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Convective Heat Transfer

Lec-03
Difference Forms of Thermal Energy Equation

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Hello welcome in the 3rd lecture of convective heat transfer in the pervious lecture we have learned about derivation of terminal energy balance equation we have seen their how thermal energy balance equation can be written in terms of internal energy equation and enthalpy balanced equation from that Same point we will continuing over here and we will see some other forms of internal energy balance equation over here.

So your topic for this one lecture is different forms of thermal energy balance equation so as I will be continuing from the previous lecture I will following the same equation numbering over here in this lecture so please refer to the pervious for the previous positions okay let us see what will be the outline of this lecture at the beginning we will be recapitulation.

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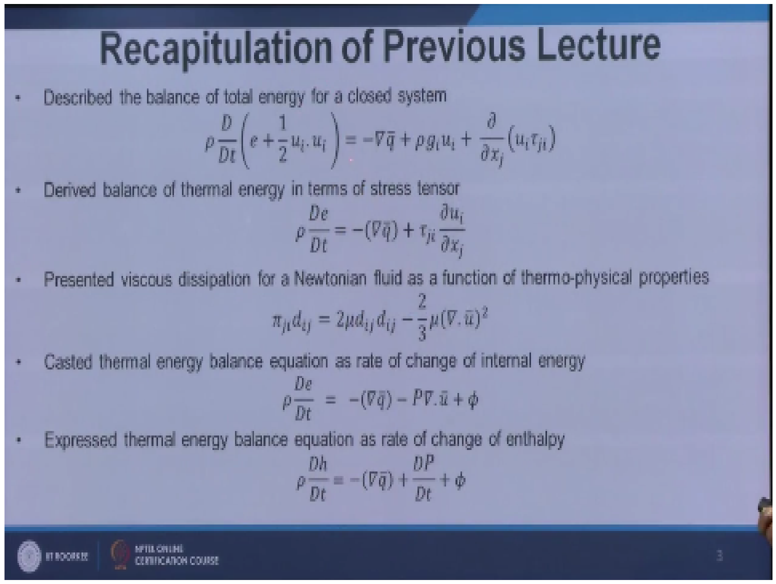

Of the equations whatever we have covered in the previous lecture okay so we will be showing you the final form of thermal energy balance equation okay in the internal energy balance form and enthalpy balance form okay we will be using Tds equation over here for derivation of entropy balance okay equations so not entropy balance I should say the terminal energy balance equation in the form of entropy okay.

The we will be using Maxwell's relationship okay to obtain a thermal energy balance equation using Cv okay also towards the end we will be converting Cv form of thermal energy balance equation to Cp form of thermal energy balance equation here also we will starting from conservation of thermal energy balance equation and from there using Maxwell relationship we will be bring that equation into simplified form using Cp right.

Okay so let us first go to the contained of the previous lecture the previous lecture we have actually described the balance of total energy for a close system so this scheme in this form in the left hand side we are having the rate if change of energy in storage and in the right hand side we are having the energies in transit first one is energy due to heat transfer and second one is energy due to body force okay which is nothing but work done due to body force and third term is actually work done due to surface forces okay.

Then we have a actually subtracted from this total energy balance equation our mechanical energy balance equation and we have obtained out thermal energy balance equation so this is balance of thermal energy so we get in the left hand side we are having ρ de dt and in the right hand side your energy in transit due to heat transfer will be remaining and this is nothing but your derivation part so you will be having τji ∇ xj of ey okay.

Next we have also shown that how viscous dissipation Π ji dij which I nothing but your deformation tenser in between we have also said that rotational tenser is anti symmetric that can be cancelled so this last term of this thermal energy balance equation which is the derived part that as simplified to Πjidij and that we have showed for Newtonian fluid how that can be connected with themo-physical property μ okay over here and here we have also derived for there and shown that how this equation can be casted in terms of internal energy balance equation so here you see e is the internal energy okay.

The term which was there as Πjidij that we have kept here as ϕ and we call that one as vicious dissipation okay also this equation we have recasted once again and we have expressed this thermal energy balance equation in the form of enthalpy so here you see h is the enthalpy okay some changes in the right hand side occurred due to the change e to h okay we have done that using the relationship between e and h in the previous lecture okay from this point we will be continuing in this lecture so in this lecture we will start first conversation of this thermal energy balance equation into entropy based balance of thermal energy equation okay.

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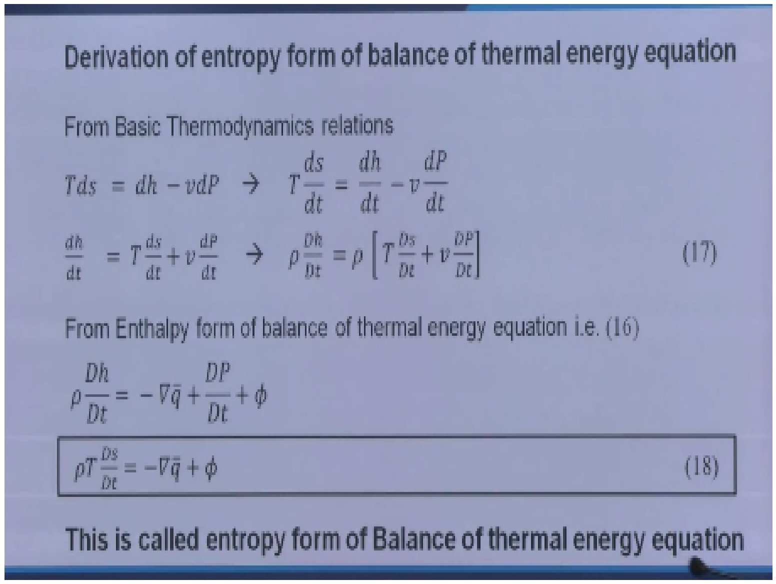

So from basic thermo dynamics okay we know that Tds relationship is like this Tds = dh= vdp okay so here you see in the right hand side we have enthalpy and left landside we are having entropy so this equation we will be using for shifting this enthalpy based equation to entropy based equation okay now from here we can write down once again Tds / dt by divide making derivative with respect to time so Tds/ dt is equals to dh dt – v Δ Δt of p right.

So before we proceed further then we can see by doing little bit change of sides we can write down dh dt = dds tddt of s + v into ddt of p okay from this equation little bit change of sides we can obtain this one next multiply ρ in both sides okay so you will be getting ρ is being multiplied both sides okay so equation come in this form here you see I have converted small ddt to capital DDT total derivative for some purpose I will be explaining later on.

Next from enthalpy balance equation whatever we have actually shown in the last lecture you can get that ρ DDh see I have used in the last lecture capital DDT total derivative so here also I have actually shifted from this small ddt to capital DDT okay so in your last lecture we have shown that enthalpy form of balance of thermal energy equation is like this ρ dh dt is equals - ∇q+ DDt of p+ ϕ was the vicious now in left hand side we had ρ into dh dt or ρ ddt of h which is equivalent to this one from basic Tds equations we have proved that this can be linked up with DDT(s) entropy, right. So we will be shifting it over here okay in the left hand side so if you do that then you will be finding out this dp, dt can be cancelled out because ρ and v they are reciprocal to each other so ρ and v can be cancelled out and we got one dp, dt in the left hand side of this equation.

And right hand side we are having dp, dt so this term from both the sides can be cancelled and you finally obtained ρ, t of DDT(s) = - ∇q + Ø viscose dissipation, so in this equation you see we have once again done the balance of energies thermal energies rather but here this we have expressed in terms of S which is nothing but entropy, right. So in the left hand side we are having actually rate of change of energy in storage in the form of entropy.

In the right hand side we are having first term is actually your energy in transit via heat transfer and next one is actually viscose dissipation, right okay. So this equation we will be calling as entropy form of balance of thermal energy equation so this is very, very important vital equation entropy form of the thermal energy balance equation, so till now we have seen internal energy balance equation or energy balance equation in the form of internal energy.

Energy balance equation in the form of enthalpy and today I have shown you energy balanced equation in the form of entropy.

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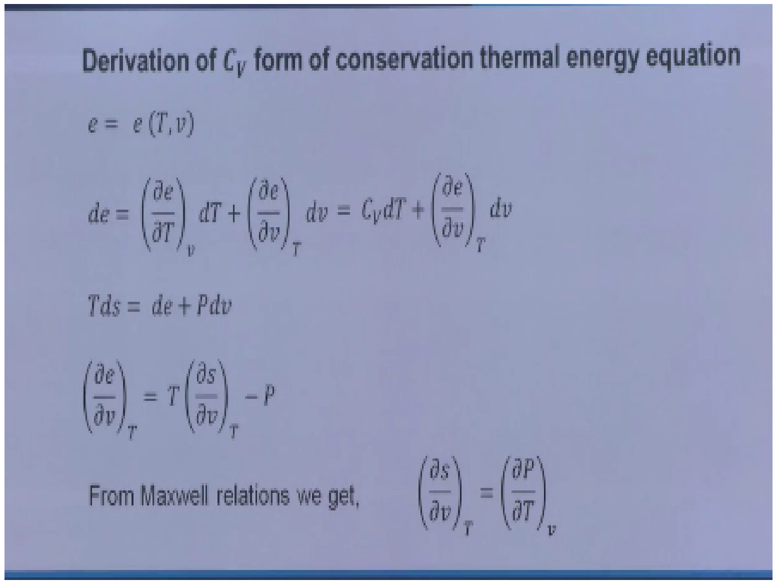

Next let us see some other forms for first I will be going for thermal energy balance equation in the form of CP sorry CV okay so derivation of CV form of conservation of thermal energy balance right so we will start this derivation from this one that internal energy E can be written as function of temperature and volume T, v right so if I write down like this and then if we try to find out the derivative of E.

As it is function of T and v this can be written as ∂T (e) at constant V .dt + ∂v(e) at constant t .dv right because it is also t and v only, okay. And here in this side we are having you see the first term is actually ∂, ∂T(e) that means you what amount of energy you required to change the temperature part unit degree okay, I am not mentioning about unit because here we are having dt so that is unit less.

So for unit degree change of temperature what amount of energy is required at constant volume, so you can relate that is equivalent to your CV so I can write down CV okay, so this is de = CV .dt will be remaining as it is and second term for the time being we have kept as it is over here, okay. So we have got de = CV. dt + (∂∂v(e) at constant temperature into dv, now let us do something else.

Let us start from once again our very famous thermodynamic relationship Tds equation so Tds = de + Pdv okay this all of you know from thermodynamics be equiseta then what we will be doing little bit change of sin and then divide by ∂v if we do then we can find out ∂∂v(e) at constant temperature is equals to T(∂∂v) of s at constant temperature –P okay, so I have actually taken this term in the left hand side.

This term in the right hand side divided by ∂v and over the whole equation I have kept the temperature constant so this equation appears, right. Next let us Maxwell relation to do convert this term into some other known form so if we do that we can write down one of the famous Maxwell relationship is like this ∂ ∂v(s) at constant T is actually ∂ ∂t(T) at constant v so you see here we are having this ∂ ∂v(S) at constant T we can substitute that one by ∂ ∂T(P) at constant V, right.

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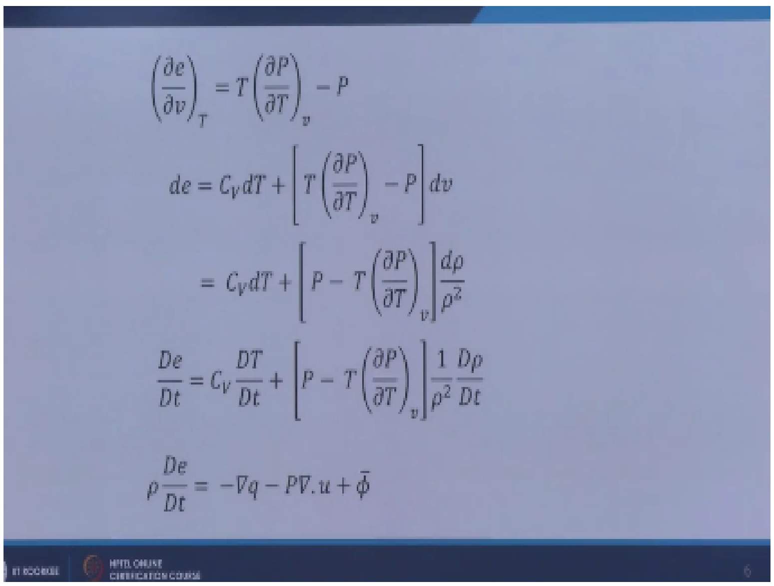

So if you do so we get this form so ∂ ∂ V(E) at constant T is equals to T x ∂ ∂ T(P) at constant V – P okay so this term we have actually got from Maxwell's relation, okay. Next if we just put this final form over here okay if we put this final form over here then we will be getting this equation so de = Cv. Dt + T(∂∂ T(P) at constant V – P and whole multiplied by dv okay, let us now do if farther.

So what I am doing over here we are having dv over here we are actually writing down that dv = $-(1/\rho^2)$ dρ because v = 1/ρ okay, specific volume = 1/ density, so we are writing this one so this has changed sign over here inside this third bracket and we have written in terms of dv, $d\rho/\rho^2$ right, so if we do it farther so de/dt so here also I have converted this small d to capital D this two match with the previous slide.

So this dt(e) is actually CV x DDT(t) here small t is time capital T is a temperature plus this term remain as it is $1/\rho^2$ I have kept dρ/dt will be coming okay. So if you remember our internal energy form of thermal energy balance equation was like this, okay. So in this we had dρ, dt(e) in the left hand side okay which was nothing but rate of change of energy in storage, okay. So here I have obtain some form of DDT(e).

So I want to substitute this left hand side whatever I have got here in the right hand side of this equation to this equation whatever we have obtained in the previous class for internal energy form of thermal energy balance equation, so if we do so then we get.

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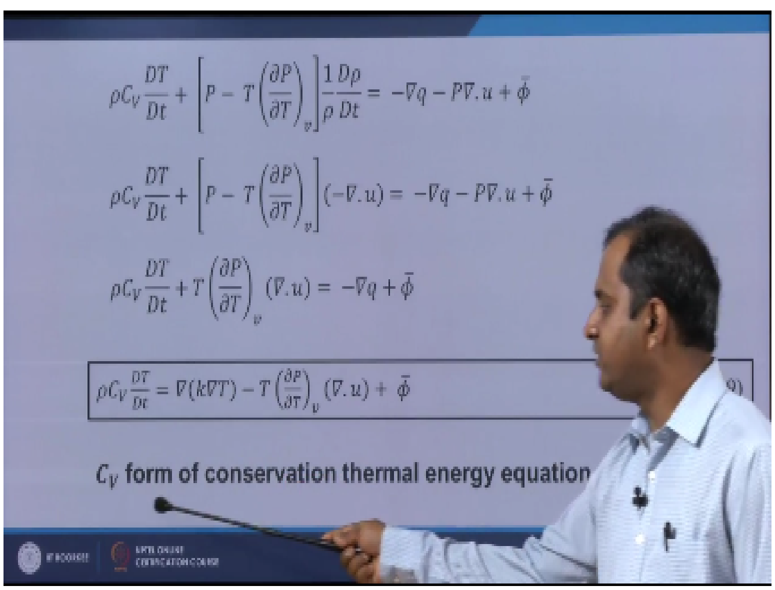

You see so this term is nothing but ρ multiplied by the left hand side term of this equation, okay. And here in the right hand side we are having all the energy in transits first one is due to your heat transfer second one is hydro statics part and the third is the deviated part right. if you simplify it further then we can find out over here this term whatever we had 1/∂dt∂ from continuation is nothing but –μ okay.

So one side write down the –μ I get PΔμ in the left hand side and I had PΔμ in the right hand side also so those can be cancelled so you get finalize simplify form ∂Cv Dt/Dt + this term and this term here T(∂P/∂T) constant volume. Heat transfer will be remaining in the right hand side and viscous will be remaining over here.

Hydrostatics part I have already cancelled from both the sides okay. Then you see if you take the second term of left hand side, so you get ρCv DT/DT which is nothing but a Cv form. In the right hand side in place of q I can write down the term. This is by following the furious law haet conduction.

Okay then the 2nd term in the left hand side actually came in the right hand side of – sign + viscous dissipation remained over here. Here it is important to mention the keys it is nothing but the thermal quantity. So you see these equation can be called has Cv form because Cv is involved over here Cv form of conversation of thermal energy balance okay.

So this is very important equation please remember this one. Next let us see that can we convert this into Cp form Cp form is nothing but heat at constant pressure okay. In the previous one we have converted that one into specific heat at constant work volume. Here let us see that can we convert this thermal energy balance equation in cp form or not.

To start with we will once again just like the previous one we have written e has a function of temperature and volume here let us write down has a function of temperature and pressure okay. And we almost follow the same analogy to make the derivative of h has

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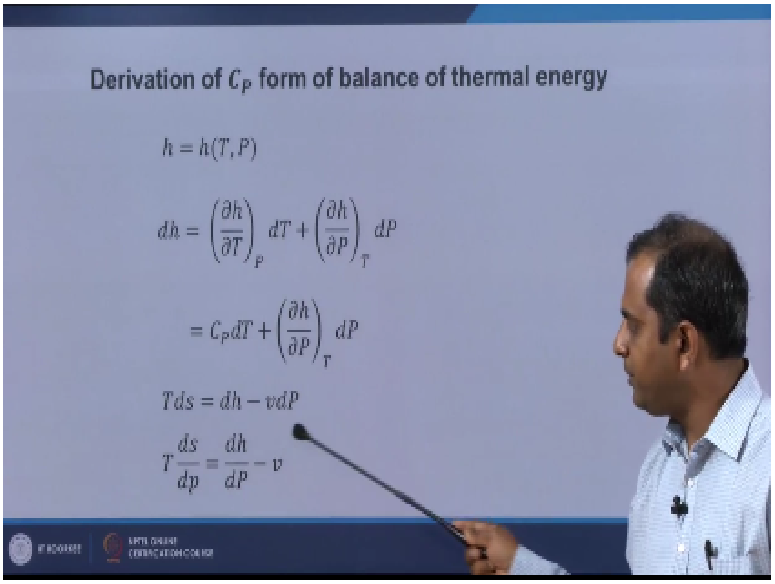

Dh=∂h/t p x dt + ∂h/∂pt dP right. so here also this ∂ of t at constant p is nothing but par unit degree change of temperature okay. What amount of energy is required h at constant pressure which is nothing but our Cp, so here I have written this term has Cp so this is Cp x dp + this term has it ∂p/ht x dp right so we have got ∂p/ht x dp. Okay next let us use once again our famous thermal energy equation but now this time we will be using the dh so Tds = dh-vdp okay.

If you remember in the previous description Cv form we have used Tds=de+vdp okay. Next here if you see we are dividing this equation both the sides by dp Tds=de+vdp t ds/dp= dh/dp –v okay. So this equation if we do little bit of side change means this you will take into left side and this will you take in right so you will be getting this equation okay. Next once again try to use Maxwell relation.

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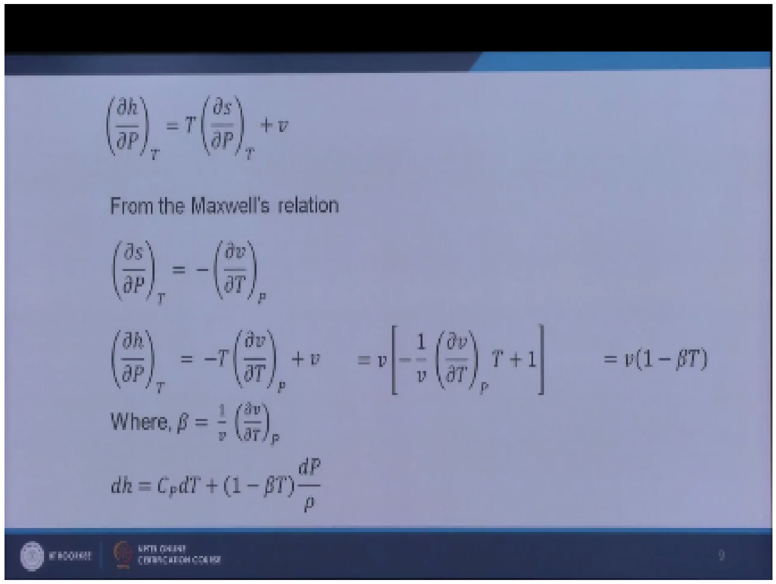

And convert this term into other term so if you do so Maxwell relation will be using which is nothing but ∂/sp t = -∂v/tp. So here you see I can substitute this term over here so this term will become over here with multiplication of d. Next you see here I have written the equation so I have used the Maxwell relationship over here.

So I have substituted this with this – sign +p okay. Little bit of modification what we are doing over here we are taking this to right hand side if you take v out of this with two terms so v comes over here. From the first term move v so this was – 1/v∂tvp T will be remaining as usual + v so here I have taken v out T +1.

Now this term it can actually be written as β, so what is β it is nothing but par unit degree change of temperature how much amount of volume has been changed at a constant pressure per unit volume so that one I am writing one has β okay volume at extenuative pressure. If I have write down this one β this whole done then I get ∂pht= v(1-βt)t.

Okay then here I have again shown what the expression of β over here which I have already used okay. Then we can write down dh which I have already showed you over here I have showed you dh=∂hp which I am replacing with ∂h/∂p by constant p v(1-Bt) then finally I will get dh= CP (dt) +(1-β t)dp / ρ.

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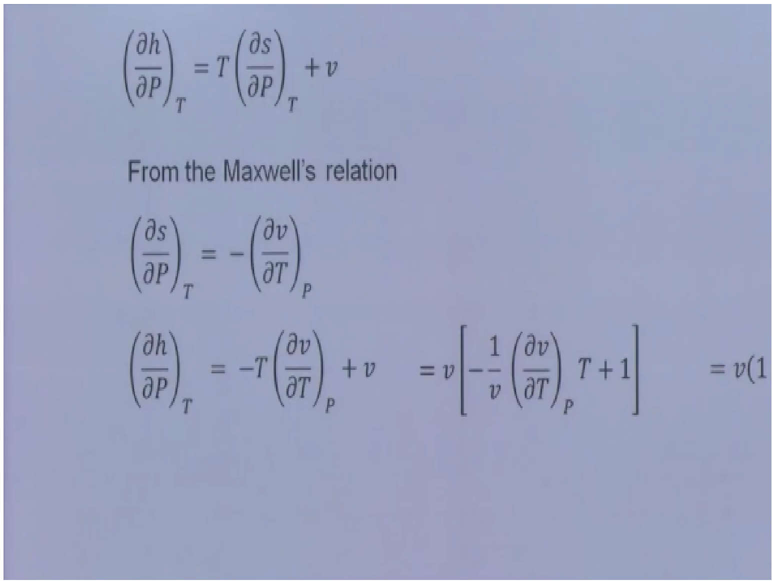

Okay next this ids the dp in to ρ and we have got they are I tried to take this ρ in this side so άdh so you see I first I diveded by the dt both the side okay and then I have axtually tried to put this equation in the multiplied by ρ if you take of άfrom both the sides if you finding out this ρ will be cancelling out which is becoming 1_βt *dp*dt then you know that enthalpy form of what thermal energy balancing vision left hand side and we had ά*dt of 8 bh.

Let us put this whole term in the thermal energy balancind equation which is having h form okay and we get in the left hand side I have substituted this ρ *in to dt of h which this term and right hand side we having same usall of q +dp/dt discuss detibation 5 band so get this equation where cp is envolvwed in the simplfication and be giving in the cp form of a thermal energy balance equation so if you see over the here we are geeting one dp in the left hand side okay 1*into dp dt and right hand side also we are havinfgin dp /dt in the send out by doing the simplification so if youy do so we will getting.

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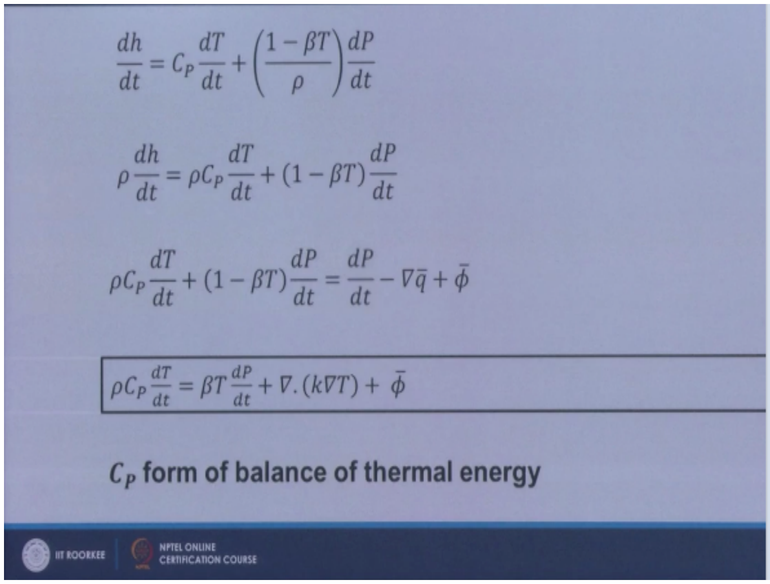

And let us take in the left hand side and in the right hand side soβt *dp the first term in the right hand side here we had -qso the -q we can right as q is nothing but into the k in to ρt and I have told you just now that I receive this by foolius law of heat conduction that can be written as .okay finally the five minutes over here as dicussed destibation so you see here it is also one energy value some energy balamce equatiopn in the left hand side we are having the CP form okay.
So this equation is also very, very important and known as CP form of balance of thermal energy okay. So what we have seen that at the beginning we have converted our energy balance equation, thermal energy balance equation in entropy form and then we have got two important forms one is CV form and another one is CP form over here okay, involving CV and CP respectively okay.

So these three forms newly we have learnt in this lecture. And in the previous lecture we have seen two more forms one is internal energy form another one is enthalpy form okay.

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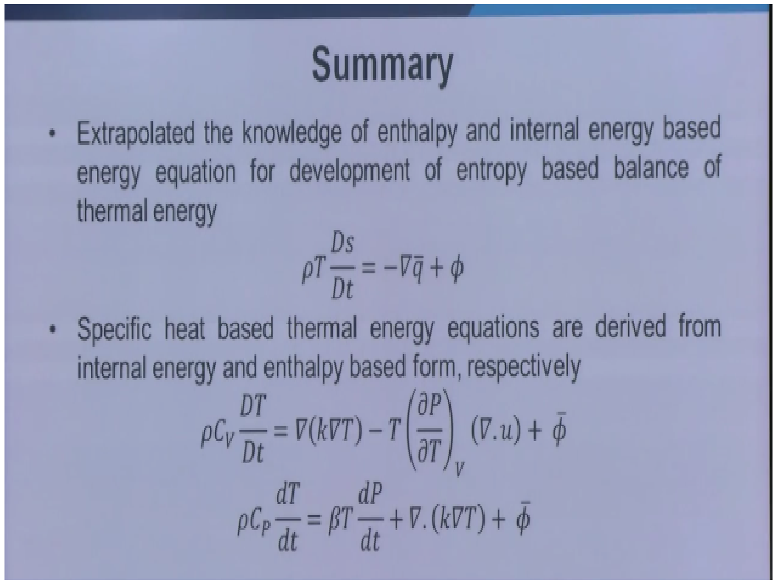

So let us summarize this lecture, so what we have done extrapolated the knowledge of enthalpy and internal energy balanced energy equation which I have dealt in the previous lecture and found out or developed entropy based balance of thermal energy equation.

So this came as final form I have shown you already. And we have also tried to see specific heat based thermal energy equations, specific heat at constant volume, and specific heat at constant pressure. So this was the final form for the specific heat based equation at constant volume and this was the final form of specific heat based thermal energy balance equation at constant pressure involving CV and CP respectively okay.

So we have dealt with five diferent forms, so first we have seen intenal energy form, then we have seen enthalpy form, and today we have seen the entropy Cv and CP forms over here. So the end of this lecture let us once again test your understanding what you have learnt over here. So we are having three questions over here for you you are having actually multiple answers you have to pick up the correct one.

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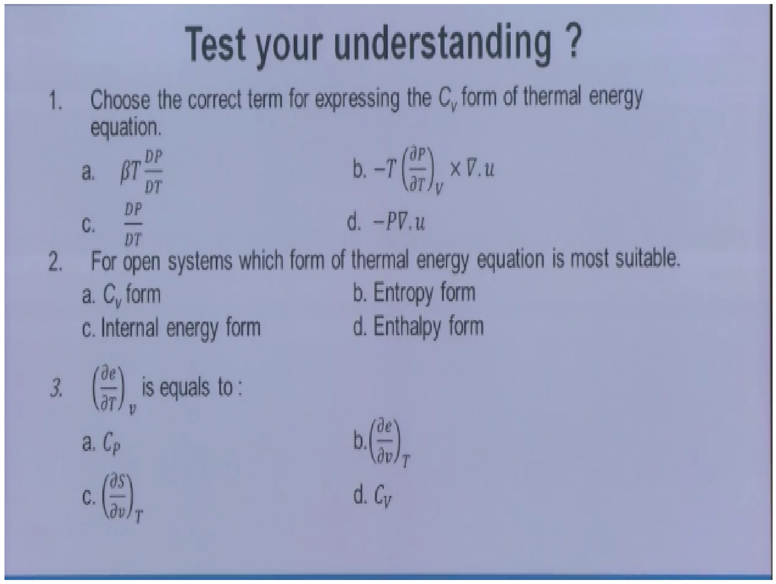

Let us see the first one, in the first one you have to find out the correct term which expresses CV okay from the thermal energy equation. So we are having four options over here βT(δT/P) DDT/P okay –T(δT/P) at constant volume multiplied by divergence of U. DDt/p and fourth option is –P into divergence of U okay. So these four options you know what is the correct one, so the correct one will be obviously option B okay –TδT/P at constant V(δ.U) right.

Next question goes like this for open system which form of thermal energy equation is most suitable, I should not say suitable this is the most suitable okay. We have seen five different forms out of that we are having options as CV form, entropy form internal energy form and finally enthalpy form okay. So all of you have guessed from your basic knowledge of thermodynamics that for open system obviously enthalpy form will be required so this is the correct answer.

Last question goes like this, δT/E at constant volume is equals to we are having once again four options first one is CP, second one is δV/E at constant T, next one is δV/S at constant T and last answet is CV okay. I think you have understood what is the correct answer, correct answer will be obviously CV because CV is nothing but power unit change of temperature what amount of energy you required at constant V that is CV, so this is the correct answer.

Hope you have got the answers correctly after reading this lecture thank you. In the next lecture we will be discussing about thermal boundary here okay. If you have any query please post in our discussion forum, thank you very much.

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