INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

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NPTEL ONLINE CERTIFICATION COURSE

Convective Heat Transfer

Lec-05
Forced Convection: Low Prandtl Number over a Flat Plate

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of technology Roorkee

Welcome in the 5th lecture of convective heat transfer in this lecture we will be discussing about forced convection low prandtl number over a flat plate so in this lecture we will be discussing about reduction of energy equation for forced convection.

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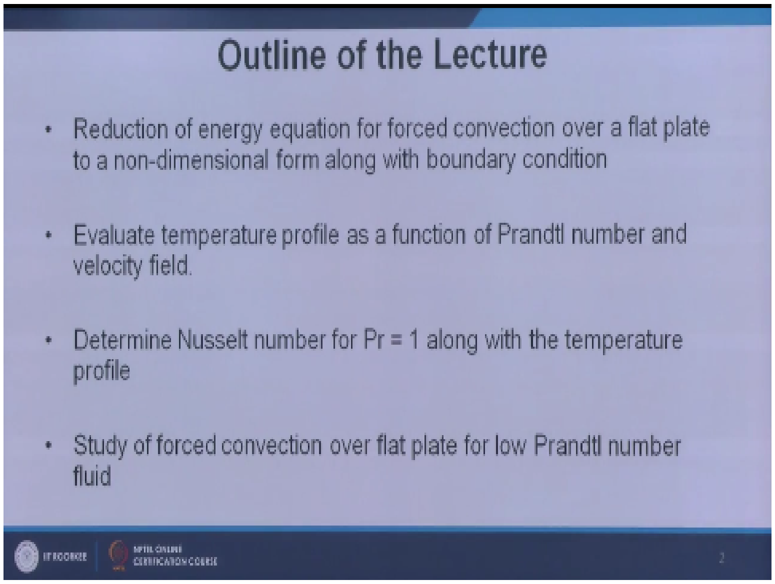

Over a flat plate to a non dimensional from along with the boundary conations so in the last lecture we have discussed about the equations here we will be specifically discussing about the boundary conditions we will be evaluating the temperature profile as a function of Prandtl number which is a very important parameter we have seen in the last lecture and the velocity field okay.

We will be determining nussel the for Prandtl number = 1 along with the temperature profile and at the end we will be studying forced convection over flat plate for low Prandtl number fluids first let me show you the figure which we you have seen in the earlier one earlier lecture but this is a flat plate.

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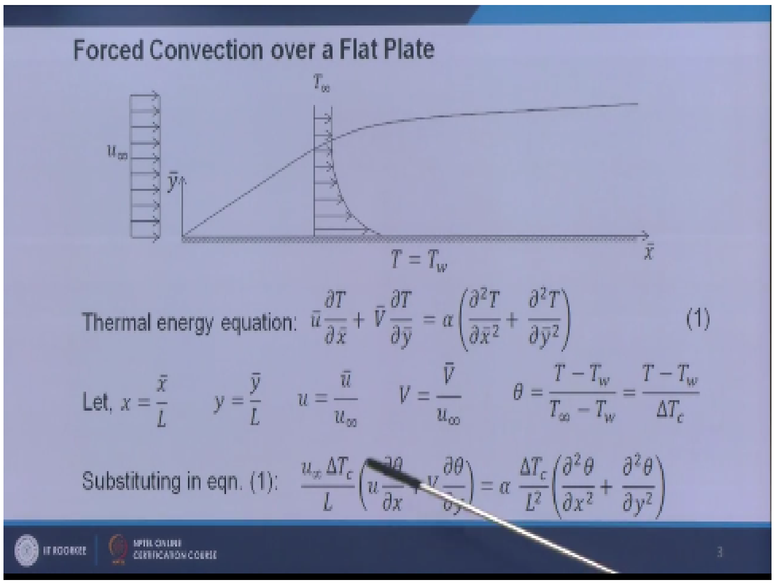

Over which the frosting velocity u_∞ is coming and at temperature T_∞ and the 1 is having Tw temperature so here you can find out the temperature corn 2 or temperature profile cokes like this okay and this is actually your locus of those points which brought 99% of the temperature of the full steam okay so this is your thermal boundary layer okay now this equation already we have see in the left hand side we are having the convection team and in the right hand side we are having the conduction term.

And α is actually in thermal diffusivity okay son this actually thermal energy equation and we have reduced in the last lecture this equation with no dimensional variables so these are the stream voice non dimensional variable cross stream voice dimensional variable the velocities in the respective directions this is very important so θ we are writing in terms of the wall temperature and full stream temperature T- Tw / T_∞ - Tw T_∞ - Tw can be written as Δ Tc okay.

So if you substitute all these then the convection team will get one coefficient like u_∞ΔTc/ L and in the conduction side we get coefficient of α of ΔTc/ L^2 abd if you try to reduce this in terms of non dimensional numbers.

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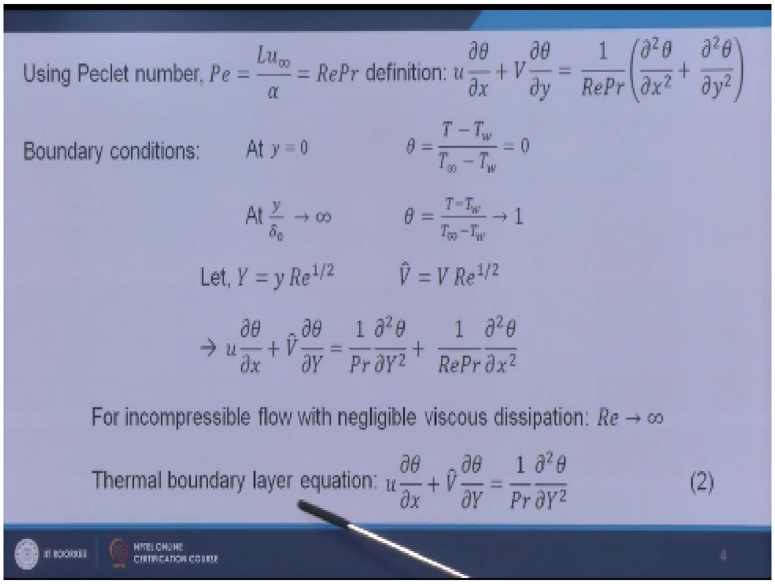

Then we have shown that Peclet number comes important so it becomes 1/ RePr okay Peclet number is nothing but RePr which we have shown in the last lecture, so this equation we have shown in the last lecture which is the thermal energy equation for flow over flat plat now let us see what are the boundary conditions so boundary conditions 1^{st} at y = 0 which is nothing but the flat plate okay.

Adjutant to the flat plate y= 0 so there we can write down θ is nothing but Tw- Tw by T_∞ - Tw so Tw – w becomes 0 so θ becomes 0 so at y = 0 θ is 0 okay and far away from the plate so which is nothing but y/ θ_0 I am saying this is the order of the boundary layer or y direction on scale you can say so y / Δ_0 is of the order of ∞ so far away from the plate.

So there we can write down θ will be actually T will become T_∞ far away from the plate so this is nothing but T_∞ - T_w/ T_∞ - Tw which is nothing but 1 okay so in case of y/ Δ tends to ∞ which is far away from the plate θ will become 1. θ which tend towards 1 okay so these are the boundary conditions next let us try to reduce this equation using some stretch variables because we know the order parameter in the cross stream direction y so using that we can write down Y = y $Re^{1/2}$ okay now if we have to satisfy the complete equation similarly we have to take the V velocity scaling so V will be V^ we have taken as the stretch variable for the cross stream velocity.

So V^ is actually V into $Re^{1/2}$ because ΔV ΔY come in your continuity equation which needs to be of the order of ∂u ∂x which is actually of the order of 1 so these two needs to be same order so this two stretched variable if you but in this equation then we get ∂u ∂ θ/ ∂x so this we meant same for this one we will be having V∂θ/ ∂y so V is over here y is over here both beat on stretched at this I and Re and $Re^{1/2}$ actually cancels out in the right hand side first turn 1/RePr ∂2 to the ∂x2 remains over here.

No change over there but for the second term here you see we are having y2 so u is actually if you try to make it stretched where it will become $Re^{1/2}$ $Re^{1/2}$ whole square so Re, Re will be cancelling now so it become 1/Pr x δθ2/ δY2 okay now if we take inside the thermal boundary layer and incompressible flow negligible viscous dissipation which we have already flow in the dissipation we need to take for.

Lower got number cases then this Re will be very beginning magician okay so if you try to take this limit are you tensed to ∞ then you can find out from this equation last term which is nothing but your steam white so conduction that can be related okay so ultimately inside the boundary or thickness will be getting in the convection term here this to terms in and in the right hand side only the crossed in viscous conduction becomes important so this term remains this terms can be cancelled up okay.

So inside thermal boundary layer so this is the equation this we can call as thermal boundary layer equation.

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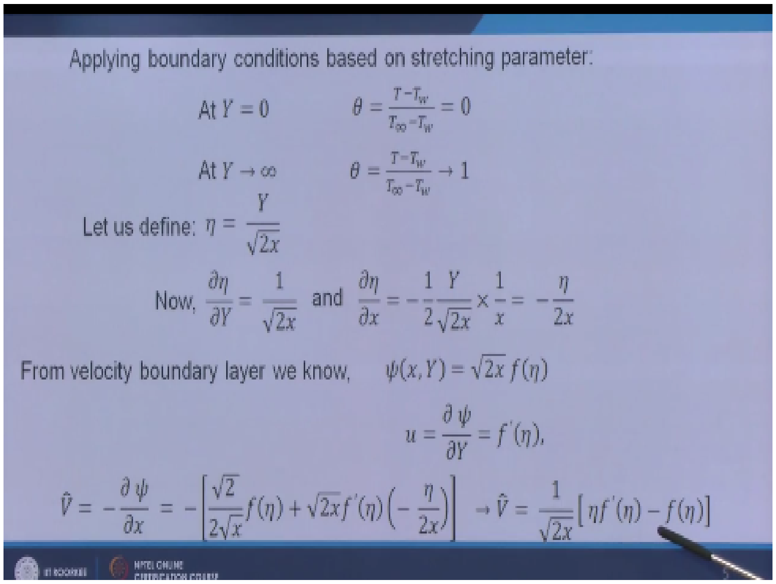

Along with the boundary conditions obviously which we have already shown at Y = 0 Y = 0 because y 0 is capital Y= 0 okay so in case of Y = 0 θ obviously will be 0 already we have shown and then Y takes ∞ if small y takes to δ_0 y/ δ_0 turns to ∞ so obviously capital Y is also tending towards ∞ so capital Y tending towards ∞ it comes out has θ takes to 1already we have shown this boundary conditions now let us define a at η okay η is nothing but capital Y/√2x okay this we can call as similarity valuable.

Which we have already shown in velocity boundary layer cases into mechanics so if you try to get the derivative of this η as it is involving Y and X so it will be having derivative because Y as well as respect to X so you can see into respective Y i becomes ½ √2x 1/√2x and in case of x it becomes if you do the derivative and try to replace η here when it will become – of η/2x okay here you see Y/√2x once can be written as η okay so this becomes – η/2 and x 2x okay so – η/2x now let us define from the velocity boundary at thickness you know.

That is high can be written as √2x into S of η function of η because already we have included y and x over here it η so now this steam lines will be of only depended on the value of η over here okay so this can be considered as √2x into f of η okay and from steam line concept you know use the ∂ψ/∂Y okay so we should do the derivative of this one it will be becoming n dashed into a simply have dashed okay and in case of V it will become - ∂ψ/∂x so if you do the derivative of this one okay if you do the derivative of this one.

So it becomes this term first derivative will be √2/2 x √x into f + √2x into f – into this will be nothing but ∂η/ηx so this is – η/2x okay so these equation we are making the derivative so two terms are there into x and f so first we have done the derivative of √2x and next we have done the derivative of f okay so as if the function of η so here bη/dx will be remaining over here okay so after further the situation we cap becomes 1/√2x into η f dash – f okay.

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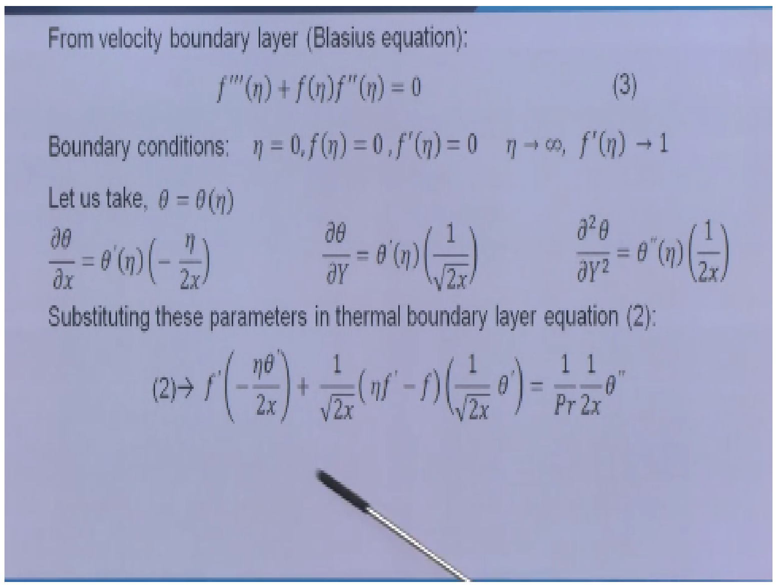

Now we know that velocity boundary layer gives us Blasius equation and Blasius equation is nothing but f’’ equals to 0 and this comes from PD of fluid mechanisms you already have studied this one okay so and these are the corresponding boundary condition this also you have rate fluid mechanics at if I goes to 0 f = 0 f - = 0 and η takes to ∞ f – takes to 1 okay now let us take for thermal case that in this course thermal bound layer will be importance so let us take θ is a function of η okay so θ is a function of eta, η is nothing but $y^2(x)$ which we have defined in the previous slide okay. So here if we try to make the derivative of θ with respect to x it becomes θ' (-η /2x).

And if we do the derivative with respect to y it becomes θ' (1/√2x) okay and second derivative with respect to y becomes θ'' 1/2x all this terms are there in your energy equation denied energy equation which I have shown in few slides back, so herein if you try to put all this derivatives of θ with respect to x capital y and capital Y two times then we will be finding that this becomes the equation, okay. So this is nothing but equation number 2 which we have shown.

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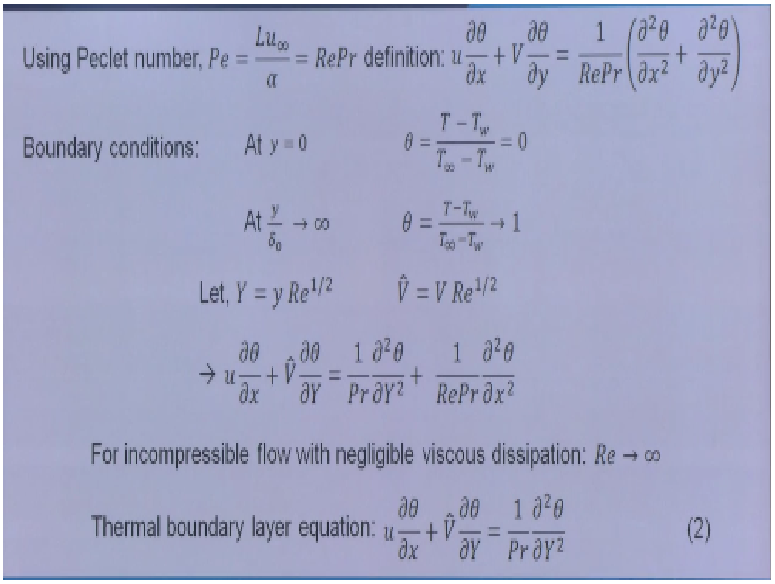

Over here okay, so here is the equation ∂ θ / ∂x we have found out ∂ θ /∂y we have find out and ∂^2 θ / ∂y^2 we have found out.

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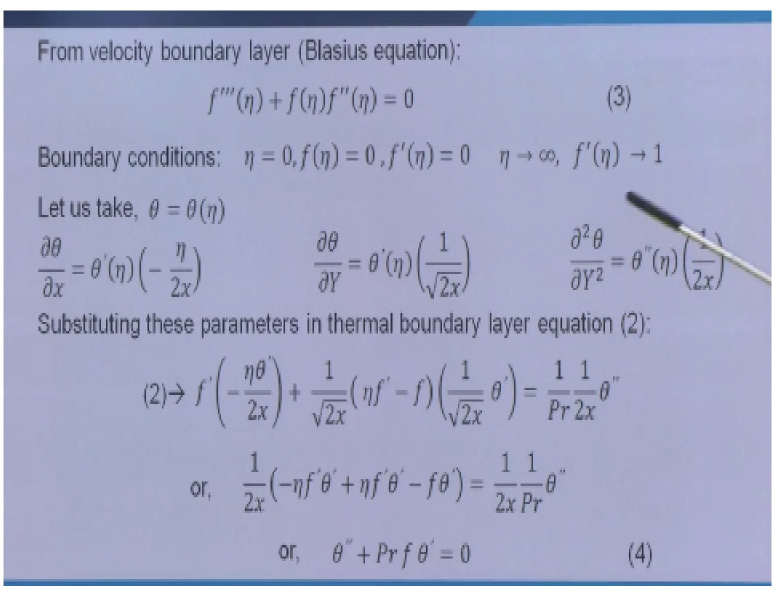

So if we put all this over here then we get equation like this, now okay so this is nothing but ∂ th η / ∂x okay, u ∂ θ / ∂x this is nothing but v. ∂ θ / ∂y and this is 1/Pr $\partial^2 \theta / \partial y^2$ okay. So if you reduce it farther then you will be finding out 1/2x can come out from here with 2x, √2x becomes 1/2x, okay. And it can be written inside $\eta f' \theta - (\eta f' \theta')$ and $\eta f' \theta'$ + so this two will be cancelling out.

Here we get $-f' \theta'$ and this side 1/2x is coming over here into 1/Pr x θ'' okay so if you reduce this equations so this will become my energy equation now in terms of the η add the function η so θ'' + random number f θ' = 0, okay. So this is my velocity bound here equation in terms of the blasius and this is my energy boundary equation, these are the boundary conditions for f's.

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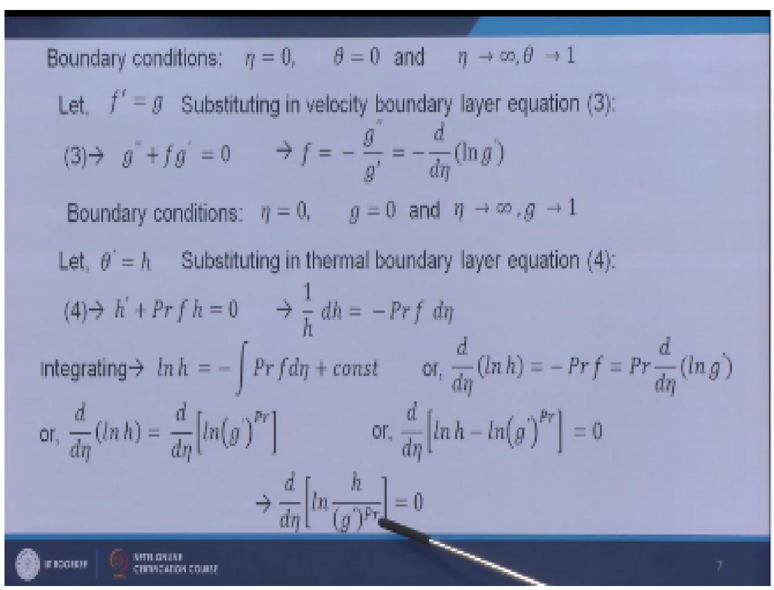

And let me show you the boundary conditions for θ now, so at θ = 0, θ will become at η will be tending towards infinity θ will be turning towards one, so this boundary condition we have introduced at the beginning of this lecture, okay. So now let me show you how this can be solved for the velocity case velocity boundary layer case we know that the process view is you have to assume $f' = g$.

And if you substitute in the $f''' + f \times f''$ equation it gets g'' s+ $fg' = 09$ and subsequently you get fin terms of ∂ η (ln g') okay and final if the boundary conditions if you plug in terms of g now η = 0, g = 0 and η → ∝, g → 1, similarly let us try to do same thing for the θ equation so let us consider $\theta^* = h$ okay, so if you consider th $\eta^* = h$ and we are substitute in the thermal boundary layer equation, this one.

What I have shown you in the last slide, this one then we will be finding out that it becomes h^* + random number fh = 0 okay, so if you do the integration then you will be finding out after integration it is becoming logarithmic of h = - ∫ Pf F d η + constant okay so this is numerical constant okay. So here we can write down that d/dn(ln h) so this integration once again we can go for the derivative term.
And d/d η (ln h) = Pr F – Pr f okay so this Pr f, this f already we have shown is actually written can be written as –(∂ /∂ n(ln g')) so here I am bringing over that one ∂ /∂ η (ln g') in place of this f okay from this velocity bound layer concept okay, so we get ∂ /∂ η (ln h) = Pr ∂ /∂ η (ln g') okay. So this Pr number can be taken as exponent over here so it came as exponent okay over here. Now if we just take in sense side so this will become ∂ /∂ η (ln h) and then ln $(g')^{Pr}$ so it can be called also two logarithms are over there so $d/d\eta[\ln h/(g')^{Pr}]=0$.

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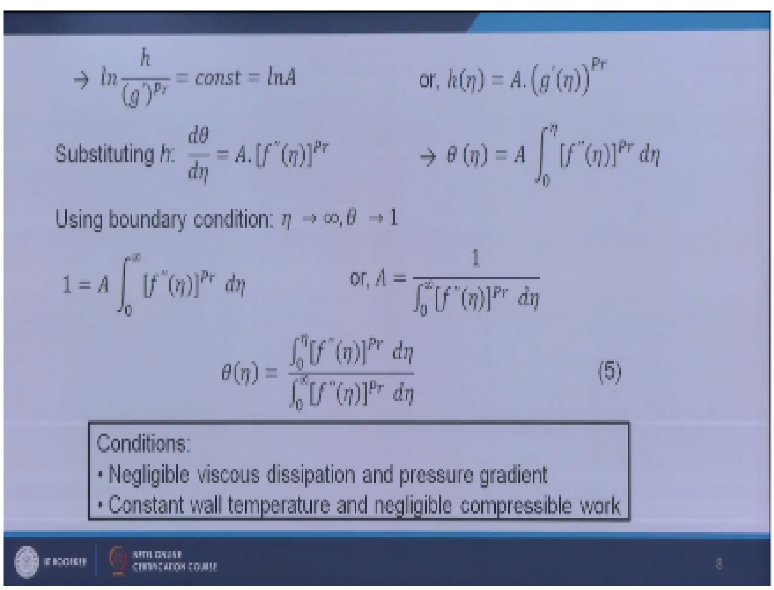

So if we integrate so this becomes $\ln[h/(g')^{Pr}$=constant this constant can be written once again in terms of logarithmic of some other constant A so we can write down $h/(g')^{Pr}=A$ so h will become

$A.(g')^{Pr}$ okay. So if we replace this h now already we have shown h is nothing but θ', so θ' that means $d\theta/d\eta$ becomes A into, now g was actually f' so it will becoming f'' so f''^{Pr} so $d\theta/d\eta$ becomes $A.f''^{Pr}$, if you integrated it so it will become $\theta(\eta)$ okay, let us integrate and put the limit from 0 to η, okay so it is becoming $\theta(\eta)$, okay.

θ_0 is actually 0 so that is why we have neglected that one $\theta(\eta)=A.0$ to η $f''^{Pr}d\eta$ okay. If you use the boundary condition over here second boundary condition η it tends to ∞, θ tends to 1 so if we put it over here so you will be getting that θ_∞ which is nothing but 1 so 1=A limit will be changing from 0 to ∞ so 0 to ∞ $f''^{Pr}d\eta$, okay. So from here we get the numerical constant A, A is nothing but $1/\int 0$ to $\infty[f''^{Pr}.d\eta]$, okay.

Now let us plot this A in the equation so in the equation of θ so we can get θ is nothing but A is 1 by this so this will be becoming 0 to η $[f''^{Pr}]d\eta/0$ to ∞ this is the constant 0 to ∞ $f''^{Pr}d\eta$, okay. So this is the profile for the temperature very, very important one, now what are the conditions required for this negligible viscous dissipation and pressure gradient which we have already seen in case of Lu A Clarke number in the previous lecture.

Constant wall temperature definitely for this derivation we have considered constant wall temperature T_w and negligible compressible work, okay so these are the assumptions required for derivation of this energy temperature profile, okay temperature profile $\theta\eta$.

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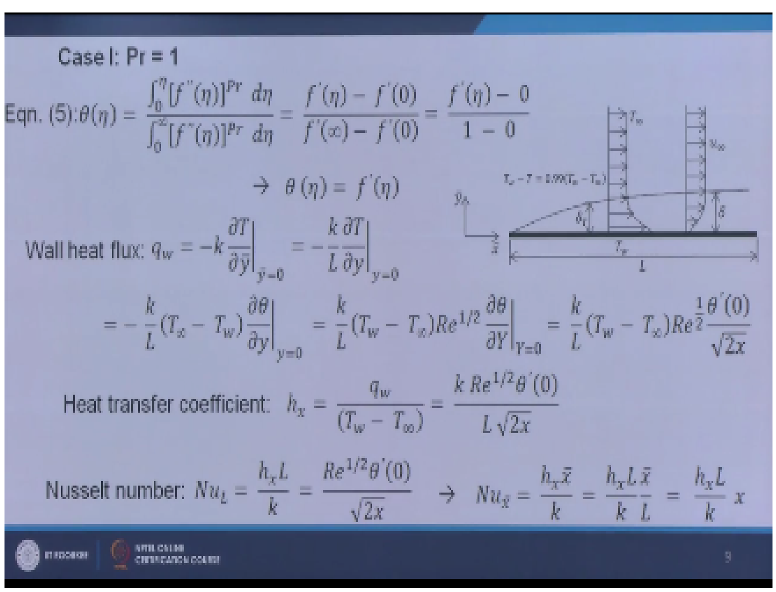

Now let us take different cases as know that thermal boundary layer always depends on the prandtl number we are having first one, which is actually prandtl number of the order of 1. Let us take Pr=1 case one Pr=1. So we will starting from the previous one so θη=0 to η f″ prandtl number is becoming 1 means it will becoming 1 and 0 to ∞ f″ prandtl number is becoming 1 now same thing I have written over here.

So we prandtl number becomes 1 and then if you do the integration it will become f″, f″ was this so it is become f′ so f′ of η upper limit minus of f′(0) lower limit divided by f′(∞) upper limit was ∞, -f′(0) lower limit is 0, okay. Now if you plug in the boundary conditions we know that this term is actually 1 from the velocity boundary layer boundary conditions and these two terms as 0, so it becomes f″ f′(η)-0/1-0 so this becomes f′ okay, so θ becomes f′ for the case of Pr=1 θ becomes f′, okay.

Now let us also try to derive the wall heat flux so wall heat flux q_w we have already shown that is nothing but -k∂T/∂y at y=0 okay, x and y co-ordinates are shown over here y is the crossed direction okay, so here if you do little bit of non dimensional relation earlier we have shown ȳ is nothing but actually ȳ/L is actually y so here we can see L is coming out over here to make this ȳ to y, okay.

So and to non dimensional this T we can have $(T_\infty - T_w)$∂θ okay, so T is becoming θ over here as the result $T_\infty - T_w$ is coming over here, okay. And to take care from y to Y this stretched variable we have already shown that $Re^{1/2}$ will be involve so $Re^{1/2}$ came over here, okay so we are getting okay and ∇θ /∇y is nothing but θ' okay θ'/ √2x okay so from this now this θ is actually a function of η okay and this is function of X and Y so this X and y we have converted to η already and that we have consider over here. So this 1/ √2x is coming due to ∇η / ∇Y okay.

Then heat transfer coefficient h_x we can write down we know that qw / tw – t infinity this is coming from the immediate at the clear convection so qw is ha x tw – t infinity okay so here we can put the value of qw / tq – t infinity from her qw / tw – t infinity so it becomes k / l $Re^{1/2}$ √2x will be coming at the bottom and here θ' 0 okay. So Nusselt number finally if you try to calculate from this h so Nusselt number is nothing but hl/k so here from l and k can go in the other side so Nusselt number will become $Re^{1/2}$ θ'/√2x okay.

Now if we try to little bit understand what the Nusselt number at a particular location is so for that we have to find out a new x̄ so x̄ is the steam voice detection so that definition can be written as h x̄/ k so a is transforming in to x̄ because this is Nusselt number at any particular location x̄, so this can be written as h x l / k x x because x̄ /l is coming over here, so l will be cancelled out to get this one okay. So Nusselt number x is nothing but Nusselt number l x x okay.

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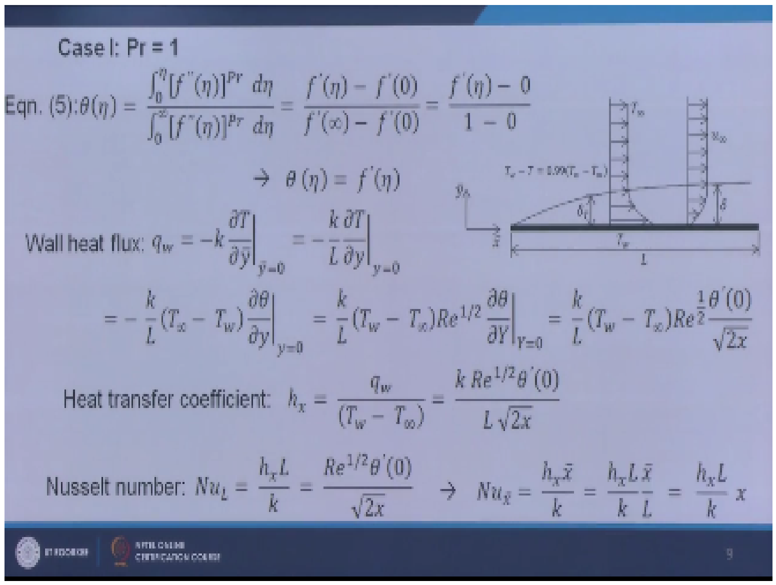

So if you reduce Nusselt number x this will become $Re^{1/2}$ θ' /√2x x x, so this was actually your Nusselt number, based on l okay. So here from you can see x and x can be can sent √x and √x can be cancelled and here we will be having √x in the upper side so here you are having Re ^{½} here you are having √x so this will become Rex ^{½} so finally we get Nusselt number and √Rex in one equation depending on the θ' value okay.

Here I have shown the reduction of the Reynolds number and Rex Okay so how these two are connected here I have shown okay. So we get from this equation Nusselt number based on xc and Reynolds number based on x -1/2 is actually dependent on θ'η θ'0/ √2 remember this θ is actually function of η okay, and depending on this will be coming from boundary condition so this is having a constant value so we can say this is actually constant o for Prandtl number equals to one case if you try to see the profile for Nusselt number and Rex Nusselt number x x Rex-1/2 which is in the left hand side of this equation.

As it is constant not dependent on Reynolds number so it is coming for a particular line as a particular line, so this line will moving up and down for different Prandtl number fluids okay and if you try to see what is the dependent of Nusselt number and Reynolds number so Nusselt number and Reynolds number if you try to see due to this minus half it will be taking a profile like this okay.

Now let us try to derive from our Blassius equation from Blassius equation we know that if double dash is nothing but 0.4696 this is the derivation of the Blassius equation and already we have shown that θ = f’ that means θ’ = f’’ okay.

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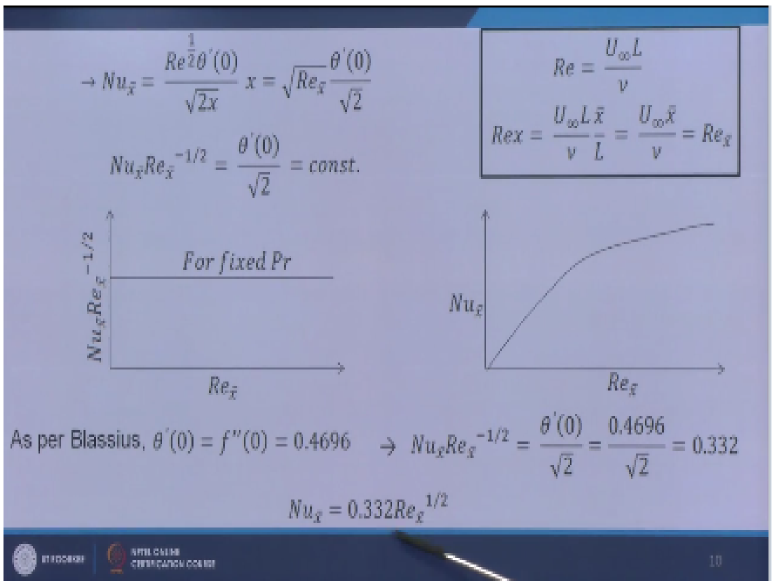

So θ’ = f’’ that means θ’ becomes 0.4696 at θ = 0 so if you put this θ’ value over here so it becomes from here Nusselt number in to Reynolds number to the power -1/2 becomes 0.4696 / √2 that means 0.332 so this is very important coordination Nusselt number x x Reynolds number x $^{-1/2}$ =0.33 it very important coordination okay. So this is very important coordination that I have mentioned right.

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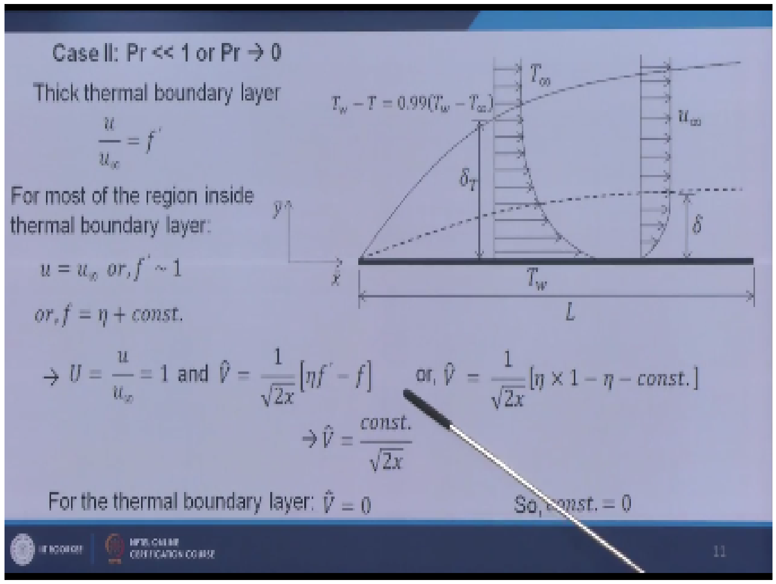

Next let us see the another extreme which is nothing but lower Prandtl number case okay Prandtl number less than one okay so Prandtl number can be taken as tense to 0 so this is for thick thermal boundary layer already in the last lecture we have discussed, so here you have see I have shown thick thermal boundary layer means boundary layer will be higher compare to the velocity boundary layer cases.

Now for thick thermal boundary layer from the velocity thermal concept we know that the $u/u=f^{/}$. Now in the most the thermal boundary layer you can find out the velocity will almost be equivalent to u∞.

So we can write down for the most of the region inside thermal boundary layer u = u∞ okay or you can write down from this equation $f^{/}$ so if you calculate you can get the f= n + const okay. So if I define U = u/u∞ at the order of one. Similarly v cap = 1/√2x (n$f^{/}$ - f) which we have already shown.

This u and v if we try to reduce here v is coming as 1/√2xη = 1 so this will η x 1 and f η + const – η – const so this η and η can be cancelled. So v cap becomes const/√2x. now for the thermal boundary layer we know that v cap will be = o inside the thermal boundary here, so this const will be 0 okay. Otherwise it will be dependent on x an d the const needs to be 0.

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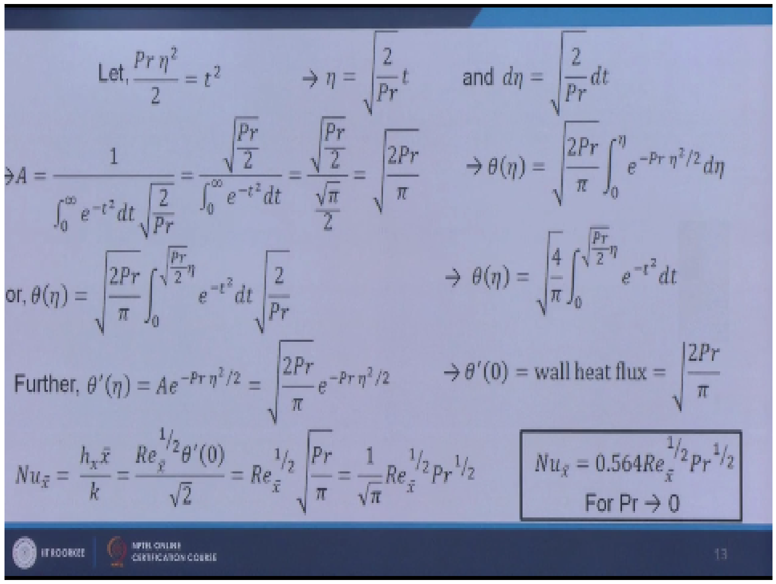

This const is 0 so this const is 0 so that means we can write f = η okay. Then let us try to see our thermal boundary equation which was ideal of this passion now in place of f we can replace η as we have proved f= η . So this compares Prη = 0. Okay now let us try to integrate this if we do a simple integration steps we will be finding out the d/dn=-Pr η okay. So if we integrate it further then you will be getting θ= $Ae^{-Prn2/2}$ simplify steps are over here okay and once we integrated another times so you will be getting second time integration and the limit has been taken from 0 to θ. B y the way this const is actually becoming 0 because θ/= η = 0 okay.

So this constant becoming 0 so here you can find out second time integration which gives θ0 = A $e\text{-}^{Prn2/2dn}$ okay. Now if you put the boundary condition this will be obviously 0 this term will be zero. Now if you put the second boundary counting which is limit θ η =1=a can be evaluated $e^{-Prn2/2dn}$ okay A becomes like this.

Let us reduce it further to the integration okay here we are having –Prn2. So let us try to consider Pr η 2 /2 = t2 okay. So here we get d η √2/Pr x dt okay so if you take out this from the integration it becomes √Pr/2 /0∞-^{t2dt} so from mathematics we can write down this is nothing but √Pr/2/√π/2 so after cancellation it becomes √2Pr/π okay.

So A value we have got so ultimately we can write down θ in terms of A. A will be replaced over here into –Pr η $^{2/2\ d\ \eta}$. So θ we have written in this fashion now let us try to put this one in the terms of t2 so we have brought e-t2 over here leaving to it changing from o to η.

And now it will be changing 0 to √Pr/2 η and dη we have changed into √2/Pr and this constant remained over here. So if you simplify it will be becoming θ=√π and this integration will be prevailing over here okay. Further if you see the derivative of this one should decide nothing but nothing a^{*2} _a*paper /2 and a will put over here become and q hit flash at the wall.

And the temperature is gradient is in the at the wall and 0 means at the wall this in nothing but wall is flak so if you put this 0 over here it will be coming 1 so it is nothing but okay so the wall heat flux is coming in this fashion now if you tried to find out a assent no so the assent no is nothing but 8gx /k and if you once again do all those all the exhibition.

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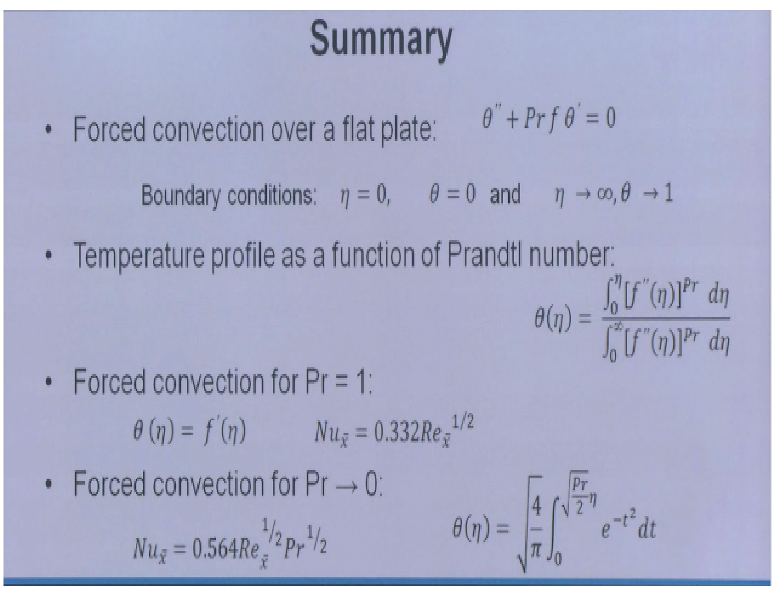

it well be first becoming and then x will be becoming in the q all this thing will you do then 1/ 1 it can be reduced final form 1/3 and external to be half but before half okay now from here if you see what you want by so it will coming by 1.564 so the accent number and we can get in the order of red to before the half and the number before to the half from here 1/root comes 1.5 and this is for the in the number case for the pedal number it comes to one we have shown in the assent no but x *in to red before half actually but here it is depend on the pandas number okay.

So summaries we have learned in this course so first convection flat plate this equation is derived it us star plus mantel number it is treated in =0 if the boundary condition q o+9and we also shown generalize temperature and profile so okay in terms of panel number so you have to so the first one is obtaining this is the equation and then all the moisturized and whish assumptions and then is manual and then okay.

So first we are having see the panel number is coming so 0to red, so 0 to becomes constant and the value is also there okay now we have studied then we have studied two pander no =to1 case so we are shown in the case in case of pander no is this is the value of a =to 1 so the q is the actually f – okay and now the accent number in to the red number –half we have shown as constant and 1.332v okay.

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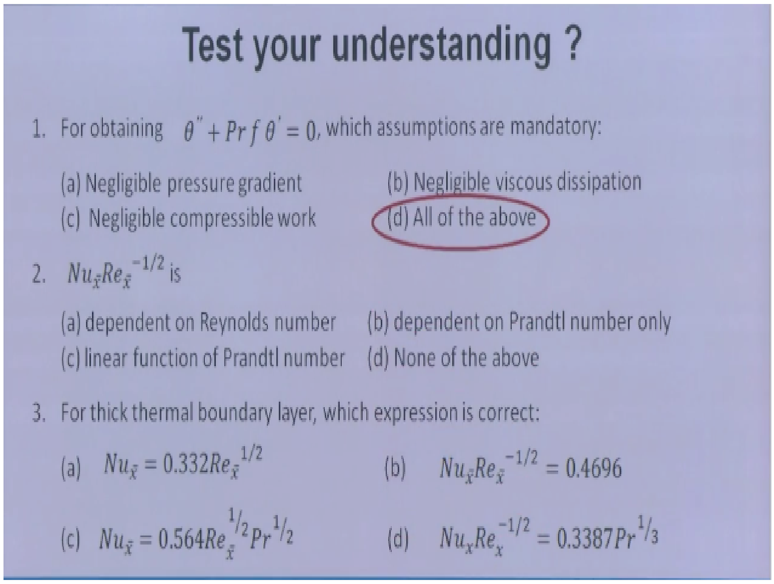

And for pander number takes to 0 that means that is low and then and o branded case we have showed at the accent number and actually in the 1.564 and then root paid by then red but half pander no and what to before half and the profile q is showered in this fashion and this was the constant and before that and then intimation and in the terms so this profile can obtained in the q by considering and in the numerical scale okay so how much they understood and then lecture in this like a people and this is terminal .

And so the discus in the compassable so the more work or a so we surely if and this all the things so okay in the a compatible silks and it will not be and it will be not there correct and service and then second portion and red to be half and actually depend on and dependent on so a linear and a panel number so that have it is dependent on the comely so if you have seen is actually in the panel number and there is the number and the panel number.

Okay for thickness for thick thermal boundary it is the correct I have given in the many expression for the rebuilt number and thick boundary number I have seen .66/2v and then this is the correct answer okay with this I will be ending this lecture so next lecture thing about the paned number so please keep in the for this hard discussion thank you.

For Further Details Contact

Coordinator, Educational Technology Cell
Indian Institute of Technology Roorkee
Roorkee-247 667
E Mail: etcell.iitrke@gmail.com, etcell@iitr.ernet.in
Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team
Sarath. K. V
Jithin. K
Pankaj Saini
Arun. S
Mohan Raj. S

Camera, Graphics, Online Editing & Post Production
Binoy. V. P

NPTEL Coordinator
Prof. B. K. Gandhi

An Eductaional Technology Cell
IIT Roorkee Production
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