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NPTEL ONLINE CERTIFICATION COURSE

Convective Heat Transfer

Lec-04
Thermal Boundary Layer

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Hello welcome in the fourth lecture of convective heat transfer. In this lecture we will be discussing about thermal boundary layer. Let me first give you the outline what we will be covering in this lecture.

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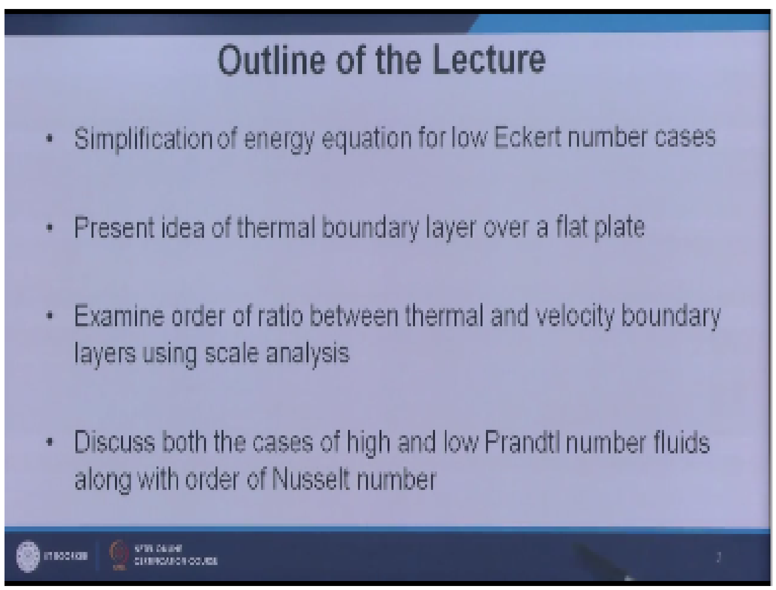

First we will be starting from energy equation and we will be simplifying this one for low Eckert number cases. Next we will be presenting the idea of thermal boundary layer over a flat plate, then we will be examining the order of ration between the thermal and velocity boundary layers using scale analysis. And at the end we will be discussing the case of high and low prandtl number fluids along with the nusselt number correlations.

So let me start from the previous lecture end points, in the previous lecture we have discussed about thermal energy equation.

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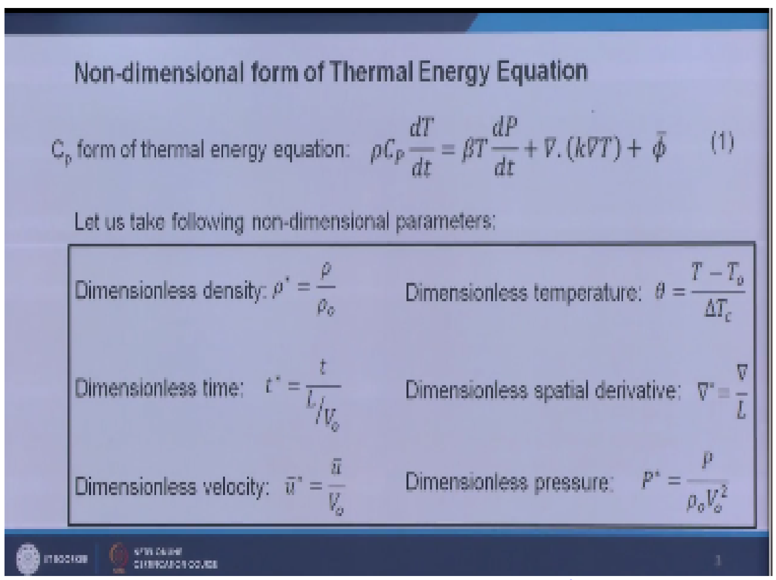

So thermal energy equation we have derived two forms will be starting from CP form and we will try to non-dimensionalize that one using different parameters. So in this equation you can see in the left hand side we are having conviction, right hand side we are having the complex ability term and then we are having the conduction and finally the viscous dissipation ϕ. So first let us see what are the non dimensional parameters we are going to use for density we are using $\rho^*=\rho/\rho_0$, ρ_0 is the characteristics density let us say for time we are using $t^*=t/L/V_0$ so V_0 is the characteristics velocity and L is characteristics length.

Then dimensional as velocity as I have already told V0 is the characteristics velocity so this is Ubar/V_0. For temperature we are using θ, so θ is nothing but $T\text{-}T_0$ which can be talked as reference temperature and ΔT_c is the gradient of the temperature. For spatial derivative, so you know spatial derivatives are coming over here as ∇, so ∇ can be non-dimensionalized as ∇*, so ∇* will be ∇/L so that means ∇∇X will be 1/L(∇∇X).

For pressure we are using P*=P/ρ0($V0^2$) so with this non-dimensional parameters later let us reduce this equation.

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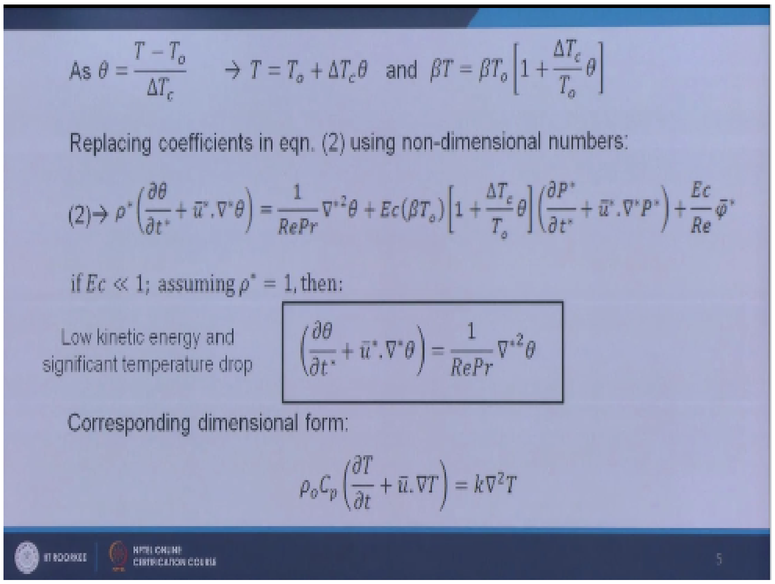

So if we substitute all those non-dimensional parameters in equation 1 what I have shown you in the previous slide, so we get this type of equation. So for the conviction term you see we are having ρ0(ρ*) after non-dimensionalization T became θ so as there is a ΔTc came over here, we are having ∇∇T and ∇ over here that gives V0/L over here. And in the right hand side for the conduction term we are having K ΔTc/L^2 for ∇P ∇T term we are having ρ0 $V0^2$ over here and for ∇∇T we got V0/L and for viscous dissipation we are having $V0^2/L^2$ okay, βT came over here which was earlier in the equation number 1.

Then let us divide the equation throughout by this term ρ0CP ΔTc V0/L if you do so then you will be finding out the equation is simplifying in this fashion where left hand side is conviction, right hand side we are having conduction, this is the pressure gradient term, total derivative has been expanded over here and this is a viscous dissipation term okay. Now let us try to find out where we are having non-dimensional numbers in this equation, equation number 2.

So first let us see what are the different non-dimensional numbers we use in heat transfer analysis. So in case of heat transfers comes the peclet number which is nothing but the ratio between the advection heat rate and the diffusive heat rates okay. So it can be written as LV0/α, α is your thermal diffusivity which takes care about their advection and diffusion heat rates okay. So here you see we have reduced this LV0/α in terms of LV0/ν and ν/α, ν is your kinematic viscosity.

So here the first part of this equation comes out as Reynolds number, because we know Reynolds number is LV0/ν and then here it is ν/α which is nothing but your prandtl number okay. So we can write down peclet number is equals to Reynolds number in prandtl number. Then another non-dimensional number will be introducing over here which is nothing but Eckert number, Eckert number gives you the inertia to the enthalpy drop ration okay.

So here Eckert number is $V0^2$/CPΔTc okay, as we are using CP form Eckert number will be very, very important okay. So here you see in the equation if you see minutely here we are having V^2/CpΔTc so we can write down Eckert term over here in front of this pressure gradient term okay. Then if we look at the viscous dissipation coefficients, so viscous dissipation coefficient is νV0/LCpΔTc so that can be reduced as V^2/CpΔTc which is nothing but the Eckert number.

And ν/V0/L which is nothing but 1/Re okay we should have introduced over here LV0/ν=Re. So this coefficient can be written as Eckert number by Re. So we have seen that different parameters can be written different coefficients can be written in terms of the non-dimensional numbers. So if new put all this non-dimensional numbers then we get this equation okay. So here you see we are having 1/Re here which is nothing bu peclet number.

Here we are having Eckert number and here we are having Eckert by Reynolds number. For βT we are doing this type of approximation from θ non-dimensionalization we know θ is nothing but T-T0/ΔTc, so T-T0/ΔTc can be reduced in this fashion T=T0+ΔTcθ and βT can be written as βT0(1+ΔTc/T0)θ so if you replace this βT in terms of this, so this coefficient comes over here right. Next let us see what happens whenever we are having very low Eckert number okay, this is our object to see that what happens to low Eckert number.

So low Eckert number means lowwe kinetic energy and significant temperature drop okay Cp(ΔTc) will be coming very big. So in that case we get the equation little bit modified so this term and this term can be neglected, because low Eckert number we are having. And if we consider that we have chosen ρ0 in such a fashion that density non-dimensionalized density ρ* becomes 1 then this equation 2 is actually reducing to this form.

So ρ* becomes 0 and these two terms can be neglected compared to the others. So we get this simplified equation where in the left hand side we are having conviction and in right hand side we are having conduction. And as non-dimensionalized number we are having Reynolds number into pandtl number okay. So this is very, very important equation. If we see the corresponding dimensional form of this equation, so it will becoming left hand side it is nothing but conviction ρ0xCpxΔTΔT+U.xΔT so this is the conduction term. And the right hand side we are having K(Δ^2/T) so this part is coming from here, so this is conviction term right.

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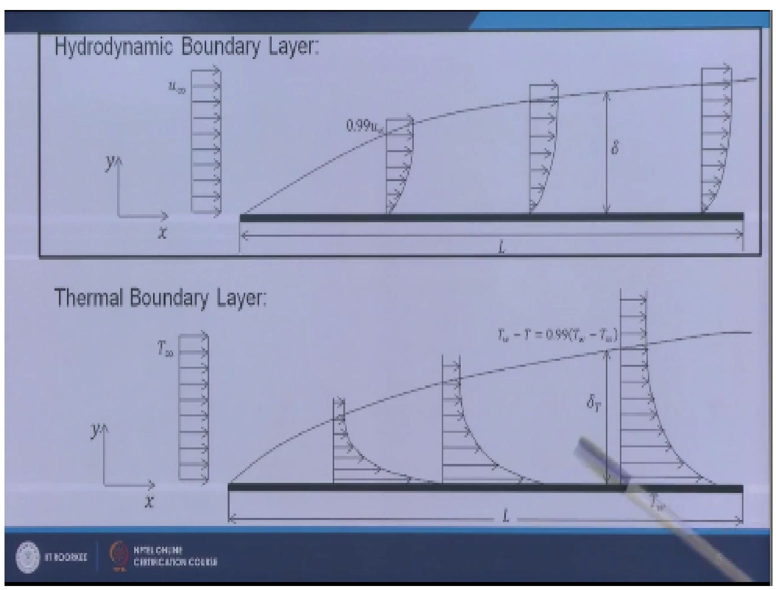

So let us now see from our knowledge of fluid in mechanics because this is the PD quiz of this course so from fluid mechanics knowledge will know what is hydrodynamic here this figure is very known to you so we are having a flat it okay so this is the flat and they first time full is coming over here having velocity you know u∞ let us consider this is our next direction in the most direction and this is the costal more direction y okay so you can find out that utility whenever the flow is coming in contact to the solid surface.

So there the velocity will be coming 0 due to mostly condition so here velocity becomes 0 if you were okay and far away from the surface will be having one second more instance so this will be your u∞ velocity okay so there will be velocity gradient and you know from our fluid mechanics knowledge that is the probably profile okay and if you see that where this velocity is becoming 99% of the fisting velocity that we need and call as hydrodynamic bound here so here I have showed hydrodynamic boundary layer in this fashion.

So this is already known to you from your few mechanics knowledge now let us see what is our thermal boundary layer okay which is the part of this course so in case of thermal boundary layer we will taking the similar type of flat layer okay so this is the flat plate and so characterize of this flat plate is having temperature T_w okay A first time fluid is coming over the flat layer having temperature T∞ okay and you see Tw is higher than T∞ for this case we have considered so whenever the fluid is coming in contact in this hot layer.

So immediately the adjacent clear of this layer will becoming very high in temperature okay so here we are having very high temperature and away from the surface we are having similar T∞ so there also will be having layer temperature gradient and this temperature slowly will be turn to this T∞ and at a point where temperature becomes 99% of T∞ okay we can call so this is your thermal boundary layer thickness okay ΔT so if you continue finding out those points which is having 99% of the this temperature and boil those like this so that is the look us of those points which is nothing but your thermal boundary layer.

So just like for velocity boundary layer you can have thermal boundary layer and just like your velocity boundary layer thickness we can have the thermal boundary layer thickness okay so this is very important concept so thermal boundary layer we can see over here.

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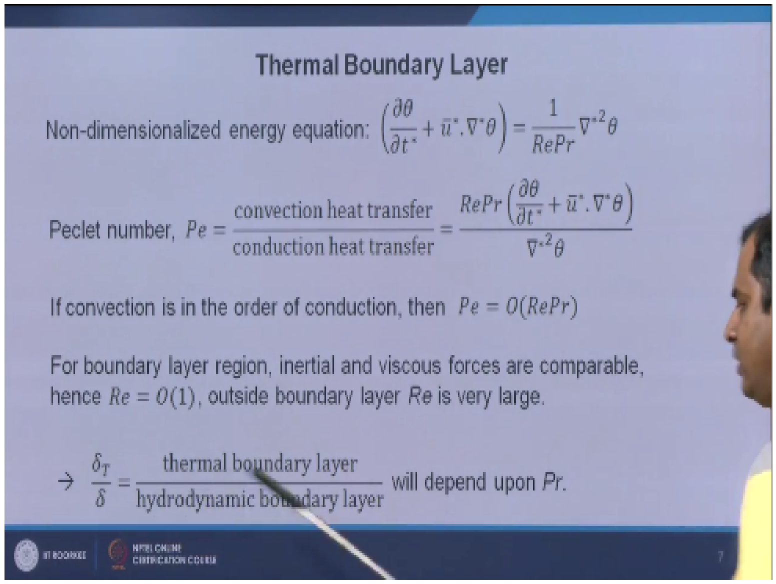

Next let us see from thermal boundary layer point of view we have earlier derived this equation Non- dimensionalized energy equation for ∇ at number cases now let us try to see over here what is the ratio between the left hand side and the right hand side left hand side we had the condition right hand side we are having conduction so convection to conduction which is nothing but the Peclet number advection to diffusion which is nothing but Peclet number so it comes as iPr x $\nabla\theta/\nabla t + \nabla^* x\nabla\theta$ which is nothing but your convection term and in the conduction your having $\nabla^2\theta$.

Now we know that Peclet number is nothing but Re x Pr okay so what we can see over here if convection is of the order of the conduction then we can write down Peclet number is of the order of Re Pr because this will be of order of this one okay so convection is of the order of conduction so Peclet number will becoming order of Re x Pr from our velocity boundary layer knowledge we know that Reynolds number inside the boundary layer Re and where inertial viscous which is our more or less comparable the Reynolds number will be of the order of one okay so Re is of the order of 1 so but outside the boundary layer but inside the boundary layer is order of one.

Now if you try to find out the ratio between the thermal boundary layer thickness and the velocity boundary layer thickness of hydrodynamic boundary layer thickness so it will be depending on all the plandle number because here we have found out Re is order of 1 so ultimately this plandle number actually defined that what will be the ratio between the velocity boundary layer and the thermal boundary layer okay so δT/δ is actually a function of pandle number.

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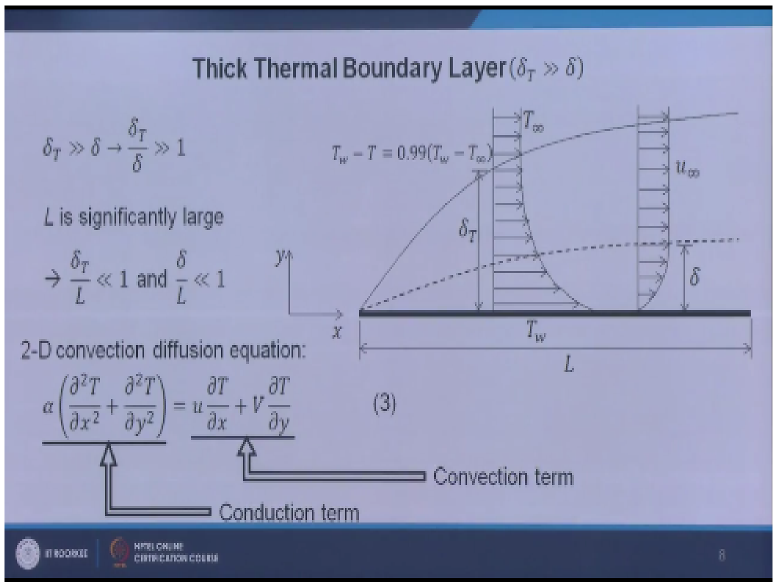

Now let us see what happens for two different cases where thermal boundary layer dominates and then we will see where velocity boundary layer dominates so first we will see a thick thermal boundary layer cases where δT is very, very higher than δ a schematic representation I have showed over here so here you see this is our velocity boundary layer okay so at this point the velocity of the fluids T will becoming almost 99% of the Piston velocity and here this is our thermal boundary layer okay so this is the look us of all those points where temperature will be coming almost 99% of the free temperature but here one important thing is here that δT is higher than the δ okay no in this case let us try to see that what happens in this competition so from the figure we have already shown δT is much larger than δ.

And we can write down that δT/ δ is actually δ greater than 1 now if you take plate very long so it is very long plate let us l is very high so δT/L is actually less than 1 and δ/L is less than 1 okay significantly large now let us see for 2-D convection diffusion equation called it about so we thinking about 2-D problem over here so this will I mean your conduction term and this is your convection term okay.

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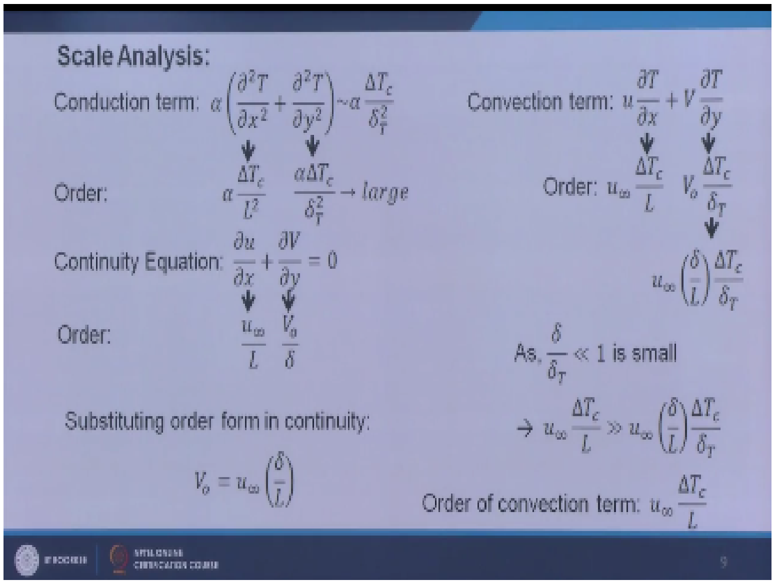

Now let us see the scale of both the cases okay so if you see the scale so in conduction term we are having α x ∂^2T/ ∂x^2 and in convection term we are having this one okay so individually if you try to find out the scale so α remains over here for the first term ∂ for this T will be having ∂Tc and for this x will be having L^2 because L is x of the steam moist reduction okay and for the second term will be having α ∂Tc and for y which is actually the perpendicular direction our length scale is α x ∂T which is nothing but your thermal boundary layer thickness.

Okay so if we compare between as L is much larger compared to α x ∂T so this with the dominant term okay so our conduction will be of the order of the α x ∂Tc / α x ∂Tc^2 similarly if we do for the convection equation by the way before going to the convection equation let us see the continuity equation first so if you see the continuity equation you know from fluid mechanics so ∂u ∂x + ∂V/ ∂y is equals to 0 so from there we know order of the first term will ∂u ∂x will becoming u∞ / L and the second term will be becoming V_0 by δ okay.

So if we equate and try to find out what is the order of this V_0 so we will be finding out V0 not comes out s u∞ δ/L but here for a y direction we have taken a two velocity boundary layer thickness δ so once you find out V0 and we can now get the order of the convections terms so here the first term becomes $u\infty$ ΔTc/L and second term become for V we are taking V_0 so Vo into the Δ_TC/ ΔT remember for here we have taken ΔT for the perpendicular direction now it is V0 and we have already found out in terms of in out let us try to reduce this so replace of V_0 we have to $u\infty$ δ/L from here okay.

And if we find out this actually becomes u0 ΔTc/ L which is this term multiplied by Δ/Δ_T now we have consider initially that Δ/Δ_T is actually very small quantity thermodynamic boundary layer actually bound here so what the velocity boundary layer, so in this case we will be finding out that this term is actually becoming the dominant term for the convection, okay. so this is lesser this is greater than this one okay, so this term becomes sorry this term become the greater term compared to this one, okay.

So our convection becomes of the order you mean $u_\infty \Delta T_c/L$ okay conduction already we have told $\alpha.\alpha T_c/\delta_T^2$. So from the equation if we see the scale analysis from both the sides we can have a comparison between the scales of conduction and convection and if we do so then we are finding out this was my convection.

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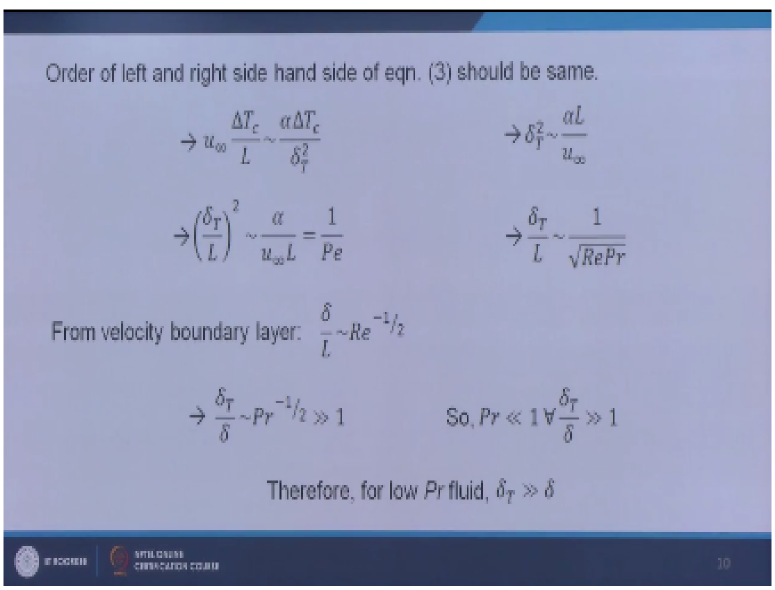

And this was my conduction okay, so from here we can get that δ_T^2 after reducing from here it cancelling out ΔT_c from both the sides, we get $\delta_T^2=\alpha L/u_\infty$ okay, so we know that $\alpha/u_\infty L$ is becoming 1/Pe number so what we have done we have divided this whole equation by L^2 so we get $(\delta_T/L)^2$=1/Pe number, right. Now we can write down δ_T/L is of the order of 1 root over prandtl number or √RePr. From velocity boundary layer we have already knowing that δ/L which is hydrodynamic boundary layer by the stream voice lens scale is actually $Re^{-1/2}$ okay, now if we mark this two equation then we can get the ratio between δ_T/δ so if you do so.

So you get δ_T/δ is of the order of $Pr^{-1/2}$ earlier also we have told you we have shown you that this ratio between the thermodynamic boundary layer and velocity boundary layer will be becoming function of prandtl number here also we are saying so but the order is something like this $Pr^{-1/2}$. Now at the beginning we have considered that the thermodynamic boundary layer actually dominates so δ_T is higher than δ, okay.

So this ratio actually is greater than 1 so that means what we have found out $Pr^{-1/2}$ is actually greater than 1, this is the case where prandtl number is very small okay, so for satisfying this one prandtl number needs to be very small. So prandtl number very low then only we will getting the thermodynamic boundary layer has actually dominated the velocity boundary layer, okay. So therefore for low prandtl number fluid thermodynamic boundary layer thickness is higher than the hydrodynamic boundary layer.

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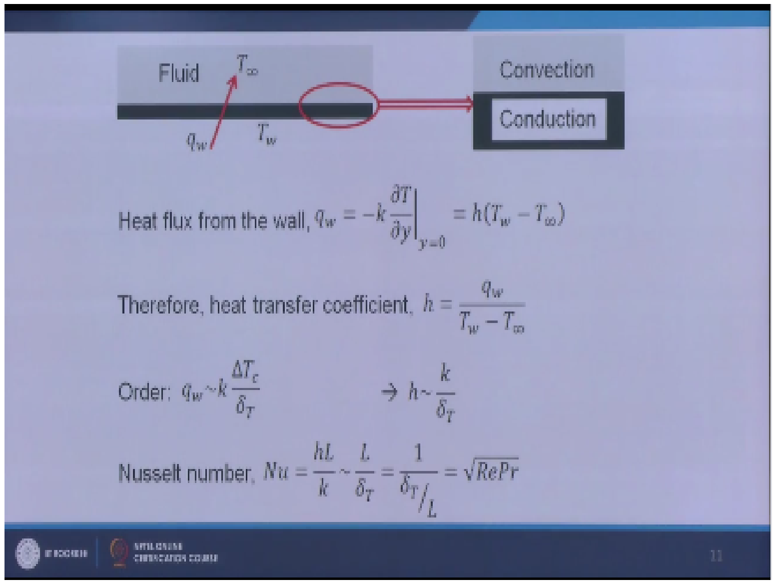

So if we try to see the heat flux so in this case so let us say the heat flux is wall hit flex is q_w so if we try to find out that what is happening in case of the heat flux so q_w heat flux is nothing but from the conduction we can write down $-k\partial T/\partial y$ at $y=0$ because here nearer the plate the conduction dominates and immediately adjutant to this plate we are having fluid layer which is having the convection so from convection we can write down $h(T_w-T_\infty)$.

So from here we can get h heat transfer coefficient is nothing but q_w/T_w-T_∞ okay, so q_w from here we can see is of the order of k ∂T and y ∂y will be coming as ∂T okay, so $\Delta T_c/\delta_T$ so this is the order of the q_w right. So if we put in this side from this two equation of this order the heat transfer coefficient find out if the order of k/δ_T okay. And if we try to see what is the Nusselt number, Nusselt number is a one dimensional number which gives rise the convection to conduction resist answers.

So Nusselt number is hL/k so from this equation we can find out what is hL/k and from the order analysis whatever we have found out over here h is the order of k/δ_T we get Nusselt number is of the order of $\sqrt{RePr}$, okay. So in case of low prandtl number fluid we have got Nusselt number is of the order of RePr, okay.

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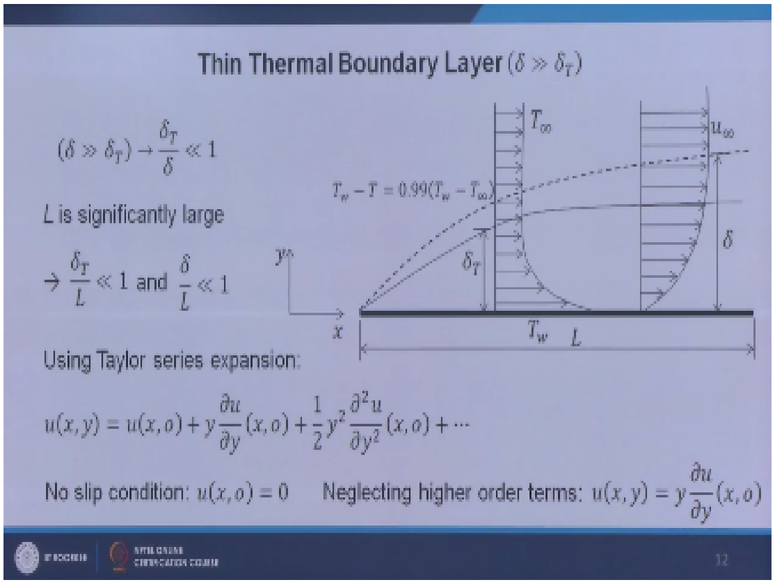

Now let us see the other horizon that means thin boundary layer thickness that means the velocity boundary layer will be actually higher okay, δ is higher compared to the thermal boundary layer thickness over here so this is the thermal boundary layer over here and here this dotted lines shows the velocity boundary layer, okay corresponding temperature profile and velocity profiles I have shown over here, right.

Now let us see this horizon so as we have considered δ is higher than δ_T so δ is higher than δ_T so I can write down δ_T/δ is actually lesser than 1, okay. So if L is significantly large than we can take δ_T/δ is smaller than 1 and δ/L is smaller than 1 and in this case we will be going for Taylor series expansion so if we do the Taylor series expansion for u we can write down in this way okay, so u is nothing but $u+y\partial u/\partial y+1/2y^2+\partial^2 u/\partial y^2+$ so on the higher order terms, okay.

Now here at the wall we know for no slip boundary condition it will be u at wall y=0 it is 0 so this term can be cancelled, okay so we get u is actually of the order of $y\partial u/\partial y$ if you neglect the higher order terms, okay.

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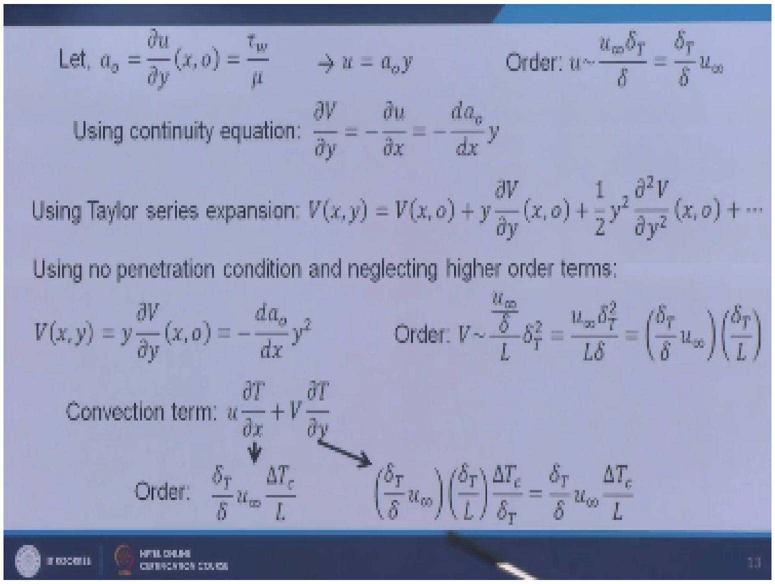

Now let us take this $\partial u/\partial y=a_0$ okay, so from fluid mechanics we know this $\partial u/\partial y$ is nothing but τ_w/μ where τ_w is the walls he has stress, okay. so we can write down $u=a_0.i$ and finally if we try to find out the order of u which is nothing but $u_\infty\delta_T/\delta$ and here δ_T/δ comes as ratio between the thermal boundary layer thickness and the velocity boundary layer thickness, okay.

Then if we try to use continuity equation so from continuity equation we have written $\partial v/\partial y=\partial u/\partial y$ this will be going in hand side so that is the continuity equation so $\partial u/\partial x$ I have, I can write down as $u=a_0y$ so $\partial u/\partial x$ will be becoming d/dx a_0 and y will be becoming as free, okay. now if we try to use the Taylor series expansion for V just like the u as I have shown in the previous slide, so here you can find out V.y $(\partial V/\partial y)+1/2.y^2\partial^2V/\partial y^2$ and so on okay, now if we use no penetration so once again you can find out that this terms will be 0 at the wall okay because no penetration will be there if it is for as then other cases whether cases will happen but as it is no penetration so this term will go to 0.

So we find out these becoming y x ∂v ∂ y okay if we neglect the higher order term And already we have shown ∂v ∂y is nothing but – of ddx (a_0) x y, so we can write down -ddx(a_0) x y x y y^2 right, so if we try to see the order of the v now so it becomes for a_0 the order will be something like u/l so u/ δ okay and here this x is coming as l and y as we are talking about thermal amount of thickness so y comes as δt^2.

So remember whenever we have considered this y, so this replace by δ and whenever we have consider this y it is replaced by δ t2 okay. So v becomes of the order of δ t / δ x u ∝ multiplied by δ t / l okay. Now if you see the convection terms for this case where hydraulic boundary are dominates so in this case the convection terms take the order of this one δ t/ δ x u ∝ x δ uc/l for the first term u is the order of this one and here we have shown and it is becoming δ tc/ l.

Second term v is becoming the order of this one so this we have put over here and δ tc/ y become δ tc/ δ t okay so this is the order of the second terms this is the order of the first term so both are same so convection terms will be taking this order okay..

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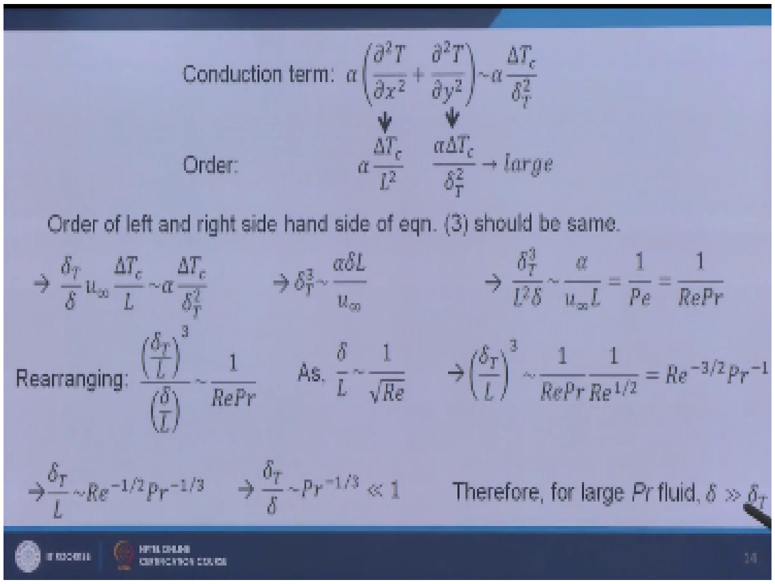

For conduction term here you see this is the order of first term α x δ tc /l2 because x we are having so this is l and for this y we can take δ t2 now we know δ t is smaller compare to l so this will be the dominant term okay. And once again this is the dominant term this is the order of the conduction term, so once again you can equate conduction and convection so you get this type of scale analysis.

So from here we can obtain δ t q is actually alpha δ l / l ∝ okay of the order of, so from here if you do little bit multiplication and modification so you can get δ tq / l 2 δ is of the order of 1/Repr okay just from this one it will be coming okay. Now you see already we know that what the ratio between δ is / l okay if you just little bit re arrange this one so this can be δ /l3 and δ / l so this δ /l already we know from our velocity boundary earlier concept okay which is the order of 1/ re √re okay.

So using this two we can get δ t/l 3 comes of the order of Re -3/2 rundle number to the power -1okay, so δ t/l comes out as re -1/2 and rundle number to the power -1 okay. Now if we compare between this two then we can get δ t / l becomes pr-1/3 okay so in this case where velocity boundary actually dominates for this case we have obtained this one now we already know that δ t / δ is actually smaller terms than one.

So this equation need to be satisfied Pr-1/3 < 1 needs to be satisfied so this can happen only whenever you can having large pr number fluid okay, so for large pr number fluid we are having δ > δ t.

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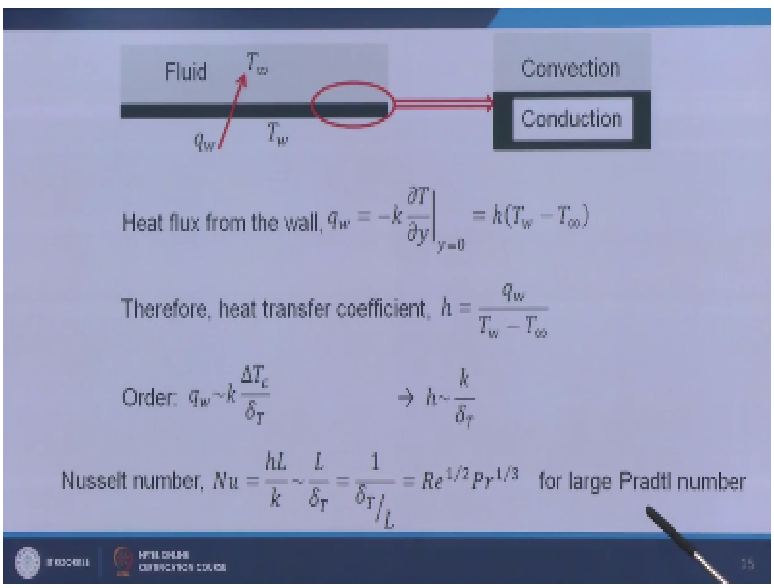

So once again if we can try to do the same analysis of heat flux so we can get heat flux is equals to conduction and convection equation just like the previous case so h becomes qw/tw – t ∝ if we go for the order this is qw comes out as k δ tc/ δ t and for the convection term it becomes k / δ t okay if you it mate the if you find out the Nusselt number, so Nusselt number becomes in terms of Reynolds number to the power half and Prandtl number to the power one third. So remember Nusselt number is comes out for a large Prandtl number right okay.

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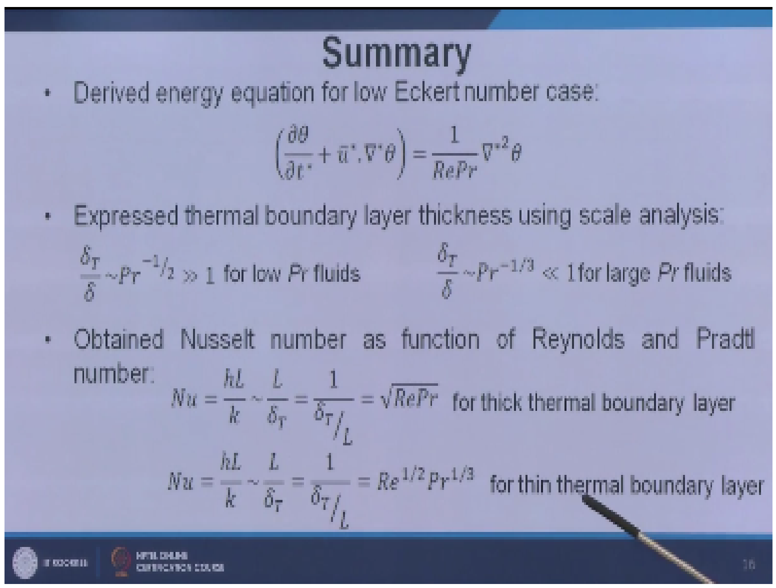

So let me summarize that what we have learnt in this so we have actually derive the energy equation for low Eckert number cases so this equation we have derive for low Eckert number cases by neglecting two terms over here okay for pressure Gradient and the discus dissipation and we have express the thermal boundary layer thickness using scale analysis okay so for low Pr number fluid we have found out it is the order of pr number to the power – ½ and for large pr number if we do have found out or this is the order of Pr number -1/3 oaky.

And we have obtain the Nusselt number function Nusselt number co relation in terms of Reynolds number and Prandtl number for thick thermal boundary layer we have found out

Nusselt number of the order of √RePr and for the thin thermal boundary layer compare to the velocity boundary always Nusselt number becomes the order of √Re x Prandtl number 1/3 okay.

So these things we have actually understood in this lecture. Now let me test how far we have understood this topic so we are having three questions over here.

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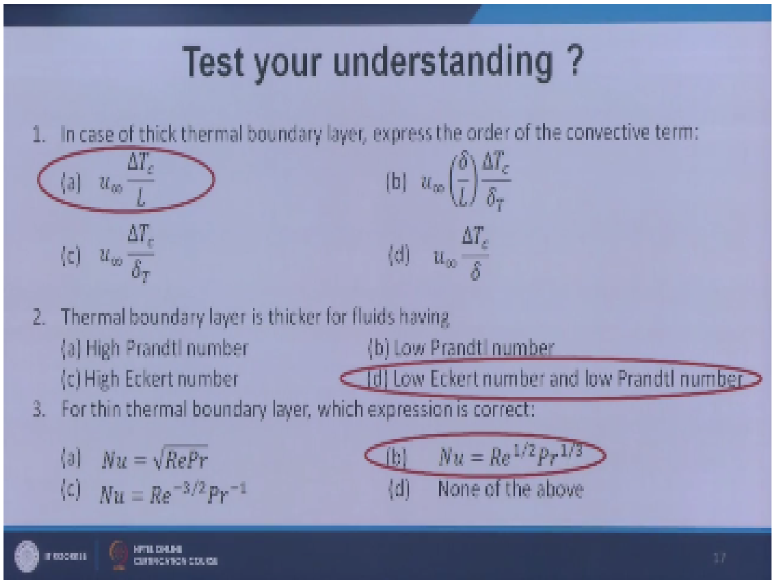

So first question goes like this in case of thick thermal boundary layer express the order of the convection term so the order of the convection term already I have discussed in the lecture four options we are having so if you go back to the discussion in this lecture earlier so you can find out the correct term is this one U ∝ δ tc / del okay. Second question thermal boundary layer is thicker for fluids having four options we are having high Prandtl number low Prandtl number high Eckert number and low Eckert number and low Prandtl number fluid okay.

So already we have seen that first assumption you need to take is low Eckert number and second assumption obviously for this thermal boundary layer thicker required is Prandtl number okay. So both the cases need to satisfied so this is the answer okay. For the third one for thin thermal boundary layer which expression is correct, so I have given four expression for the Nusselt number and last one is obviously of the above.

So if you see my nuclear correct answer is this one for this thermal boundary layer okay so Nusselt number is the order of RE1/2 and Prandtl number to the power 1/3 okay. So thank you at the end of this lecture please visit our next lecture which we will be discussing about low Prandtl number over a flat plat okay and if you are having any question please keep post in our discussion forum than you.

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