we are having a velocity profile like this, okay. We are not saying that this velocity profile will be symmetric or it can be you know taking high value very near the wall and then slowly going as towards the 0 velocity, okay.

So just like your concept of velocity bound layer in case of forced conviction here also velocity boundary can be constructed but in this case as there is 0 velocity in the freeze team so we have to reach something around 0, 99% of the 0 velocity so those points if we find out along the axial direction axial location of the plate then and then join those points will be finding out the velocity boundary layer.

So this inner curve is actually our velocity boundary layer and outer curve is our temperature or thermal boundary layer, okay. But here I have shown you only the representative case quickly I will be showing you what happens in case of you know 2 different types of extend of fluid high prandtl number and loop prandtl number of fluids, where you will be finding out there is contest between the velocity boundary layer and the thermal boundary layer.

Sometimes velocity boundary layer it is becoming dominant and sometime thermal boundary layer becomes dominant okay, so this is one representative where I have shown velocity boundary layer thickness Δ is smaller than thermal boundary layer thickness Δt but reverse cases also possible, okay. Quickly we will be coming to those situations by the way let us then quickly try to understand that what are the governing equations involved with this one.

So first we will be introducing from fluid mechanics what is the continuity equation, okay. We are concentrating that we are having incompressible fluid over her around this vertical plate so we are actually as neglecting the density changes so if you consider that one then continuity equation simplifies to δu, δx + δu δy = 0 okay. Then if you see the momentum equation let us consider two dimensional plates over here in x and y coordinates x-bar and y-bar coordinator rather.

So we will be finding out that our x momentum equation it goes like this so this is your inertia term pressure gradient term, viscous term and subsequently this term will be there due to bouncy okay now as the plate is actually aligned with the x- axis so the whole term of the bouncy will be coming in the x momentum equation there will be no representation of the bias in the y momentum equation.

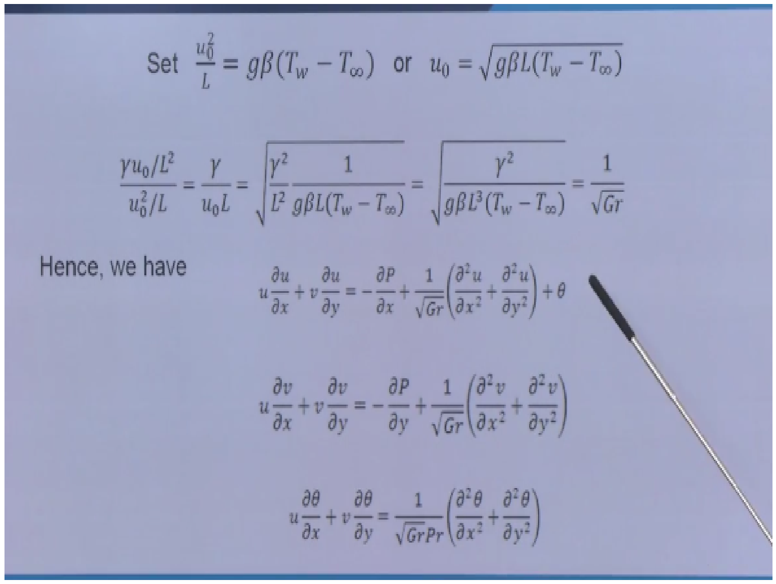
Y momentum equation is very simple in terms of v – bar so this is the inertia pressure gradient in terms of subsequently the viscous terms, okay. So after this continuity and momentum equation as we are in convictive heat transfer so energy equation will be important so let us see the energy equation, so energy equation you see in the left hand side we are having as usual conviction and right hand side we are having as usual conduction term.

They are linked up with the α which is nothing but your diffusivity, okay thermal diffusivity then as we have done earlier let us try to non dimensionalize this equations by taking different scales, so first for the locations x-bar and y-bar will be non dimensionalizing with L which is the characteristic length scale or you can length of the plate okay and we actually construct x and y as non dimensionalize parameters, non dimensionalize lengths scale, okay.

In plate direction and cross plate direction, okay. For temperature let us consider θ, now we have mention the temperature of the plate is TW and freezing temperature is T infinity so with the help of this two we are non dimensionalize and temperature T so T – T infinity/ TW – T infinity this we are considering as θ , okay. So as a result near the wall θ will become 1 and away from the wall θ will become θ will tend towards 0, okay.

For velocity we are considering some characteristic velocity scale U_0 okay this will be also U_0 so some characteristic velocity scale U_0 we are considering so U and V these are U- bar/ U0 and V = V bar/ U0 okay, pressure we are characterizing by P-bar / ρu^2 so ρu^2 is having actually a unit operation so p-bar/ ρu^2 is actually P which is non dimensionalize pressure, so if you take all this non dimensionalization schemes and try to modify this continuity momentum and energy equations.

(Refer Slide Time: 09:51)



Our system of equations will be changing, so let us try to see from there what this equations we will be taking the form but before that let us see what is this U_0 because U_0 is some fictitious quantity what we have taken okay so to take this U_0 what we are considering that what is the order of your inertia term so if you see the x momentum equation the inertia term will be becoming $U_0{}^2$/ L okay.

So this is U_0 x U_0 and here it is x so that means L, so $U_0{}^2$ /L is the dominant term over there, so what we are considering $U_0{}^2$/L which is the inertia term order is equals to the last term that means the bias term over here G ß T – T infinity, okay. So if you consider that so g ß (T – T infinity) = $U_0{}^2$ /L so here actually we have equated the inertia order along with the bias order if you do so, so you can get the idea what will be the characteristic velocity U_0.
So U_0 will be √g ßL(TW – T infinity) okay, next let us try to find out that what will be the what will be the coefficient in our viscous term so if you see the viscous term in the previous equation I will show you in the viscous term we will be having Γ and then which is connecting viscosity from there here we will be getting U_0 and here you will be getting L^2 and from this side we will be getting U_0^2 / L so if you divide the whole equation by U_0^2 /L the coefficient in front of the vicious term will become ΓU_0/ L^2 and U_0^2 / L okay.

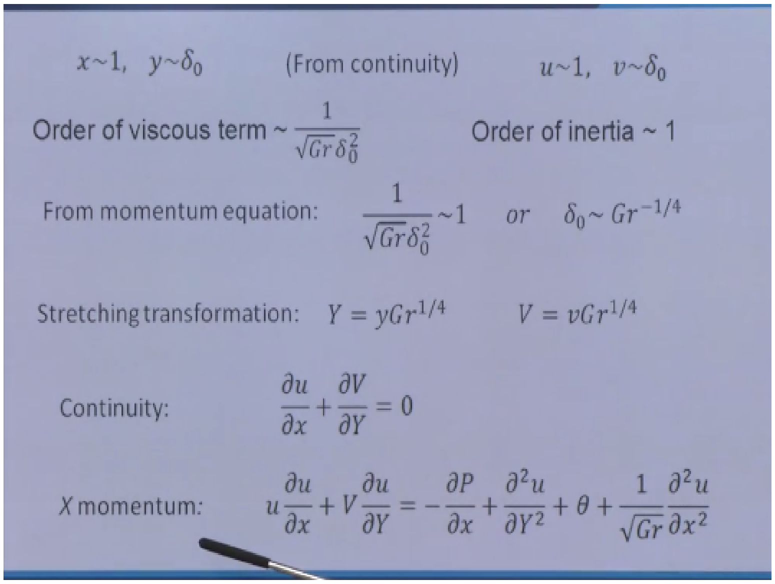
So after reduction you will be getting this is nothing but Γ/U_0 L okay, so as we have already found out what is U_0 we can straight away put this U_0 value over here to get what is the coefficient of the viscous term, so if you see the coefficient after cancellation and putting this value of U_0 it is becoming $\Gamma_0{}^2$ /g ßL^3) TW – T infinity. Now this g ß L^3 (TW – T infinity)/ $\Gamma_0{}^2$ is actually special quantity or special non dimensional number we call that one as actually gross of number, okay.

So this is kinematic viscosity so this is gross of number so we are getting this coefficient of the viscous term is becoming 1/ √grass of number okay, so grass of number gives the physical significance that what the effect of bias and what is the effect of viscosity, okay. So that is actually non dimensional number, so once if we see that what our equations are taking the form after non dimensionalization so you will be getting inertia term is having simplified U ∂u/ ∂x + V ∂u/ ∂y because we have divided with the coefficient whatever we had over there throughout so we have got 1/√Gr in front of the viscous term and as we have equated the buoyancy and inertia force so there is no coefficient in front of the buoyancy force.

Because we have divided with the coefficient of the buoyancy term so coefficient of the inertia term so you are finding out that here it is throughout divided so it is becoming one okay, why momentum equation similarly you too will be getting U δv/ δx + V δv/δy = - δp/δy + 1/√Gr δ^2v/δx^2 + δ^2v/δy^2 okay, here v term will not be there as plate is vertically oriented, okay. Similarly for energy equation you will be finding out along with 1/√Gr we will be having Prandtl number also over here.

Because in this cases in front of energy equation in place of Γ we had α so to bring the effect of the grass of number what we can do, we can write down α / Γ x Γ okay so that Γ will be giving the effect of the grass of term but an α/ Γ will become the Prandtl number, so here you can see √Gr x prandtl number is coming over here, okay.

(Refer Slide Time: 14:19)



Next step let us try to do some sort of scale analysis from all these three equations whatever we have derived, so let us start from continuity equation now we know that what we can do, for the along the direction of the plate we can assume that x which is along the direction of the plate is of the order of 1 and for the cross direction of the vertical or the horizontal direction we can call y is of the order of the δ_0, δ_0 is actually the some thickness.

Whether it can be velocity boundary layer thickness or thermal boundary layer thickness that comes later on whenever you are considering different prandtl number, now from continuity we can straight away write down that du, δu/ δx need to be in order of 1 so if x is order of 1 then obviously u will be order of 1 and in case of Y if it is order δ_0, δv/ δy needs to be in order of 1. So V becomes order of δ_0, okay.

Now let us see the viscous term if you see the viscous term what is the order of the viscous term let me show you the viscous term once again, so if you see the viscous term once again over here 1/ √Gr then we are having δ^2u/ δx^2 + δ^2 u/ δy^2 okay, so in this case you can find out that the order will be coming will be finding out that this term is becoming dominant, okay. Because Y is of order δ_0.

So this term will becoming dominant but x is of order 1so this term is becoming dominant and order will become 1/ √Gr x 1/ δ^2, okay so this is the order of viscous term 1/√Gr x 1/ $\delta_0{}^2$ okay. In a similar fashion if you try to see from the inertia term so in case of inertia you will be finding out the order as we are having V and Y over here, so this will be cancelling out okay, so δ_0 and δ_0 will be cancelling out.

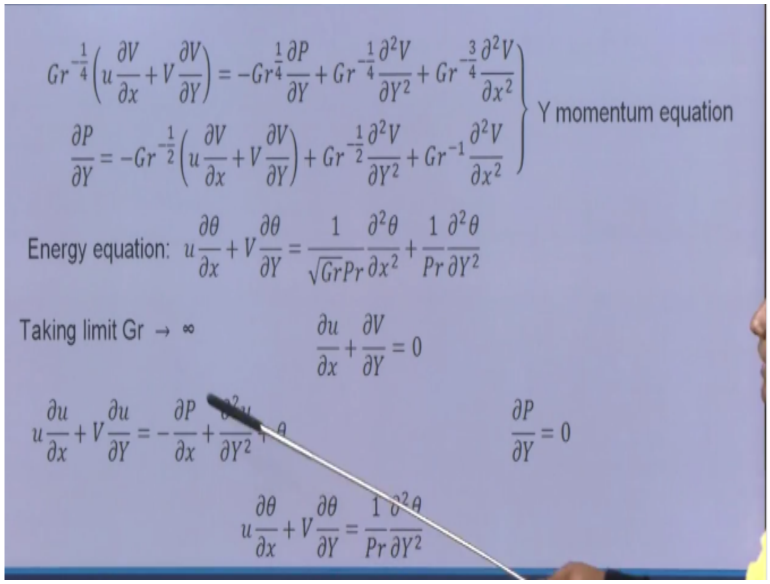
U is of order 1 so this becomes order 1 similarly all this quantity U and X these are of order 1 so the total inertia term is becoming of order 1, so inertia term is having order 1, okay. Whereas the viscous term is having this order right. So if we equate the inertia and viscous term to get the momentum equation back, so we get $1/\sqrt{Gr}\ \delta^2$ is of order 1 and from here we get δ_0 is actually $Gr^{-1/4}$ okay.

So we have got the thickness of the boundary layer is of the order of grass of number to the power -1/4. So let us take some stretching terms formation, so from small y as we have already known that what is the order in the y direction we can take y into grass of number to the power ¼ is actually capital Y and similarly to satisfy the continuity we need to take for small v also, so small v into grass of number to the power ¼ is actually capital V, okay.

So if we use this stretching transformation our continuity equation will not be changing because in the first term there is no capital Y or capital V, in the second term we are having capital V and capital Y but this grass of number to the power ¼ will be cancelling from denominator and numerator okay, so continuity equation will not change, in x momentum equation we can find out in inertia term there is no change.

As it is of order 1 already we have told, okay. In case of your viscous term you can find out that your cross wise viscosity that means $\delta^2 u/\ \delta_y^2$ is actually releasing one $Gr^{1/2}$ so that $Gr^{1/2}$ and its coefficient are earlier coefficient $1/\sqrt{Gr}$ is cancelling out and there is no coefficient in front of the $\delta^2 u/\ \delta_y^2$ term but $\delta^2 u/\ \delta x^2$ will be keeping the coefficient $1/\sqrt{Gr}$, okay buoyancy remains as usual as θ and pressure gradient remains as – $\delta p/\ \delta x$, okay. As we have not done any transformation for x so this term will be remaining same so this is my x momentum equation.

(Refer Slide Time: 18:31)



In a similar fashion Y momentum equation can be constructed so this is Y momentum equation so as we are having over here V so what we have done the coefficient of V actually has taken this one $Gr^{-1/4}$ similarly over here we are having two V's and one Y so ultimately $Gr^{-1/4}$ will be coming out, so that we have taken common over here, in the right hand side for the pressure gradient term we will be having $Gr^{1/4}$ okay because Y is over here.

For the viscous term so for the first term δ^2 v/ δx^2 there will be only V over here so $Gr^{-1/4}$ will be coming out and earlier we are having $Gr^{-1/2}$ so that will be giving rise $Gr^{-3/4}$ and for the last term δ^2v/ δy^2 so we will be having one V at the bottom sides so that along with 1/ √Gr will be giving rise to the $Gr^{-1/4}$ okay. Subsequent changes of side will be giving you δp/ δy = something in the right hand side which is having everywhere grass of number, okay. So this is the final form of the momentum equation we can write down, okay.

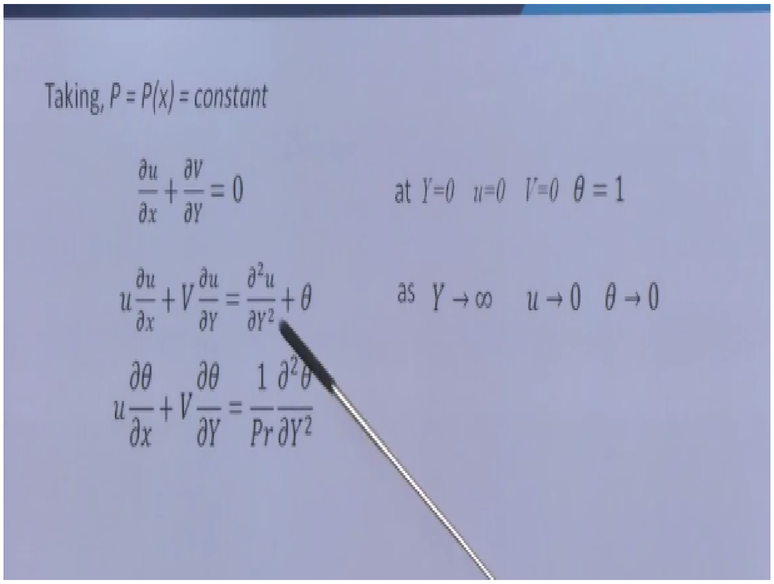
Similarly energy equation so you see left hand side will be having no change because this is your conviction side having order 1 so U and X those are of order 1, V and Y they are having stretching's but grass of number is releasing in denominator and numerator those will be cancelling out, in the right hand side conduction term you see for δ^2 θ/ δ y^2 if we convert that into capital Y, capital δy^2.

So one grass of number to the power ½ will be releasing so that grass of number to the power ½ and 1/ √Gr will be cancelling out and leaving out this 1/Pr x δ^2 θ/ δ y^2 but same thing will not happen for δ^2 θ/ δ x^2 so you will be finding out 1/ √Gr Pr okay δ^2 θ/ δ x^2 right, so these are the equations form equations of different forms after this stretching term formation. Now let us take the limit of grass of number tends to infinity.

So this grass of number tends to infinity comes out whenever you are having significant amount of temperature drop between the wall vertical wall and the freeze team compared to the you know viscous effects, okay. So here we take this Gr number tends to infinity if you take grass of number tends to infinity in all the equations wherever you are having grass of number to the power minus power.

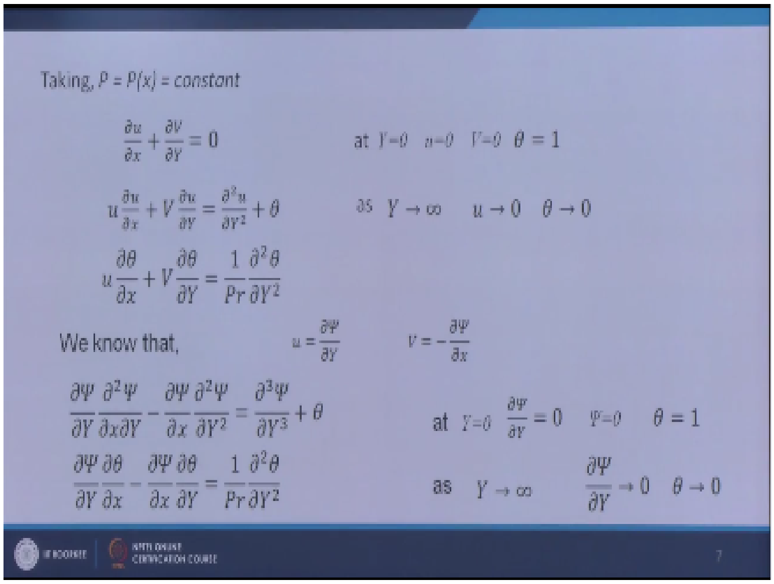
So those terms can be cancelled, so you see from x momentum equation last term can be cancelled okay and from Y momentum equation whole right hand side can be cancelled okay everywhere we are having minus power okay of the gross of number and from the energy equation this δ^2 θ/ δ^{x2} can be cancelled, so ultimately the equations comes out to it continuity in this fashion x momentum in this fashion this is your Y momentum equation, simply δp/ δy = 0 and finally this is my energy equation which is important for this course okay. So δ^2 θ/ δ^{x2} is absent over here, right.

(Refer Slide Time: 21:54)



So let us take for the simplicity let us consider that from here we can get δp/ δy = 0 so that means P is a function of x. So let us take for the simplicity, let us consider that. From here we can get ∂P/∂Y=0 so that means P is a function of X.

(Refer Slide Time: 22:04)



And let us take that P which is function of x is actually a constant okay, so if we take a constant then obviously in x moment and equation -∂P/∂x can be dropped down okay, so here you see from x moment and equation ∂P/∂x we have dropped down to make our equations simpler in look okay. So this is my x moment term equation, y moment term equation we have not shown over here because this derivation now we have actually taken from y moment term equation.

This is my energy equation as usual whatever we have derived in the last slide, let me tell you the boundary conditions also so at Y=0 which is nothing but adjacent to the wall okay, adjacent to the wall obviously no sleep and no penetration will be giving me u=0, V=0. And from the non dimensional relation of the θ we will be finding out that at the wall temperature is t_w so this is becoming t_w-t_∞/t_w-t_∞ which is equal to 1, okay.

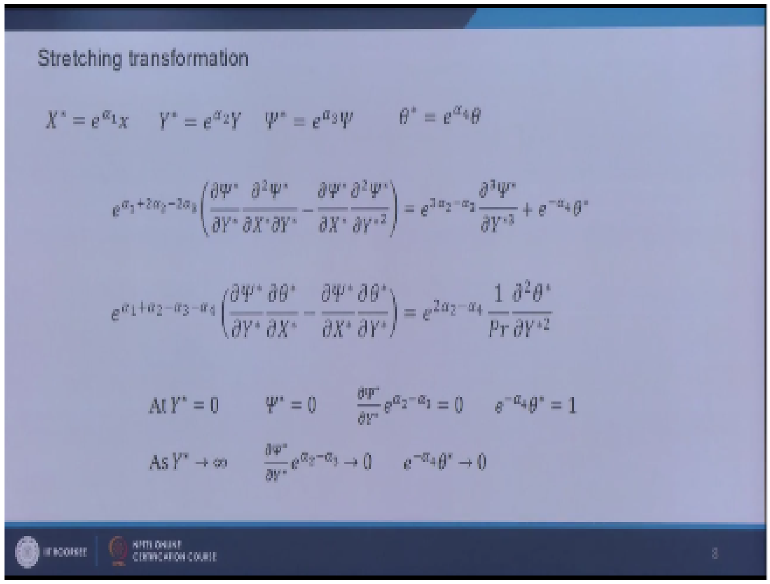
And away from the wall so Y tends to ∞, Y tends to ∞ we will be finding out there is no velocity so u=0 okay, and θ will be becoming 0 because t_∞-t_∞/ t_w-t_∞ is the θ which is nothing but 0 okay, so these are the equations and boundary condition subsequently. Let us try to modify this things using our stream function concept, all of us know that u is nothing but ∂ψ/∂Y and V=-∂ψ/∂x where ψ is the stream function, okay from fluid mechanics we know this.

Let us try to get that what will be the equation forms involving this stream function, so obviously continuity equation will not be giving us something so I am not writing continuity equation over here, basically this things have been derived from the continuity equation only so let us see the momentum equation x momentum equation so if you plug in this u over here and V over here then we will be writing down $\partial\psi/\partial Y.\partial^2\psi/\partial x\partial Y$.

So this is nothing but your $\partial u/\partial x$-$\partial\psi/\partial x$ this is nothing but your V and $\partial^2\psi/\partial Y^2$ this is nothing but your $\partial u/\partial Y$. On the right hand side we are having $\partial^2 u/\partial Y^2$ is nothing but $\partial^3\psi/\partial Y^3$+ your by in θ miss giving θ. And in energy equation first term is actually your u $\partial\psi/\partial Y$ is actually your u and $\partial\theta/\partial x$ is remaining like this for V we are writing $-\partial\psi/\partial Y$ and this is your $\partial\theta/\partial Y$ which is coming from the second term of the convection.

In the right hand side we are having $1/Pr\ \partial^2\theta/\partial Y^2$ okay as usual from the energy equation. Boundary condition will not be changing much, so at Y=0, u=0 means $\partial\psi/\partial Y=0$, V=0 means from here we can get if we integrate once we will be finding out $\psi=0$ and energy equation temperature boundary condition is not changing θ=1. And for away from the wall Y tends to ∞ we are having $\partial\psi/\partial Y$ tends to 0 and θ tends to 0, okay. So this is your u tends to 0 and θ tends to 0 right.

(Refer Slide Time: 25:10)



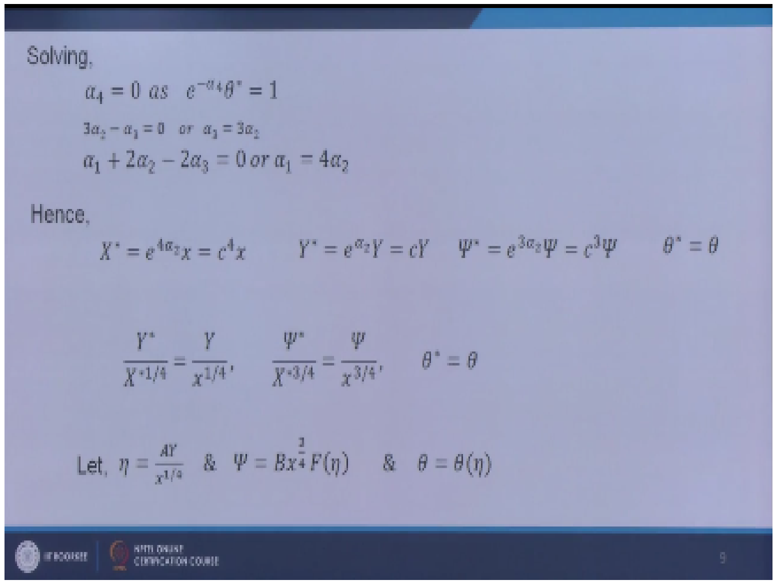
Let us do now little bit further try to have some stretching transformation from this stream function equations, let us define that $X^*=e^{\alpha 1}x$ okay, so we are taking this exponential relationship where α1 is one constant, okay. Let us do similar things for Y*, ψ* and θ* okay, so $Y^*=e^{\alpha 2}Y$, $\psi^*=e^{\alpha 3}\psi$ and $\theta^*=e^{\alpha 1}\theta$, this we are doing for finding out the similarity variable in terms of x and Y, okay.

So here we are having four unknown α1, α2, α3 and α4 we will putting back this stretching variables in your vowelling equations to find out the similarity variable. So if we put all these stretching transformation in our equation and convert all those without star terms to star terms we will be having one co-efficient of $e^{\alpha 1+ 2\alpha 2-2 \alpha 3}$ from our inertia term in x momentum equation. And in viscous term we will be having $e^{3\alpha 2- \alpha 3}$ okay, as we are having Y^3 so 3α2 and as we are having ψ it is becoming α3 okay, α3 and α2 like this, okay.

And for the θ term we are having $e^{-\alpha 4}$ okay, similar thing can be done for the convection side so our equation is becoming $e^{\alpha 1+ \alpha 2- \alpha 3- \alpha 4}$ okay, so ψ,θ,Y and x so all α1, α2, α3, α4 will be coming into picture, and in the conduction side we are having $e^{2\alpha 2- \alpha 4}$ okay, so θ is giving rise α4 and Y^2 is giving rise to 2α2, right. And from the boundary condition so Y=0 means, Y*=0 okay, and ψ*=0 means ψ=0 but here the second u=0, V-0 boundary condition gives rise ∂ψ sorry, u=0 boundary condition gives $\partial\psi/\partial Y.\ e^{\alpha 2- \alpha 3}=0$, okay.

So and your temperature boundary condition gives $e^{-\alpha 4}\theta=1$ okay, in a similar fashion away from the wall Y* tends to ∞ we can find out $\partial\psi^*/\partial Y^*.e^{\alpha 2-2 \alpha 3}$ tends to 0 okay, so this is nothing but your u tends to 0 and $e^{-\alpha 4}$ θ tends to 0 this is nothing but your θ tends to 0, okay. So now from here all these things if we equate the order from both the sides we would like to find out what is the relationship between α1, α2, α3 and α4, okay.

(Refer Slide Time: 28:02)



So from the boundary conditions, first boundary conditions clearly we can see that α4 leads to be 0 otherwise θ* cannot be equals to 1 okay, because θ=1 or the boundary conditions otherwise we will be making it complicated. So α4 definitely needs to be equals to 0 okay, as we need to have this one equals to 1, okay. And then from here we can see that this 3α2- α3 definitely needs to be – α4 okay, as we have already prove that α4=0 so 3α2- α3 also needs to be 0, so 3α2- α3 needs to be 0, okay.

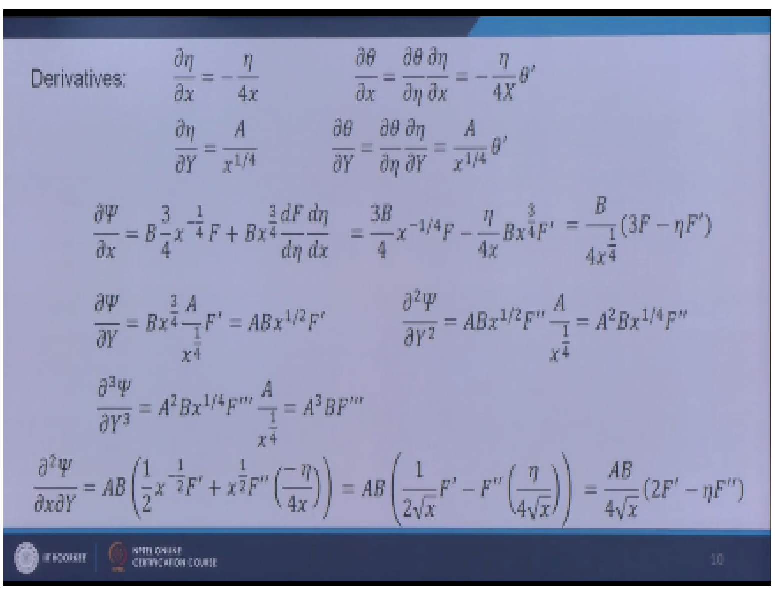
So from here we are getting α3=3α2 okay, in a similar fashion if we equate the inertia and viscous terms co-efficient then we can find out relation between α1 and α2 in this fashion, so α1+2α2-3α3 needs to be equals to 0, because inertia and buoyancy terms these two needs to be of say molder okay, so here you can find out we are getting α1 in terms of α2 as α1=4α2 okay. So here we have obtained X8=$e^{4\alpha 2}$.x, Y*=$e^{\alpha 2}$.Y, ψ*=$e^{3\alpha 2}$.ψ and θ*=θ as α4=0 okay.

Now let us try to consider that $e^{\alpha 2}$=c, so ultimately we will be getting x*=c^4, Y*=cY, ψ*=c^3 and θ*=θ. Now from here let us try to construct the similarity variable, so we can easily write down from this two equation Y*/$X^{-1/4}$ so Y*/$X^{*-1/4}$ will become y/Y/$X^{-1/4}$ okay, so here you see c and $c^{4/1/4}$ is actually cancelling out it side, so this becomes the similarity variable, okay.

On the other hand if you see the second, third equation and the first equation here from also we can write down $\psi^*/X^{*3/4}=\psi/x^{-3/4}$ okay. So here also we can get that what will be the functional dependence we do in the ψ and X and as usual $\theta^*=\theta$ will be remaining over there. Now as we have got the idea that what can be my similarity variable, so let us write down this similarity variable as $Y/x^{1/4}$ okay, so I am writing over here η which is a similarity variable $Y/x^{1/4}$ and as I do not know what is the constant term will be coming in front of that let me keep that one as A.

So I am defining $\eta=AY/x^{1/4}$, in the same fashion let us define ψ as a function of η so what we are writing ψ is actually $x^{3/4}$ from here we can get that one, $x^{3/4}F$ okay and as we do not know what will be the constant we are keeping the constant as β okay. And θ always from this one we can write down θ can be expressed as a function of η, so θ is actually $\theta(\eta)$, right. So now our task simplifies to finding out the values of A and B okay, to explicitly find out the similarity variable and stream function dependence over η.

(Refer Slide Time: 31:54)



So let us do that so in order to do that first we need to find out the derivatives of different terms and put in the equations, so first let us find out what is the ηs derivatives so similarity variables derivative, so $\partial\eta/\partial x$ is $-\eta/4x$ and $\partial Y/\partial\eta$ is $A/x^{1/4}$ okay, and let us side by side do $\partial\theta/\partial x$ also, so $\partial\theta/\partial x$ will be important for your energy equation, so this becomes $\partial\theta/\partial\eta$ $\partial\eta/\partial x$ okay, so $\partial\eta/\partial x$ I can pick up from here and $\partial\theta/\partial x$ will become θ', okay.

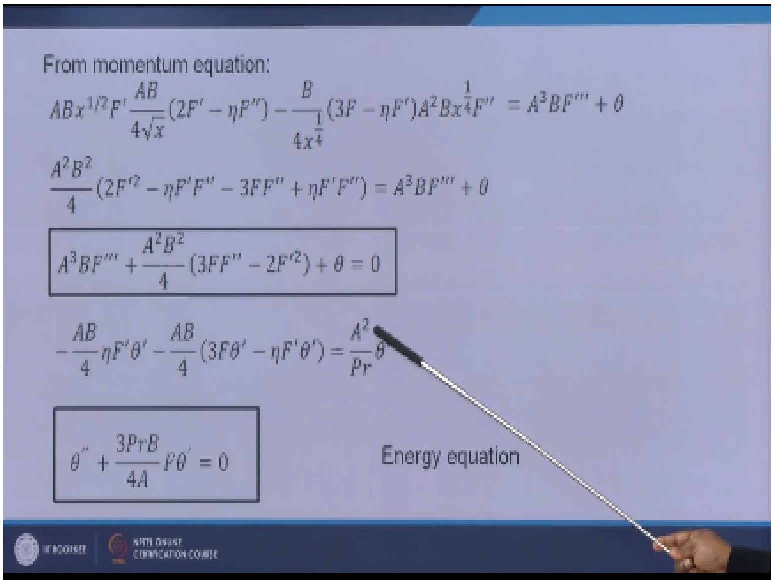
So $-\eta/4X\theta'$ in a similar fashion $\partial\theta/\partial Y$ we can write down $\partial\theta/\partial\eta$ into $\partial\eta/\partial Y$ okay, so these becomes θ' and this $\partial\eta/\partial y$ is nothing but $A/x^{1/4}$ so this is the $\partial\theta/\partial y$ coming out to be. So let us now put everything in, at first everything let us put in no, before that we need the value of $\partial\psi/\partial x$ also so this is $\partial\psi/\partial x$ okay, so for finding out $\partial\psi/\partial x$ what we are doing the previous equation whatever we have over here we are making the derivative with respect to x, so this is also function of x and F is also a function of x so we are having derivative of two multipliers.

So we are using that multiplier rule, so here you see first we have kept f constant and we have made the derivative of $x^{-3/4}$ okay, $x^{3/4}$ and here we have kept $x^{3/4}$ constant and here we have made the derivative of F, as F is the function of η so $d\eta/dx$ also comes into picture using the value of $d\eta/dx$ we can simply this and it will be coming as $B/4x^{1/4}(3F-\eta F')$ okay. In a similar fashion $\partial\psi/\partial y$ can be calculated this is simple, so $Bx^{3/4}$ as x is not a function of Y so this will not be making derivative this time.

So only F' is having and $A/x^{1/4}$ is coming due to $B\eta/dy$, okay, so simplified form is $ABx^{1/2}F'$. second derivative of ψ with respect to Y you will giving $ABx^{1/2}F''.A/x^{1/4}$ okay, so ultimately it is becoming $A^2Bx^{1/4}F''$. Third derivative of ψ which is also coming in our viscous term so this is, this will be becoming A^3BF''' okay, and cross derivative of ψ $\partial^2\psi/\partial x\partial Y$ will be, this you can achieve by making derivative with respect to x of this term or derivative of this term with respect to Y, okay.

So both will be giving you same result and final simplified answer you will be getting as $AB/4\sqrt{x}(2F'-\eta F'')$ so all this derivatives we have obtained for ψ okay, and θ also we have obtained only term left is that $(\partial^2)^3$ so all this terms you can get in this fashion.

(Refer Slide Time: 35:22)



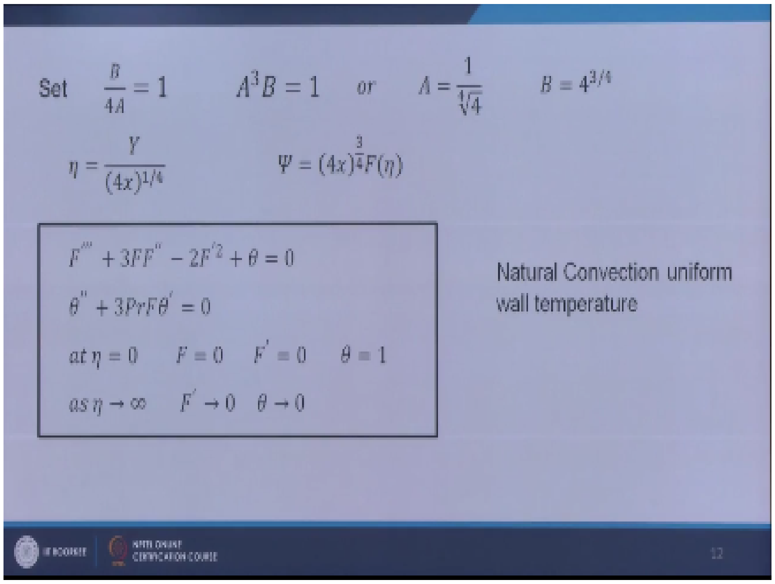
So let us put everything in our momentum equation so this we have put in the momentum equation, so you can find out this is your u okay so this is your $u\partial u/\partial x$ okay, and this one is your V so this one is your V okay, and this part is u actually your $\partial u/\partial Y$ okay, and in this side we are having over here $\partial^2 u/\partial Y^2$, $\partial^2 u/\partial Y^2$ and θ term will remains like this okay, in the momentum equation.

Further simplification of this one u will be giving you in this fashion the equation will come out in this fashion okay, where this two term can be cancelled okay, so if you do the simplification this is our equation involving A and B. Remember here also still a and B prevails. From your energy equation if you do the same things so this is your u, this is $\partial\theta/\partial x$, this is my V and this is my $\partial\theta/\partial Y$ and in this side we are having $\partial^2\theta/\partial Y^2$ into 1/Pr number.

Simplification of this one if you do for the simplification this will come further one steps simplification if you do then it will becoming like this and at the end you can find out this term and this term can be cancelled, so $\theta''+3PrB/4AF\theta'=0$ becomes an energy equation. So this is the momentum equation this is the energy equation, but still here A and B terms remains so let us try to find out A and B.

So first let us see that what is happening over here, here in both this cases AB/4 is the co-efficient and here we are having A^2/Pr.

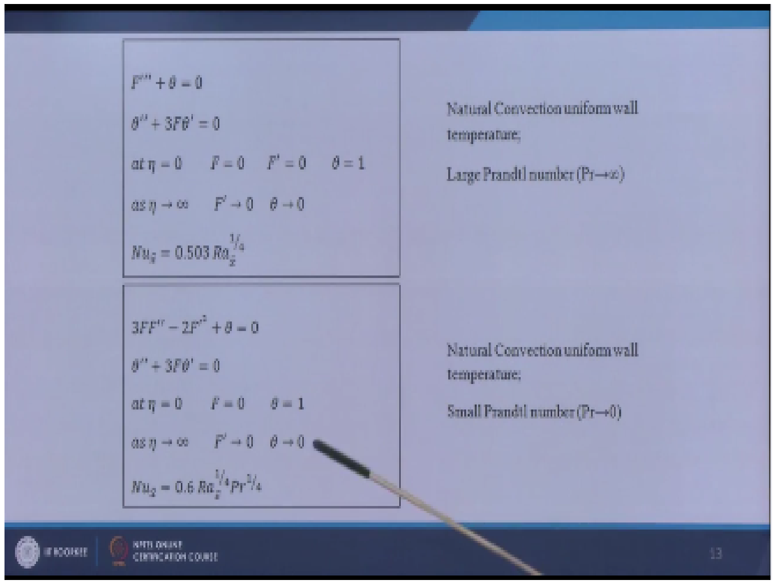
(Refer Slide Time: 37:03)



So first let us try to see from B what we can say it is that $B/4A=1$, so $B/4A$ you see here in the energy equation we are having $B/4A$ if we can make this one this equation look like a simplified one, so first let us try to make $B/4A=1$ and also try to make that A^3B is actually equals to 1, A^3B is here so $A^3B=1$, so if we make like this then from this two equations we can get the value of A and B by replacing A from here to here we can get the value of A and B and that comes out to be 1/ condrupile $\sqrt{4}$ and B it will becomes out to be $4^{3/4}$.

Once we know the value of A and B getting the similarity variable is not difficult so similarity variable becomes $Y/(4x)^{1/4}$ and ψ becomes $(4x)^{3/4}$ F okay. so once again if you give back all this A and B values in your equations whatever we have shown over here this two equations so my final form of equation comes out to be $F'''+3FF''-2F'^2+\theta=0$ this is the momentum equation and energy equation comes out to be $\theta''+3PrF\theta'=0$ okay, so this and this we have made actually one okay and boundary conditions that $\eta=0$, $F=0$, $F'=0$, $\theta=0$ okay, and at η tends to 0 F' is 0 and θ tends to 0 okay, so these are the equations and boundary conditions for natural convection from uniform wall temperature, okay.

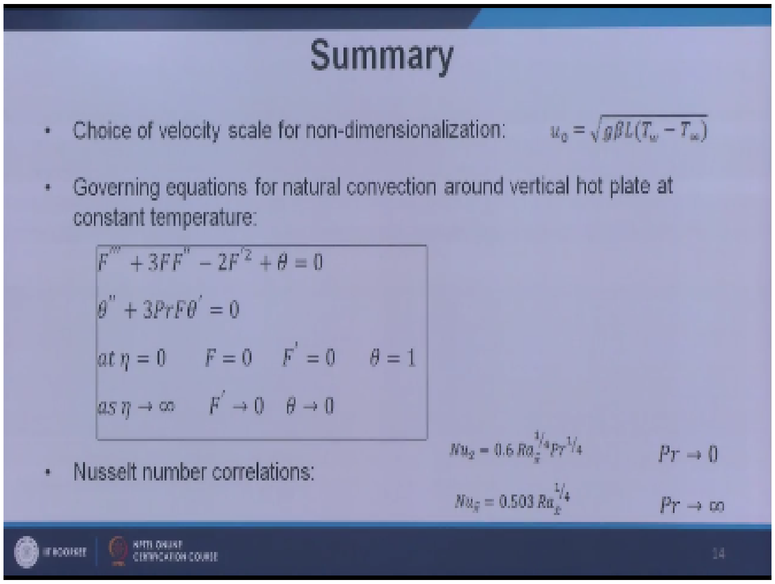
(Refer Slide Time: 38:47)



Next let us show you that what happens in case of large Prandtl number case and small Prandtl number case okay, in case of large prandtl number case the previous equations will be changing in this form, $F'''+\theta=0+\theta'''+3F\theta'=0$ okay, and for small prandtl number these will be becoming $3FF''-2F'^2+\theta=0$ and $\theta''+3F\theta'=0$ corresponding boundary conditions I have written over here which are more or less same okay, at η tends to 0, $\eta=0$ F and F' both are 0, $\theta=1$, η tends to ∞ F' and $\theta=0$ and for prandtl number tends to 0.

At $\eta=0$ F is 0 and $\theta=1$ and as η tends to ∞, F' and θ both tends to 0 corresponding Nusselt number also I have also written if we solve this two equations numerically then we can find out Nusselt number becomes 0.503 rally number to power ¼ okay, so this is rally number is very important this is nothing but grass of into prandtl number, okay. And here in this case prandtl number tends to 0 we give Nusselt number = 0.6 $Ra^{1/4}$ $Pr1^{/4}$, okay.

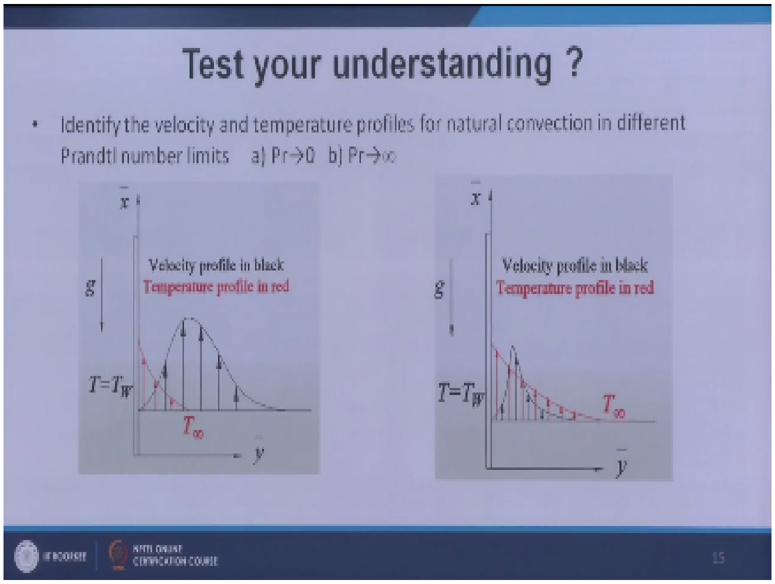
(Refer Slide Time: 40:12)



I will summary let us see what we have learnt we have seen what is the choice of velocity scale for non- dimensionalization, we have seen u_0 is nothing but √gβLδt okay, so δ is (Tw-T∞) as wall temperature was known. Governing equation for natural convection around vertical hot plate at constant temperature so this was the generalized governing equation, this is from the momentum equation, this is from the energy equation and corresponding boundary conditions this we have derived.

Different Nusselt number correlations also we have mentioned for low prandtl number and high prandtl number case, in low prandtl number case 0.6 was the co-efficient and for large prandtl number case it was 0.503. And in case of large prandtl number rally number came into picture, okay here also rally number came with Pr as having power of ¼, okay. Now let us test your understanding what you have understood in this lecture.

(Refer Slide Time: 41:10)



Help me in identifying the velocity and temperature profiles for natural convection in different prandtl number limits, okay. I have shown here two cases so red lines are for temperature profile and black line is for the velocity profiles, okay. So here temperature profile very promptly goes to 0 okay, but velocity profile prevails and it is having a very high velocity profile so δ is high compared to δt.

And in this case we are having δt higher compared to δ okay, so you need to tell me which one is for large prandtl number, which one is for small prandtl number so I think all of you can guess the correct answer is first this one is for large prandtl number okay, so that is why velocity boundary layer is having higher thickness compared to thermal boundary layer just opposite happens over here in case of low prandtl number, okay.

So with this I will end this lecture, thank you in my next lecture we will be having natural convection from a plate which is having uniform hit flex in between if you are having any query please keep on posting in the discussion forum, thank you.

For Further Details Contact

Coordinator, Educational Technology Cell

Indian Institute of Technology Roorkee

Roorkee – 247667

E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in

Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team

Sarath K. V

Jithin. K

Pankaj Saini

Arun. S

Mohan Raj. S

Camera, Graphics, Online Editing & Post production

Binoy. V. P

NPTEL Coordinator

Prof. B. K. Gandhi

An Educational Technology cell

IIT Roorkee Production