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Convective Heat Transfer

Lec – 06
Forced Convection: High Prandtl Number over a Flat Plate

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Hello welcome in the 6th lecture of the convective heat transfer in this lecture we will discussing about forced convection at high Prandtl number over flat plate.

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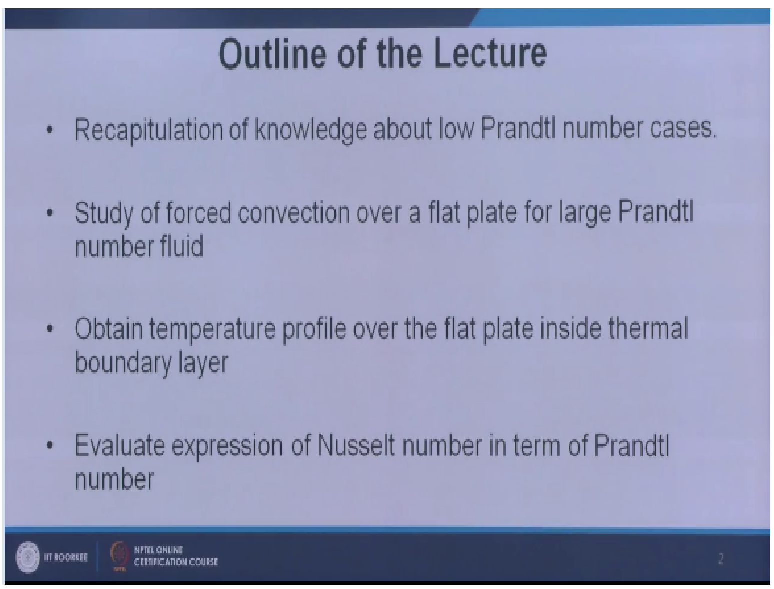

First let me discuss about the outline of this lecture first we will recapitulating the knowledge about know Prandtl number case which we have discussed in the previous lecture okay and then we will studding forced convection over a flat plate for large Prandtl number fluids okay we will obtaining the temperature profile over the flat plate inside thermal boundary layer and we will be expressing the nusselt number for this high Prandtl number cases okay in terms of Prandtl number we will be expressing that one.

So as first point is to see that what we have discussed in the last lecture if you remember in the last lecture we have discussed about the energy equation.

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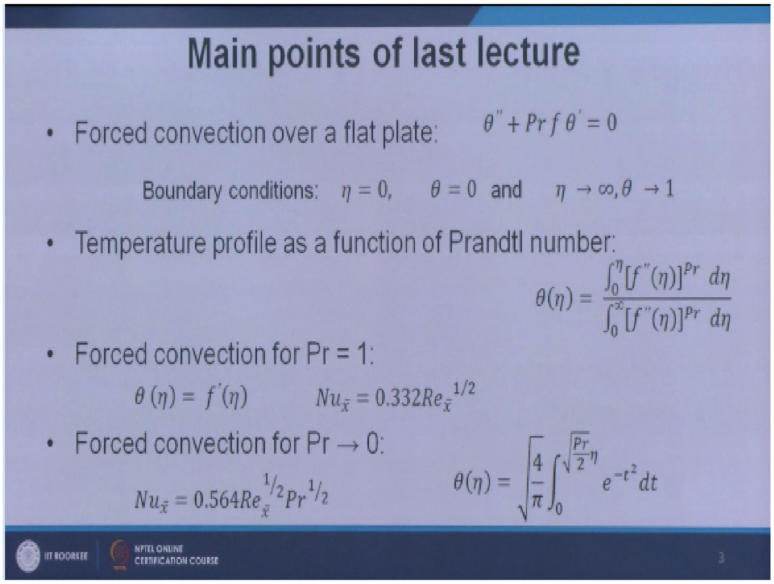

Reduced energy equation θ ``+ prandtl number if θ ` is = 0 here θ is actually function of η we have provide that boundary condition will be coming has θ 0 is 0 and θ infinity is actually 1 okay, from where we have shown that what is the temperature profile coming out okay temperature profile generalized temperature comes out in the form of this one which depends on the prandtl numbers okay and then we have taken 2 cases prandtl numbers = 1 case and prandlt number tends to 0 case.

So power prandtl number = 1 case where fluid is having more or less equivalent to 1 prandlt number in those case we have found out nusselt number into Rex – of is actually a constant 0.3332 okay if prandtl number is = 1 then this becomes 0.3332 and of the order of prandtl number = 1 then this value will reduce changing okay and for force convection at law prandtl number case prandtl number tends to 0 so we have soon that nusselt number into Rex – is actually deponent on prandtl number to the power ½ so this is point 564 x prandtl number the power ½ okay and here the temperature profile we have show in this fashion where θ η is actually √ over 4/ Π 0 to √ over of Pr / 2 x η so this is the upper limit e^{-t2}dt okay so this we have shown in the last lecture for low prandtl number cases okay.

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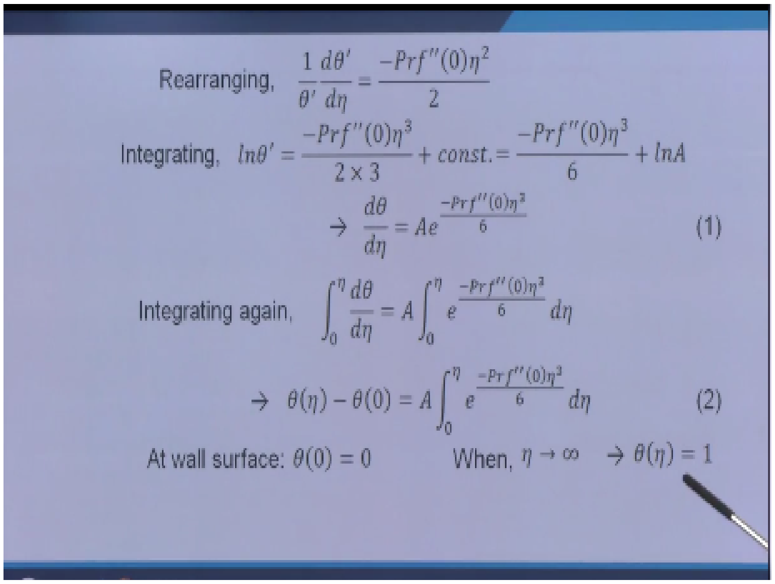

In this lecture let us start with high prandtl number cases so prandtl number is very high >> 1 or we can say prandtl number tends to infinity so this will be the case for thin thermal boundary layer okay so already we have showed that this is the case boundary layer so in case of thin thermal boundary layer δT will be smaller than the velocity boundary layer δ okay so these are the velocity boundary layer profile and thermal boundary layer profile with dotted line respectively okay.

The concept of velocity and thermal boundary layer as been already discussed now let us see that inside this thermal boundary layer what is happening so inside this thermal boundary we are actually η is very small because thermal boundary at thinness itself is thinner that the velocity boundary thinness so that actually limits with in a very small value of η okay so η is very small what we can do we can write down the expression of f which is function η from velocity boundary layer and the function of Taylor series expansion so you see this Taylor series expression of f and here η is very small quantity okay now from the boundary conditions of velocity boundary layer we know that f (0) is 0 and f' it is 0.

So first two term actually cancels out so we get f is actually of the order of ½ if '' η^2 if we live the higher order terms okay so f order we have got like this now in our course thermal boundary layer will be important so inside the thermal boundary layer we have found out θ'' + Prf θ' is equals to 0 this we have already derived on the last lecture and there this f can be replaced as

$1/2''\theta^2$ because the order we have brought from velocity boundary layer order and this scale analysis so here we can write down θ Pr number in terms of in case of f we have written ½ if '' θ^2 θ' comes over here = 0 so for Pr number greater than 1 Pr very high value this equation actually is very important okay.

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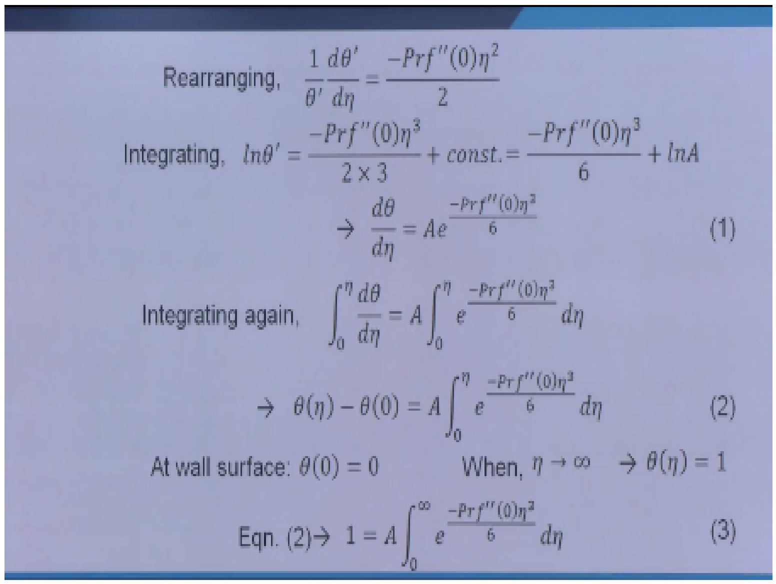

Now let us try to rearrange and try to do by integration of the pervious equation whatever we have shown over here if we do the side change little bit we can find out one 1/ θ' x θ'' to which si nothing but d θ'/ d η is actually on right hand side – Pr f'' θ η2/ 2 okay so this we can do by rearranging and now if we intimated it once two this becomes ln θ' the left hand side and the right hand side it becomes θ^2 becomes interactive by 3 + a constant okay now this constant can be written as ln A okay so if we plug these two ln the3n we can write down d θ/ d η will be nothing but $\theta' = A$ into $e^{-Prf''(\theta)\eta^2/6}$ this 6 came by 2 x 3 so this is 6 okay.

So if you do one once more integration and put the limits form 0 to η which is nothing but the thermal boundary layer then you w2ill be getting the right hand side 0 to η e-Prf'' η/ 6 x d η okay so if we took the limits over here so this becomes θ η - θ 0 is = A x this term periods over her okay now let us but the boundary condition of θ so this term we know this is θ0 which is at the wall so this we have taken the non dimensional of temperature 2 θ in such a fashion that at the wall θ becomes 0 so θ 0 is 0 and away for the wall θ η is actually 1. So if we put this things over here so it becomes θ infinity okay which is nothing but 1 is actually A 0 to infinity $e^{-Pr f''(0)/ 6 x d \eta}$ okay.

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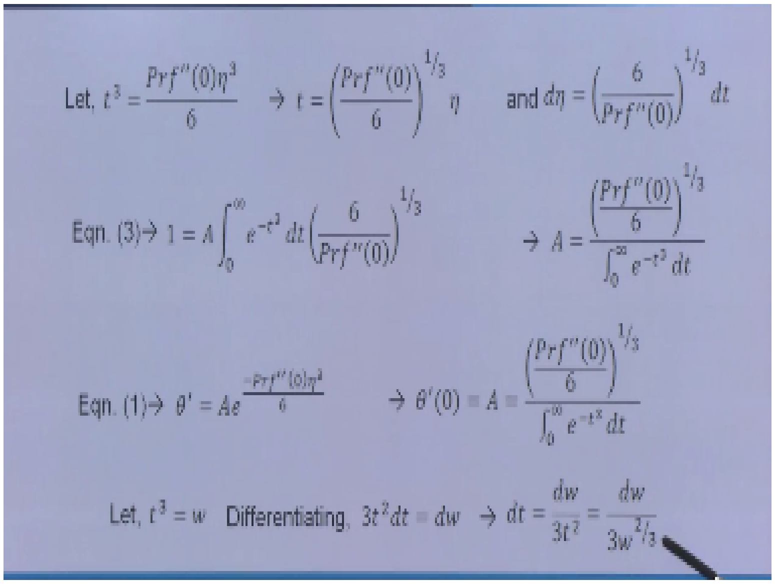

Then let us try to simplify further as you done in the previous case and the number low Pr number case so we are using we have considered this expression which was there just as a questioned of the e okay as a ^{Pr} so that take as t^3 so if you take as t^3 this 1 can comes this 1 and d η can be this one and d η can we written as 6/ Pr f'' to be power 1/3 into dt okay now what we can do for finding out the value A we can replace η t and d η by dt x 6/ pr f'' to the power 1/3 as we have derived over here okay so A becomes Pr f'' by 6 to e power 1/3 and this integration will be reaming 0 to infinity e power –t3 x dt okay also we know θ' okay.

So from the previous slide we can see θ' is nothing but A e power this term Pr f'' θ3 by 6 okay so from there we can find out what is the θ' at wall okay so θ' at the wall will be helping us to find out the heat flux so θ' at the wall is actually A into which is A because η = 0 means this term will be canceling out so this is A that means at wall θ' will be actually be coming this turn okay, where this e^{-t3} integration means to be evaluated okay, let us try to do that so first what we will be considering this t^3 is = w okay if yu consider this one then $3t^2$ dt becomes dw and dt becomes dw / 3 $w^{2/3}$ okay.

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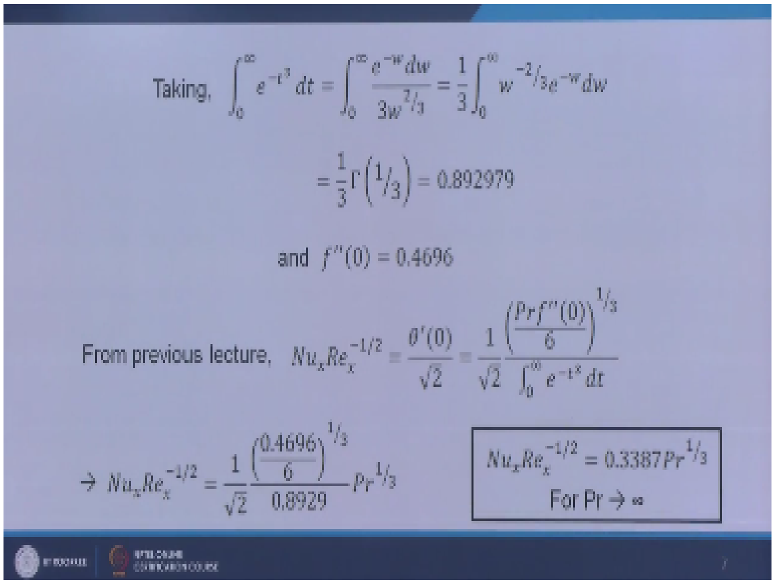

So if you put everything over here in this integration it becomes 0 to infinity e^{-w} dw /3w $^{2/3}$ okay, further reduction will be giving you this type of integration 0 to infinity w $^{-2/3}$ e^{-w} dw okay. Now we know that this is nothing but actually γ function okay this is integration will be giving rise to γ function it will be coming from mathematics okay. So this is actually γ function of 1/3 okay, so this 1/3 is remaining over here and this integration gives rise to γ function of 1/3, if you get little bit knowledge of γ function from mathematics then you can find out that γ function of 1/3 is having a constant value and multiplying that one with 1/3 we get 0.892979.

So the value of integration we obtain as 0.892979 okay, also we know from velocity boundary linear concept that if double dashed is equals to 0.4696 by the way why we have evaluated this two constants because if you see here we require f'' as well as this integration value okay. So both the values we have found out now we can put this over here okay in θ' okay which was actually earlier linked up with Nusselt number and Reynolds number in previous lecture okay.

So this θ' is having this part okay where we are having f'' and e^{-t3} integration from 0 to infinity so this we have found out has 0.892979 and this f'' is 0.4696 and this 1/ √2 remained earlier from the relationship between Nusselt number and Reynolds number with θ' which has been shown in the last lecture. so if you put all this values over here like this okay so we can get Reynolds number Nusselt number and Pr number equation okay and this constants will be giving raise to 0.3387$Pr^{1/3}$.

So this is the very well known equation for large Pr number cases okay, where velocity boundary layer will be dominating over thermal boundary layer cases okay.

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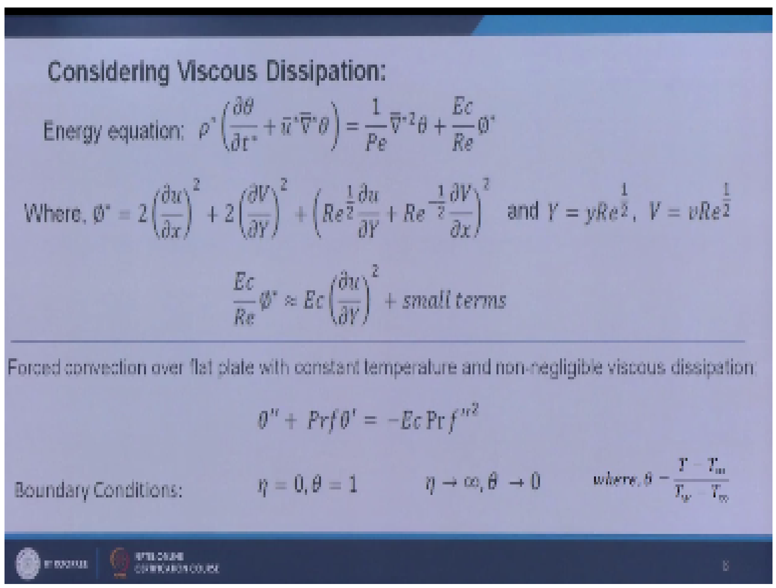

So in this one till now we have discussed about cases where viscous dissipation we have neglected okay but here if we have to consider this viscous dissipation, so if you remember earlier we have shown that with viscous dissipation terms will be coming like this Eckert number by Reynolds number in to five star okay so here let me consider the viscous dissipation this term cannot be neglected.

So let us keep this term this is for high Eckert number cases so if we keep this term along with convection and conduction this terms will be also becoming important okay, so let us see first that what is there inside this five star which is from fluid mechanics once again so this viscous dissipation can be written as 2 x $(\partial u\ \partial x)^2$ + 2 x $(\partial v\ \partial y)^2$ + (re ^{½} $\partial u\ \partial y$ + Re $^{-1/2}$ ∂v vx $)^2$ by the way this term actually came after conversion of small v x V, so actually this terms is $2x\ (\partial u / \partial x)^2 + 2(\partial v / \partial y)^2 + (\partial u\ \partial y / \partial v\ \partial x)$ okay.

So from there if you take this relationship between Y and y V and v with respect to Reynolds number then this viscous dissipation will be reducing in this passion. Now let us see which term is dominant over here, so here you can find out l is quite large so x is actually of the order of l so this terms can be neglected. Here both v and y all are same order okay so if ∂ if y is actually in the terms of ∂t so v will be of the same order so this term is also not significant okay.

Now you can take $Re^{1/2}$ come on outside so you can find out that it is becoming a ∂u ∂y over here and it will be becoming re^{-1} ∂v ∂x over here. So the important ∂ will be becoming then this one because V is very small. Inside this thermal modular this is. So this term this term and finally this term these are not that much important. The important are the dominant are will de ∂ u by ∂ which is over here. So Re to the ½ we have taken out side this whole square so this will becoming Re a and Re and Re can becomes in ultimately it becomes at ∂u/∂y whole square plus these term will becoming inside this small terms okay.

So Ec Ø is finally this into dominant term. So if you incorporate this term in your energy equation okay do the similar analyses whatever I have shown in this lecture. So you can find out the non dimensional equation will be coming in this form θ" are plus Prf θ' which was all are here there. Is equal to Ec Prf" to the whole square. So this is the new term will be coming to will discuss which in this term we cut number Del u del y the whole square okay. The corresponding boundary conditions will be θ is actually one. And θ infinity is actually 0 okay.

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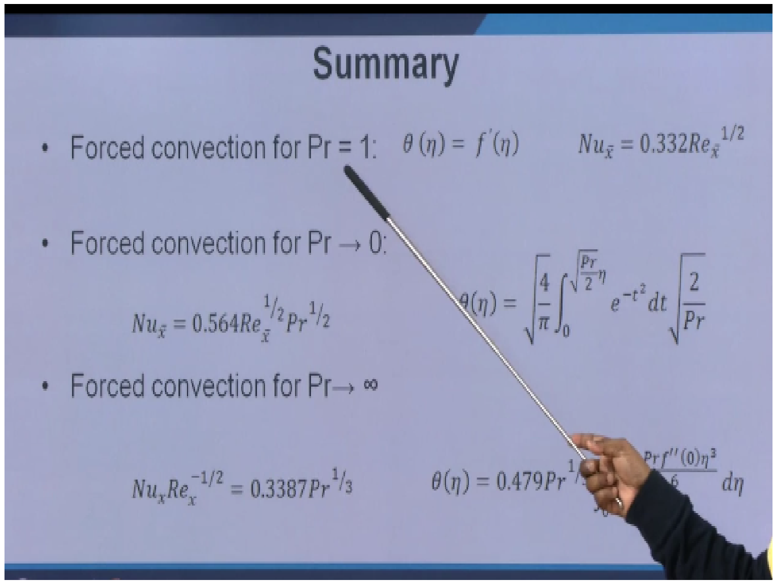

Next will be summarizing so force convection for pr number is equal to 1 so in this case we have shown θ is a best and Nux number we have actually returned as 0.332 into Rex1/2 for prandtl number tends to 0 we have shown. Nux number is equal to 0. 564 into Rex1/2 Pr number to the power ½. θ becomes this fashion this we have shown in the last lecture. √4/Π o to root over of Pr by 2 into η e to the power – t2 into dt √2/Pr okay. And this lecture we have shown that Pr number tends to infinity very large Pr number till thermal modular cases. Nux number into Rex to the power -1/2 is equal to 0. 3387 Pr to the power 1/3.

Okay and the temperature poof that what in this fashion. θ is equal to 0. 479 Pr to the power 1/3 to the integration o to η it will be for – Prf” (0) η 3 by 6 into d η okay now as like the hardly lecture here also I understanding from the lecture we have three questions over here. First one Nux number into Rex to -1/2 = 0.3387 Pr to the power 1/3 is expression for which cases or which case: for having four options high prandtl number.

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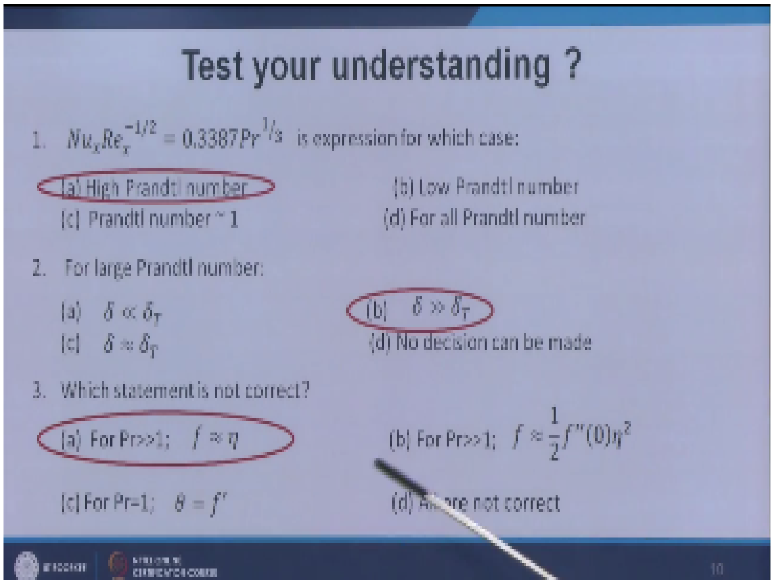

Low Pr number pr number of the order of one and for all pr number it is valid okay so it is very simple just now we have discuss so the correct answer is obviously for high Pr number case thin thermal boundary layer case okay, next question is for large pr number case which one we can take δ velocity boundary layer is smaller than thermal boundary layer velocity layer boundary layer is actually larger than thermal boundary layer both are of same order and no decision can be made okay..

So this is also very simple you know that in this case for large pr number case obviously velocity boundary layer will be dominating and δ >> than δ t, last one which statement is not correct remember not correct okay, we are having four options for pr number >> one f will be becoming of the order of η, okay for pr number >> one then large pr number f will be becoming of the order of ½ f'' θ^2.

For pr number = 1 θ becomes f' or all are not correct all are wrong okay, so obviously you know the correct answer if this one is not correct because this is for the this f will becoming η for low pr number cases okay. So race to r is correct okay. So with this I will be ending this lecture thank you, and the next lecture we will be discussing about force convection over flat plate at uniform heat flats, so keep on posting your quarries in our discussion forum. Thank you.

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