INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

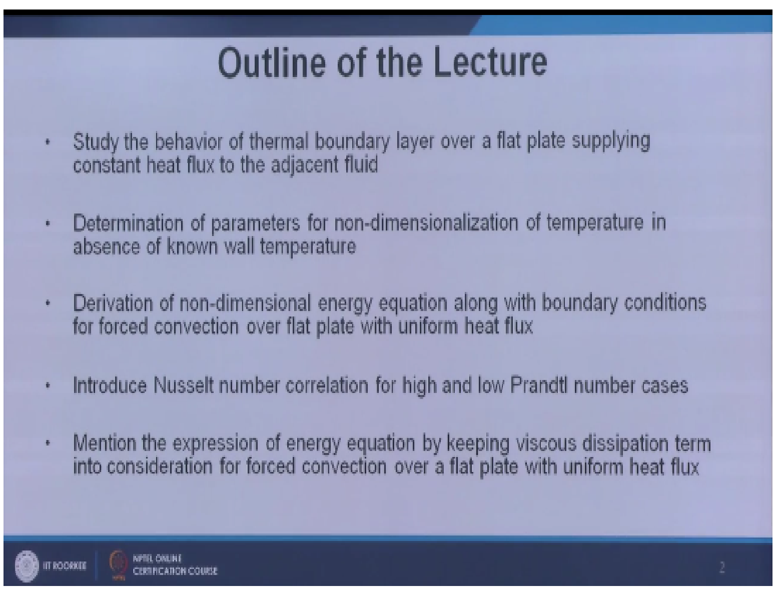
Convective Heat Transfer

Lec – 07
Forced Convection over a Flat Plate: Uniform Heat Flux

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee

Hello welcome in the 7th lecture of the course convective heat transfer. In this lecture we will be discussing about forced conviction over a flat plate. But this time we will be considering uniform heat flux is applied at the plate okay. In the previous lectures we have discussed the wall was at constant temperature, here the wall will be supplying uniform heat flux okay. So let me show you what things we will be discussing over here.

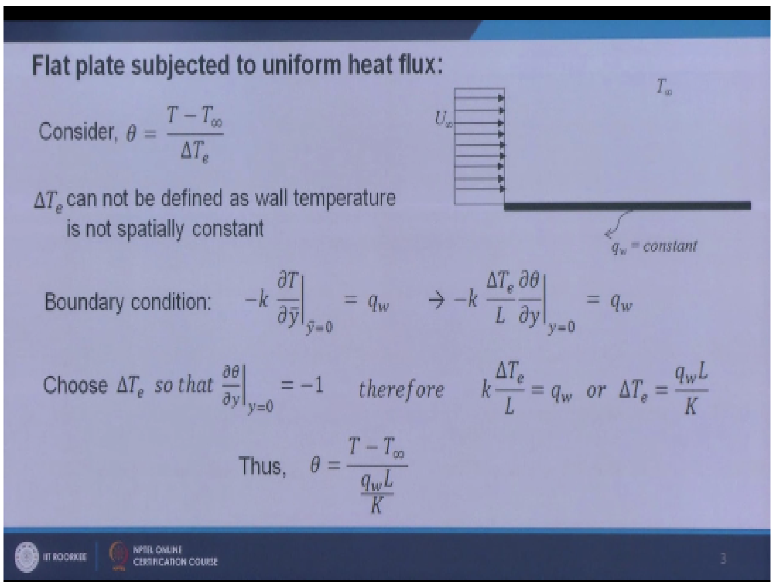
(Refer Slide Time: 00:52)



First we will be studying the behavior of thermal boundary layer over a flat plate which supplies constant heat flux to the adjacent fluid. Then we will be discussing or we will be determining the parameters for a non dimensionalization of temperature, because in earlier case temperature was straight away temperature non dimensional came straight away because the wall temperature was known.

But in this case the wall temperature is not known, because we are supplying the constant heat flux, so what will be the parameter for non dimensionalization that we need to see over here okay. Then we will deriving the non dimensional energy equation along with the boundary conditions for uniform heat flux case okay. We will be showing both the limits high and low prandtl number in this case okay. And we will be mentioning the energy equation if we consider the viscous dissipation with constant heat flux.

(Refer Slide Time: 01:52)



Now let me show you the situation what is happening over here so this is the flat plate let us say okay. Velocity is immune fluid over here and the temperature is T infinity, but in place of wall temperature TW now we are having the heat flux known which is Tw and it is constant in magnitude okay. So let us consider that we have to non dimensionalize the temperature, so earlier we have done T-T∞/ some temperature drop okay, earlier it was Tw-T∞, but in the absence of Tw my constant is what would be this ΔTe okay.

So let us try to find out that one, for that we will be starting from the boundary conditions, because now the boundary conditions we will be varying earlier it was T at wall it was Tw, now it will be -k ∂t/ ∂T at Y=0 that means the wall here Y is in this direction positive direction

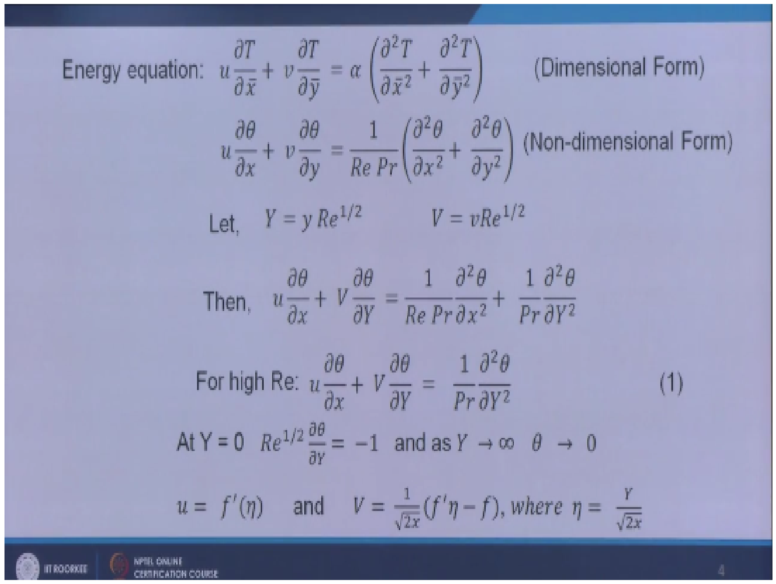
$-k\partial T/\partial Y$ is actually equals to a constant we are taking that 1 as Qw okay, so this is the boundary condition at 1 we are having right.

So if you do the scale analysis of this one so for T we have seen it is a border of ΔT_e which is for still unknown, so ΔT_e we have taken, for Y we have taken L okay so $\Delta T_e/L$ is actually Qw okay. So this is the order of the equation okay, if you non dimensionalized θ and Y in this fashion. So Ybar has actually replaced over here by Y, because Y is actually we have considered Ybar L/L in the previous lecture okay.

So from here you see this became very complicated boundary condition, we can reduce this boundary condition in a simplified form just if we can shade of this part and we can simply write down $\partial\theta/\partial Y$ is equals to some constant that will be convenient for us. So we can choose this ΔT in such a fashion that it captures all this other constants and make this boundary conditions very simplified.

So in order to do so what we are writing $\partial\theta/\partial Y$ at $Y=0$ is equals to -1 we want let us say okay. So we want to shade up all these coefficients. So in that case ΔT_e needs to become QwL/K okay. So here we have got some idea of the temperature scaling okay in terms of the wall heat flux Qw okay. So this temperature non dimensionalization scale we will be using for the rest part of this lecture okay. So have I had showed so θ will be $T-T_\infty$ and ΔT_e we have replaced by QwL/K over here okay.

(Refer Slide Time: 04:48)



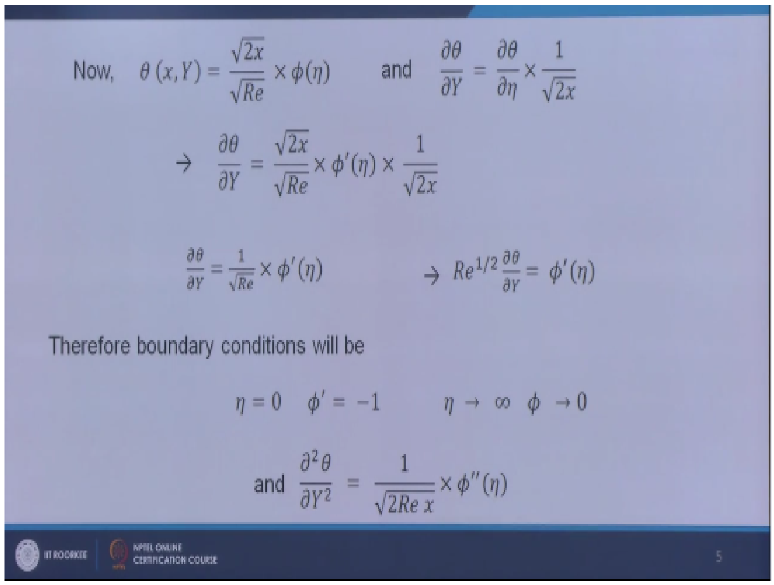
So let us try to see that how this is affecting the equations, energy equations. So we will be starting from the generalized equations so this is the dimensional form of the equation having conduction in the right hand side, and conviction in the left hand side and again by air we have not considered the viscous dissipation part. So now if you try to do the non dimensionalization in that case it is becoming 1/Re Pr.

So what we have done essentially over here is that this X we have converted Xbar we have converted to X, Ybar we have converted to Y as a result this Re Pr came over here okay. So α is actually giving rise this peculiar number Re Pr okay. Now let us try to go for this steps variables, so Y is becoming y and y is becoming y into $Re^{1/2}$ and V is becoming $vRe^{1/2}$ already we have seen in the previous lecture.

So if we do so then we can actually neglect the stream wise conduction term okay. So this 1/Re pr$\partial^2\theta$ ∂x^2 can be cancelled and finally this equation already we have shown in the previous lecture okay where stream wise conduction has been neglected okay. Important thing is boundary condition earlier we have seen that ∂θ/∂y was -1, now as we have used Y and y relationship like this so here $Re^{1/2}$ is coming in front of that. So $Re^{1/2}$ ∂θ/∂Y = - 1, this is Y remember okay. And far away from the plate the condition will not be changing, so θ∞ is actually 0 and ending towards 0.

Now as we have done in the previous lecture also that η will be taking as $Y/\sqrt{2x}$ derivative variable and using this what we can do, we can write down $U=f'(\eta)$ and V is actually $1/\sqrt{2x}$ $f'\eta-f$ okay, so this we can take from usually mechanics knowledge okay.

(Refer Slide Time: 07:03)



\

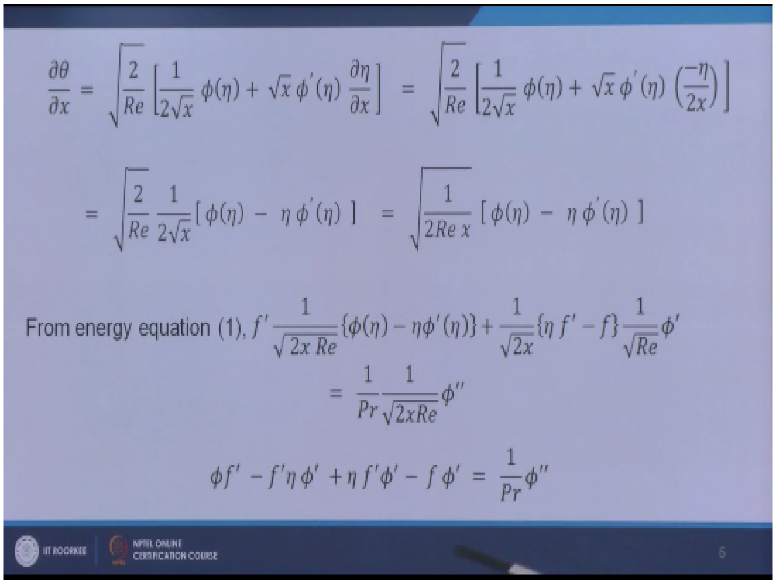
Then here let us define θ as a function of η in this fashion θ is nothing but your temperature to find non dimensional temperature profile in X and Y coordinate and the we are writing $\sqrt{2x}/\sqrt{Re}$ x $\phi(\eta)$.So all this X and y is coming in η okay, though $\sqrt{x}$ is over here in this equation okay. Now if you try to do the differentiation of this one with respect to x and y because those times are there in your energy equation as we have shown in the previous slide.

So here you see $\partial\theta/\partial y$ first it becomes $\partial\theta/\partial\eta$ $\partial\eta/\partial Y$ which is nothing but $1/\sqrt{2x}$ okay. So this $\partial\theta/\partial\eta$ actually becomes $\sqrt{2x}/\sqrt{Re}$ x $\phi'(\eta)$ because these are not function of η okay so only this is the function of η so this became ϕ' okay. And $1/\sqrt{2x}$ remains over here okay, so and these two terms can be cancelled from here in this equation it simplifies to $\phi'/\sqrt{Re}$ okay. So it can be written $\phi'=Re^{1/2}$ $\partial\theta/\partial y$ okay. So boundary condition whatever we have showed in the last slide so the boundary conditions as we have seen here it is $\partial\theta/\partial y$ at Y=0 is -1 and which we have shown over here after making y to Y $Re^{1/2}\partial\theta/\partial y=-1$ so that can be now converted to θ can be converted into ϕ using this equation.

So $Re^{-1/2}$ $\partial\theta/\partial y=\phi'$ so ϕ' will be now becoming -1 okay. So ϕ' to 0 is actually -1 is the boundary condition at the wall okay. And η from the wall remains same so ϕ tends to 0 at when η tends to

∞ okay. Also we can go for the higher order derivatives of this term which will be requiring for the energy equation, so ∂θ ∂y was here in this fashion if you do the derivative once more with respect to Y so this become 1/ √Rex (ϕ’’) okay.

(Refer Slide Time: 09:48)

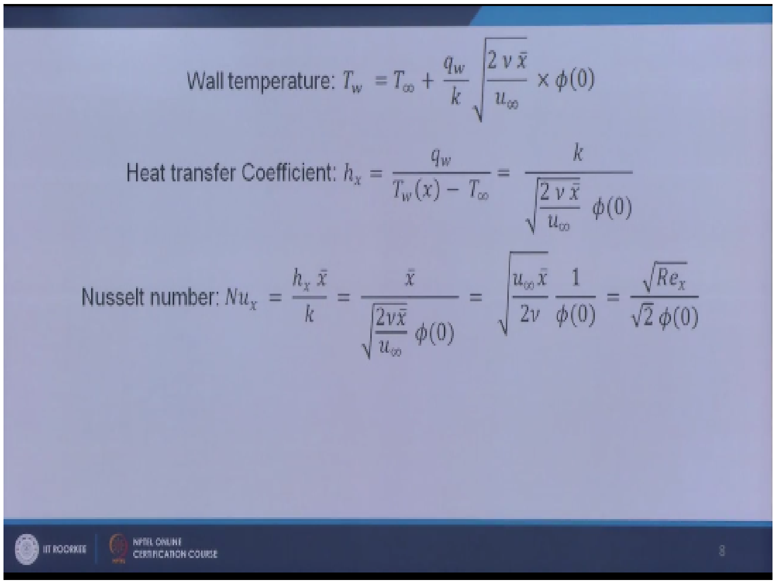


And if you go for the ∂θ/∂y this is also a term in the conviction side of the energy equation, so this derivation gives you the final form of this fashion. So here you can find out the final form is coming as √1/2Rex (ϕ- ηϕ’) okay. So these are simple states you have to do the derivation of θ with respect to X we are having two terms actually √2x and ϕ both are function of X. So if you see over here first we have kept ϕ I have done the differentiation of √2x and here √2x has been kept constant and ϕ derivation we have done with respect to first η and then η is derivation with respect to X okay.

And further integration has given rise to this one okay. So all three terms we have found out ∂θ ∂x, ∂θ ∂y Y and ∂θ $\partial^2\theta$ ∂θ Y^2 so let us put in the equation okay. So energy equation which we have already shown. So first this is your U, U is nothing but f’ okay and this term is actually your ∂θ/∂x okay here I have derived so ∂θ/∂x is over here then plus V, so this is your V okay and then this is your ∂θ/∂y okay ϕ’/√Re I mean the right hand side conduction side 1/Pr then we are having $\partial^2\theta/\partial Y^2$ so this is the second derivative of the θ with respect to Y okay.

So this equation we can further simplify and cancelling different terms will be giving us ϕf’-f’ηϕ’ + ηf’ϕ’ - fϕ’=1/Prϕ’’okay. So if you just do the multiplication all these terms will be coming okay ηf’(ϕ’) and fϕ’ okay, so these two terms can be cancelled.

(Refer Slide Time: 12:06)



So finally the equation will be coming as ϕ’/Pr so Pr can be taken multiplication in this side so Pr(fϕ’=ϕf’) = 0 okay and boundary condition as we have already derived that ϕ’0 will be equal to -1 and ϕ of ∞ will be equals to 0 okay. So this is the equation and these two are the boundary conditions we have obtained okay for constant heat flux cases. As we have taken this equation θ to ϕ relations we have taken in this fashion, so here let us try to see that how heat flux can be actually link up with ϕ okay.

So we know that ϕ already we have seen is nothing but T-T∞/QwL/K so we say that till ΔTe our non dimensional term, our dimensional term for the scaling of the temperature okay. And in the right hand side we are having the function of the ϕ as we have considered θ and ϕ equation like this okay. So if you proceed for the ϕ can be written ϕ can be written in terms of T-T∞/Qw/K √2vxbar by ∞ okay. So ϕ we have written in terms of temperature okay.

So if you try to find out ϕ0 okay so what happens at ϕ0, so ϕ0 means at wall what is the profile of the ϕ okay what is the value of the ϕ. So we can find out, so this will become actually Tw because T at wall is Tw so Tw-T∞/Qw/k √2vxbar/U∞ here why we are doing this because we want to find out actually what is the value of T_w because in this case wherever we are having constant heat crossing this case the wall temperature is unknown okay.

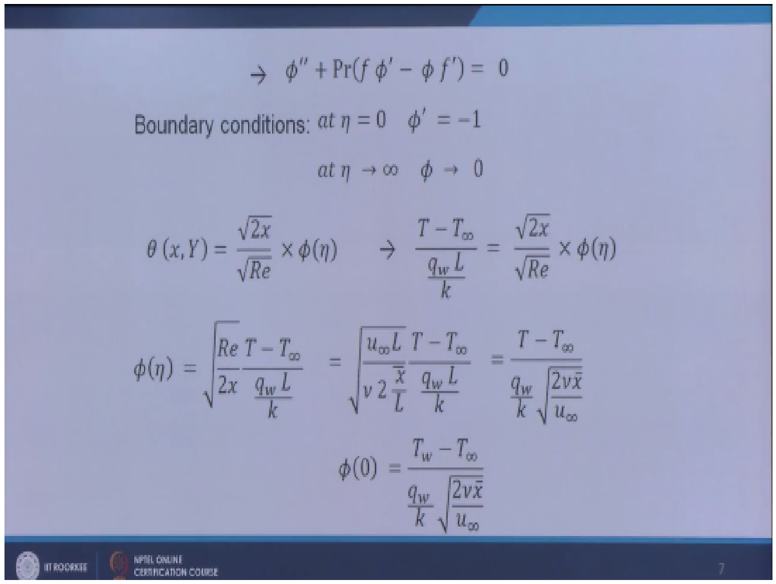
So from here we will be trying to get what is the value of this wall temperature right. So wall temperature if you do the site changing and little bit of simplification of the previous equation so wall temperature can be written in terms of $T_\infty + Q_w/K \sqrt{2\nu\bar{x}/U_\infty}(\phi_0)$ okay. So main task here as T_w is constant we have to find out what is the value of ϕ okay at the wall once we can find out ϕ we can get the value of the wall temperature okay.

Let us also see what is the heat transfer coefficient h_x so h_x is nothing but we know from adjacent layer which will be actually taking the heat by a conviction so Q_w is the wall heat transfer which is constant divided by $T_w=T_\infty$ okay. So T_w we have already defined over here, so T_w can be found out in this fashion so this is $K/\sqrt{2\nu\bar{x}/U_\infty}(\phi_0)$ okay. So T_w-T_∞ can be found out from here so this term will be coming over here okay.

So heat transfer coefficient we can obtain in terms of ϕ only okay. If we proceed for that to obtain the nusselt number of heat transfer coefficient so then we can find out once again nusselt number can be linked up with ϕ over here because nusselt number is nothing but $h\bar{x}/K$ okay. So h already we have found out an xbar and K if you link up with this one cancellation of this xbar and xbar $\sqrt{\bar{x}}$ and this x bar will be giving you $\sqrt{\bar{x}}$ at the denominator and then we can find out that this term is $U_\infty\bar{x}/\nu$ this will be giving you actually Re_x.

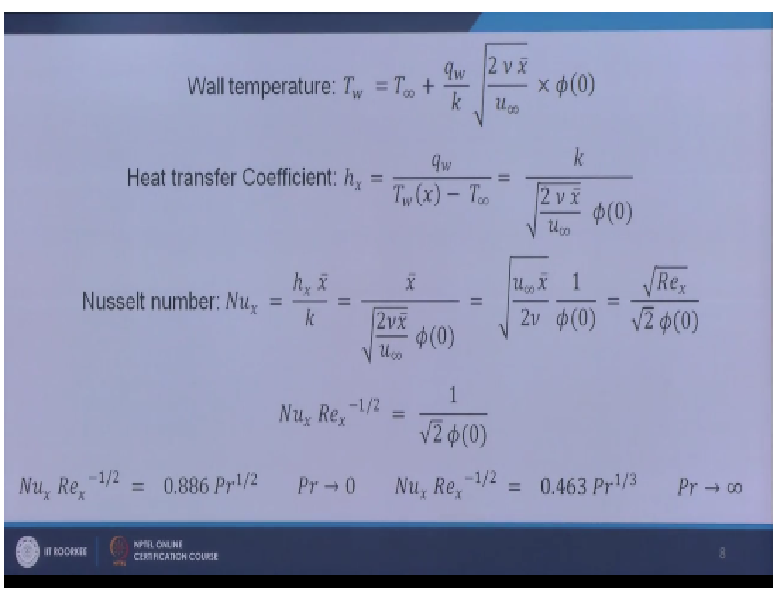
So $\sqrt{Re_x}/\sqrt{2}\phi_o$ any how this nusselt number is actually linked up with ϕ_0 once again.So we have found out if we can get this ϕ_0 okay so all these wall temperature heat transfer coefficient and nusselt number can be calculated.

(Refer Slide Time: 16:23)



By the way to obtain this ϕ what we need to do we need to actually do the solution of this equation along with the boundary conditions okay. Solution of this equation is actually coupled with your basic equation is equation which we have already showed in the previous lectures which is nothing but f’’’’+f’’=0 along with the boundary conditions okay. So these equations are couple equations and needs to be solved numerically taking iterative solver.

(Refer Slide Time: 16:53)



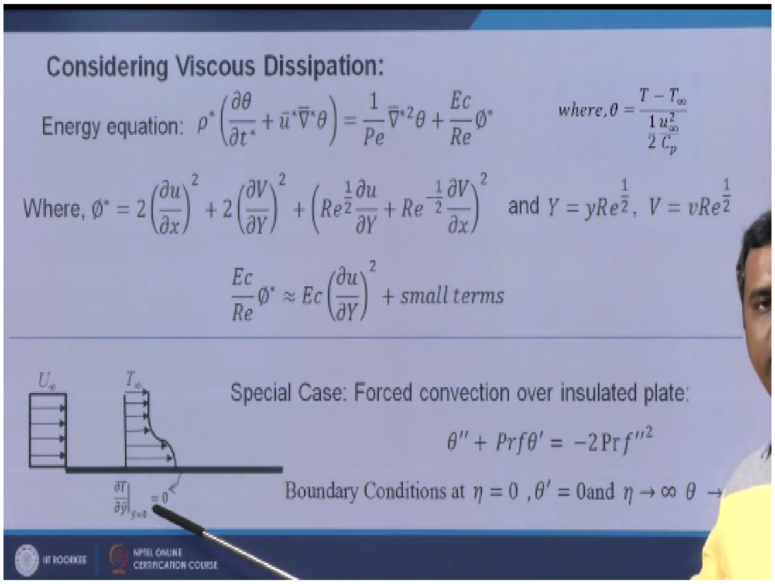
And if you do that and find out what is the value of ϕ at wall that is ϕ0 then you can obtain this wall temperature heat transfer coefficient and nusselt number respectively okay. Little bit of site change from this nusselt number and Rex can give you Nux $Rex^{-1/2}$ is actually a sole function of ϕ0 okay. Now here I will be introducing two different cases one is for a loop and a number limit another one is for high prandtl limit. So if you do the solution coupled solution for this ϕ equation and f equation for a loop and a number limit you will be finding out ϕ at wall and put in the equation of this nusselt number.

So then you will be finding out that this is actually the correlation what we get for lower prandlt number, nusselt number $Rex^{-1/2}$ = 0.886 prandtl number to the power ½ okay, using your compression of this dynamics knowledge if you just do the analysis of these two coupled equation ϕ equation and f equation you can find out what is ϕ0 for low prandtl number limit and this constant 0.886 will be coming from there okay. And similarly for the high prandtl number, prandtl number tends to ∞ it will become 0.463 into prandtl number to the power one-third okay.

Interestingly you can see that prandtl number to the power half for low prandtl number and prandtl number to the power of one-third for high prandtl number already we have shown in our constant temperature cases. So similar dependence also we are finding out over here only the values are changing over here from your constant temperature cases only because the high value we need to obtain from activated solvers okay.

So this part I am not going but ultimately I am showing you that these are the equations we can obtain for the nusselt number derivations for two different limits okay.

(Refer Slide Time: 19:12)



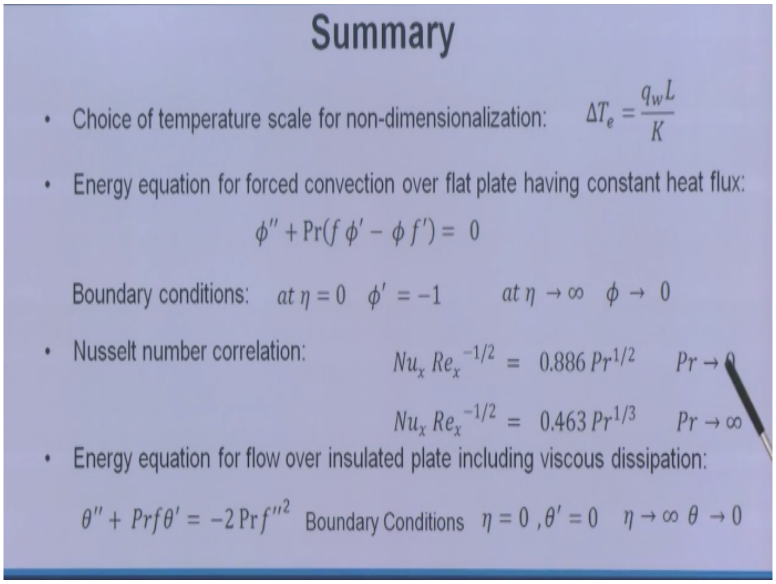
Next let us see that when we are having discuss dissipation then how this equation is going to change okay. If you remain but till now mwe have eliminated the viscous dissipation by considering very low Eckert number cases, here this is very high Eckert number cases let us say we are having viscous dissipation which cannot be neglected so this term will be remaining over there okay.

In the previous lecture I have showed you that ϕ^* can be written as this one okay, so $2(\partial u/\partial x)^2 + 2(\partial u/\partial Y)^2 + (Re^{1/2}\partial u/\partial Y + Re^{-1/2}\ \partial V/\partial x)^2$ okay. So here we have taken the transformation of y to Y in this fashion $Y=yxRe^{1/2}$ and transformation of v to V we have taken in this fashion $V = vxRe^{1/2}$ okay. So here if we try to find out that which term is dominating once again it is like the previous one you will be finding out that this term is dominating, because x is very high compared to your Δs okay.

V and Y they are of same order and here also V is fixed compared to X okay. So all these terms will be actually not dominating with respect to this one okay. And if you take this $Re^{1/2}$ outside with whole square this will become Re and this Re and 1/Re can be cancelled from the equation. So ultimately Eckert number $(\partial u/\partial Y)^2$ becomes the dominant term okay. All other small terms can be clubbed in this plus small terms okay.

Not if you plug in this one in the equation of your energy form in case of constant heat flux so the equation will be finding out this changing in this fashion θ’’ + prandtl number fθ’ = $(-2Prf'')^2$ so this extra term will be coming due to your viscous dissipation term okay. And subsequently boundary conditions are like this θ’ 0 is 0 and θ ∞ is actually 0 remainder were θ’ is coming in case of constant wall temperature there was θ okay, because θ need to keep for constant heat flux case okay. So heat flux is constant okay.

(Refer Slide Time: 21:25)

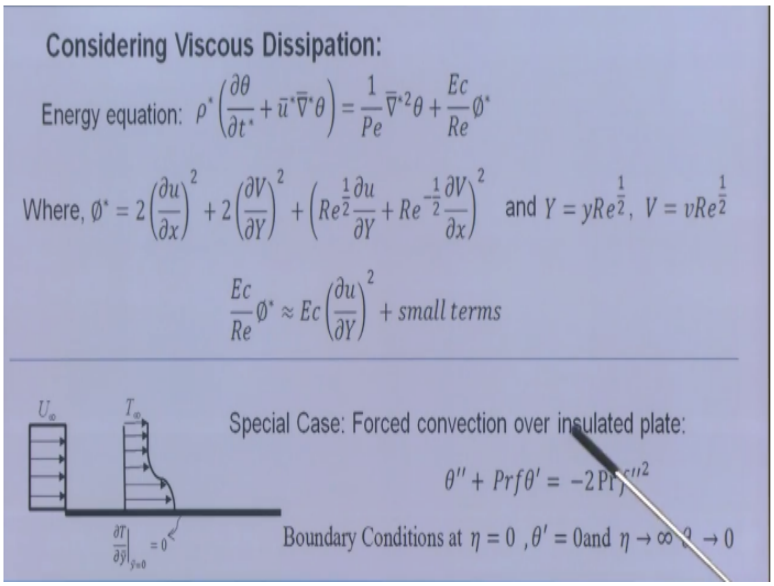


Let me summarize what we have learnt in this lecture first we have say what is the choice of what temperature skilled for non dimensionalization of temperature. So unlike the previous one where we have seen the constant wall temperature case the scale was Tw-T∞ here we cannot take that also we have chosen intelligent temperature scale like QwL/K where K is the thermal conductivity, Qw is the constant wall heat flux.

And using this temperature scale we have shown that force conviction of flat plate having constant heat flux can be reduced in this form in terms of ϕ okay, so ϕ’’ + prandtl number of ϕ ϕ’ – ϕf’ = 0 okay. We have also derived the corresponding boundary conditions rather I can say that this boundary condition has been simplified from this temperature scale non dimensionalization. So at η=0 ϕ=-1 and η tends to ∞ ϕ tends to 0.

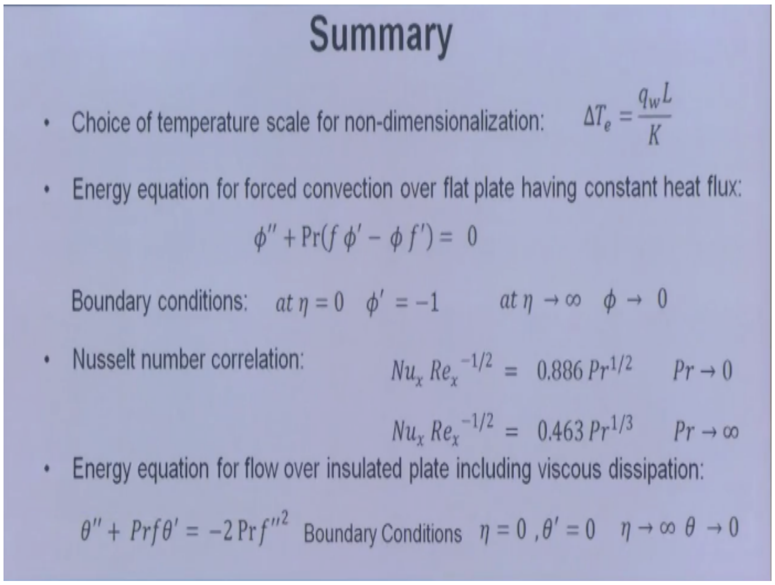
So this is for updated value okay so this becomes 0 okay. We have also mentioned the nusselt number correlations for low prandtl number limit and high prandtl number limits okay. So prandtl number tends to 0 we have shown that nusselt number into $Rex^{-1/2}$ is actually 0.886 prandtl number $2^{1/2}$ and for prandtl number tends to ∞ we have shown nusselt number into Reynolds number to the power of ½ is equals to 0.463 into prandtl number to the power one-third okay. And the end also showned in what ever types of discuss in the exibition and the contribution so in the energy equation so the insulated plate okay.

(Refer Slide Time:23:22)



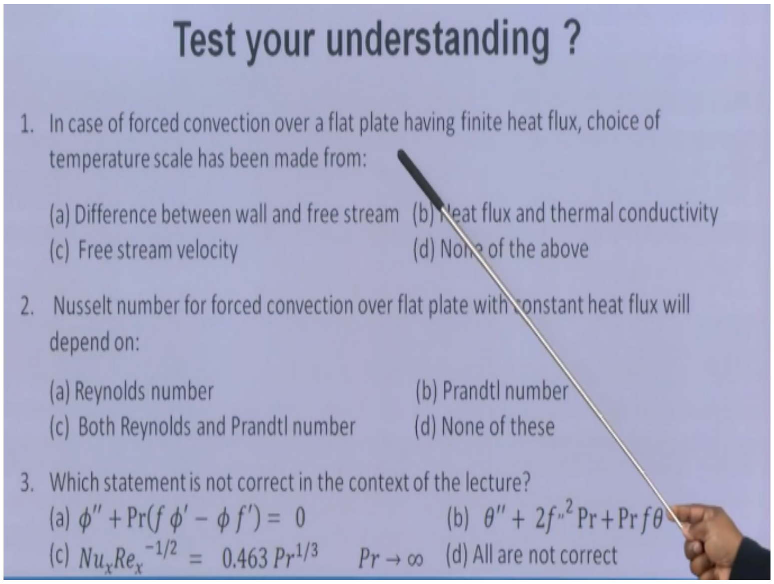
By the by if you go back to the previous one here I have not mentioned this is the the insulated plate okay, so already we have till now we discussed the constant heat flux the here is the insulated plate so the king W actually=to 0 .so in that case find out the this equation we can come in this form which we also summarize.

(Refer Slide Time:23:39)



In the next slide the Q double –the prandtl number Qis equal to the in the two prandtl number if the two whole square okay the coresopnding boundry we call the confrugations are in the Qo is actually be 0 in the infinity is tendings to the 0 let me also in the end of this lecture we are having the many questions okay in the source of the convectuions.

(Refer Slide Time:24:04)



In the flat plate finally the box is the made of the mean from the temperature is are the temperature scale is very important we are having four option difrence between the wall and the velocity and the heat and the none of the above in the thermal conductivity and the stem velocity and then none of the above im the temperature scale okay.obsivouly we have understood this is the thing about in the lecture and this QW and K QW/l okay.

The second questions is the assend number for the heat flux and the thermal cobnductivity will depend on four options so the prandtl number both the in the prandtl number can also in the prandtl number and none of this so this is for the constant heat flux of the flate plate so the last question here I have shown three equation over here first number is the prandtl number and 5 and the second one is 2 double ^{2} and the prandtl number iss the thrid one id the prandtl number tends to the infnity but it I can be obsivouly that all the equation are not correct okay.
It is not the correct answer so thank you please visit our next lecture national conduction from and please keep on posting in the enquiry of our discusssion for us thank you.

For Further Details Contact

Coordinator, Educational Technology Cell

Indian Institute of Technology Roorkee

Roorkee – 247667

E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in

Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team

Sarath K. V

Jithin. K

Pankaj Saini

Arun. S

Mohan Raj. S

Camera, Graphics, Online Editing & Post production

Binoy. V. P

NPTEL Coordinator

Prof. B. K. Gandhi

An Educational Technology cell

IIT Roorkee Production