**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Convective Heat Transfer**

**Lec-02**
**Balance of Total Energy**

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Hello welcome to the second lecture of convective heat transfer. Today we will be discussing about balance of total energy. In the previous lecture I have dealt about different modes of heat transfer and we have seen that what is the essence of convective heat transfer in that, few applications of convective heat transfer also we have introduced over there. Today let us see the mathematical formulations related to convective heat transfer. So first we will be dealing about energy equation.

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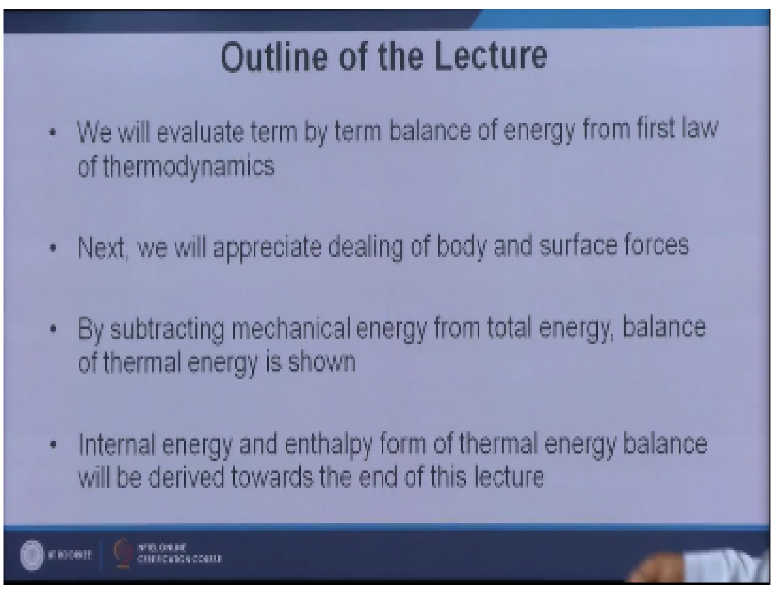


So as outline of this lecture we will be understanding what is the balance of energy from the first law of thermodynamics. So as thermodynamics is prerequisite of this course, so all of you know that what is the first law of thermodynamics from there we will be finding out the balance of energy term by term we will be evaluating the balance of energy. Next, we will appreciate dealing of body and surface forces.

So what are the basic differences of body force and surface force we will be discussing as well as we will be subtracting mechanical energy from the total energy balance and find out the thermal energy balance. Towards the end I will be showing you internal energy and enthalpy form of thermal energy balance equation. So all of you know what is internal energy and enthalpy as thermodynamics is a prerequisite of this course.

So we will be expressing the thermal energy balance equation in the form of internal energy as well as enthalpy. So to begin with let us start how to do this total energy balance equation. We will be starting from a closed system.

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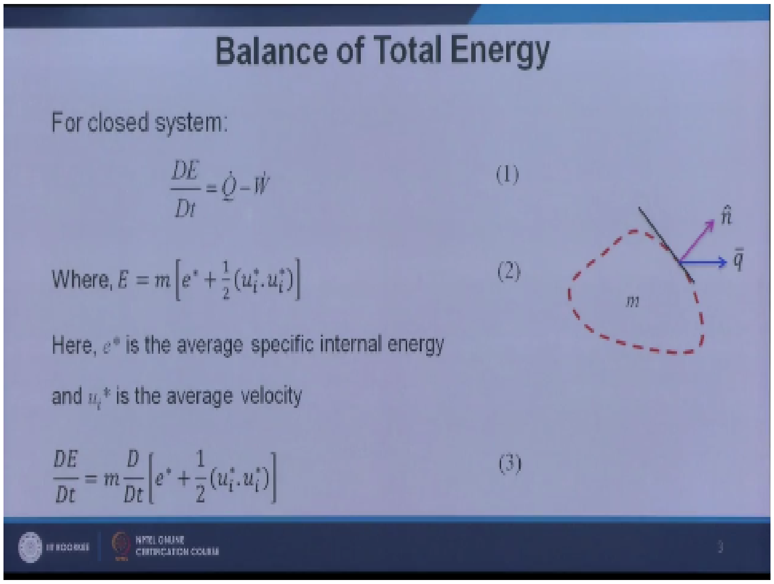


So here I have shown a closed system you see this is a closed system bounded by a boundary, boundary here I have shown by a dotted line and the system is having mass m okay. at any arbitrary surface if you draw the surface normal that is N^ and let us consider as we are dealing about the energy balance equation. Let us consider through that surface Qbar amount of energy is being released.

Now all of you know as per the first law of thermodynamics for a closed system we can balance the energy in transit as well as the energy in storage. So first let us talk about energy in storage for the system, let us consider that energy is capital E, so energy in storage will be changing in the form of DE DT okay. So in the left hand side we are having storage energy change with respect to time and in the right hand side we have all the energies which are in transit.

So transit energies can be in terms of heat transfer $\dot{Q}$ as well as the work transfer $\dot{W}$. If you take the sign convention in a proper manner you will be finding out they are opposite in nature and if you just reverse the sign convention these signs will be changing, but overall the equation can be written in this fashion okay. So we have actually equated energy in transit and energy in storage and energy in transit over here okay.

Next let us see what is this energy in storage inside this system T, so this E will be actually summation of the internal energy e, and its kinetic energy here UI is a vector, so if we are talking about a 3 dimensional plane uy will be having 3 different components okay so here you see e I have motioned that internal energy now here as it is a finite volume so we have consider about the average internal energy of this system so here I have del with $e^*$so star simplexes the average specific internal energy okay similarly $ui^*$ is actually average velocity.

Okay next if you try to do the temporal derivative or derive with respect to time of this term e then you will be getting DE/Dt or DDT of e if = m mass is not going to change for this control volume or for this control system so you will be finding out m is coming out and DDT is remaining over the term inside this base $e^* + ½\ ui^* .uy^*$ okay so here what we have done we have found out is the balance of energies in a closed system and energy in storage we have found in this fashion.

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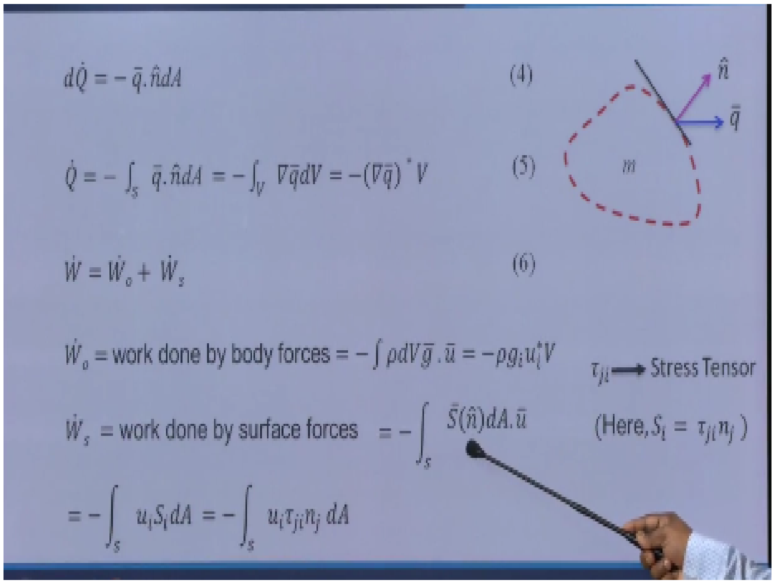


Next let su see what about energy in transit first we will be discussing about energy wire heat transfer so let su consider that for a small amount of sun phase a very small infant amount of surface over here which is having surface area dA trough that dQ. Amount of heat energy is being released okay and that energy is being realized in any arbiter reduction so for finding out the energy balance we need to found out that what is the component, of the energy being realized normal to the surface dA.

So what we have done over here . product of q^ which I over here okay . n^ is a normal direction okay of the infinite symbol surface over here n^.dA that will give us the account of ∇ Q. right then for finding out the overall Q. which is being realized from the overall surface of this control volume we need to integrate this term whatever we have got for this infinite symbol small area over here of the overall area surface area so here we have done the surface integral of that term integration of s of this term Q. n^ into dA okay.

Here at this movement we have applied divergence to your and from vector calques so you can find out over has per divergence surface integral can be converted to volume integral okay and after this conversion the inside term will be converted to divergence of Q into dV ∇ Q *dV okay in a similar fashion here what we can do ∇q we can write down as average property as I have shown you in the previous slide for small e and ui.

So that average property can be once aging represented as q* which can be take out from integration because it is averaged okay so if you take out this ∇q bar out of this integration by giving this * then integration of dV over the volume will become total V so what I can write down - ∇q* . V so total heat emery will be coming as -∇ q* , V right next let us see the work transfer which is another form of energy in transit so we can have the work transfer as two parts first one is work transfer due to body forces.

So $\dot{W}_o$ and work transfer due to surface forces which is W.s right so here let us see that how force the body forces can be evaluated so work done by body forces $\dot{W}_o$ can be written as integration of ρdVg.u bar okay here g and u are the vectors okay so g is actually the body force acceleration term okay in case of gravitational problem so you can find out this g will be equivalent to 9.81 okay so there can be some other accelerations also apart from gravitational acceleration so that is why we have kept this one has vector over here and depending on the magnitudes of different directions.

Or accelerations this will be getting components okay so work done on body work done by body force can be written as ρdVg.u bar okay whether we this negative sign is very, very important why because we have consider that if we have them work on the system that is negative and if the system is we worked that is actually a positive one okay. So here you see we have found out that this term after integration one second can be average dot so this term we have average dot ρ and gi will be coming out and as usual uiV have average.

So this is $u_i$ x $u_i$* and then this will be coming V integration of V will be coming V right whatever about the surface forces so in case of surface sources $\dot{W}$s we can write down that one has surface integral of S (n) now this S(n) is actually surface stress vector okay into dS.u bar okay so this surface case vector can be also express in terms of states so let us see what is that $S_i$ which is a component of this surface stress vector so Si can be written has $\tau_{ji}$ x $n_j$ here j is a dummy index.

Which will be running from 1 to 3 or let say xy and z coordinated okay so this Si is equals to $\tau_{ji}$ x $n_j$ here this $\tau_{ji}$ is actually the stress tensor okay now if we further simplify this term work done by surface force then we can write down this S in cap dA x u as $u_i$ x Si x dA so we are dealing here component wise $i^{th}$ component wise $i^{th}$ component if we write down and then if we do the integration over the surface then will be getting the total work done by the surface forces now here already I have showed that Si can be related to stress tensor $\tau_{ji}$ so I am writing down over here Si equals to $\tau_{ji}$ into $n_j$ following this relationship so you can get $W_{\cdot s}$ can be written as surface integral of $u_i$ $\tau_{ji}$ $n_j$ x dA.

Right so we have got some sort of idea about work done by body forces as well as work done by surface forces now if you one second apply on this surface force the divert density or from a vector calculus so this was surface integral we can easily convert that one into volume integral so here you see from surface integral to volume integral w can write down ∂xj of $u_i$ τji x dV so this was on dA now it will be converting to dV okay so here we have got one second some problem which can be averaged out okay just like the previous cases so here what we have done, we have done the volumetric average of this one and this term we have written as star term taken out of the integration so you see δui, τji or δxj or δδxj(ui τji) as has been written as star term.

So which is volumetric average of this term multiplied by volume whenever we have taken this term out so it will be integration of dv only over the volume so it becomes V okay so this term we have got for the work done on the surface forces, so till now we have seen that energy in transit can be written as Q. which is simplified in this fashion, W. which is having two components W.O which can be simplified in this form.

And W. S which can be simplified in this fashion surface forces okay work done on the surface by the surface forces, okay.

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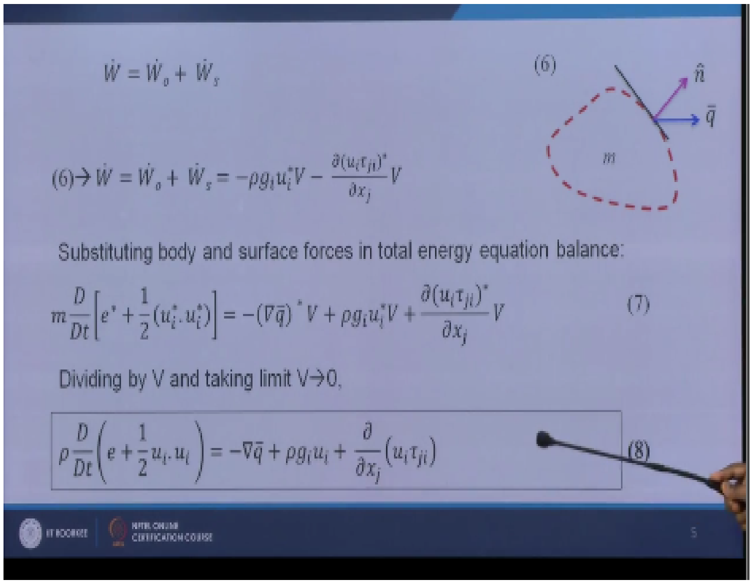


Let us first add both the terms of the body forces and the surface forces okay so W. as we have already shown will be equivalent to W. O + W.s so it can just add the previous two terms so that means this term as well as this term so you can get this minus of this okay first one is for the body forces second one is for the surface forces okay, now if you substitute this body and surface forces in total energy equation along with the Q.

Which already I have simplified into a vector form so then we will be getting you see this left hand side was actually your energy story rate of change of energy storage okay, so and in the right hand side first term is coming from capital Q. which was heat transfer okay and this term is coming from your work transfer first part is actually work done due to body force and second part is actually work done by the surface forces, okay.

Now let us take some simplifications, so first what we have done this whole equation I have divided by V that means volume so if you do so M/V will be becoming your density ρ, here also V and V will be cancelling in the second first term in the right hand side, second term in the right hand side also V will be cancelling and here also the V can be cancelled if I divide it by V the whole equation, also let us take the limit of V tends to 0 that means very infinite simple volume, okay. So from control volume now we are moving to the infinite decimal volume problem, okay.

So if we do so then you can find out this M/V will be coming as ρ rest terms will be remaining same now as we have taken the limit V→ 0 average property can be now written as simple properties okay so I have dropped the star terms over here and written in terms of simple terms where E has been converted to E star has been converted to E, Ui has been converted top Ui* has been converted to Ui and so on.

In the right hand side after dropping the V every term is coming as simple term and also we have dropped the star due to this limit in V → 0 okay so from this equation by dividing V throughout in the left hand side, right hand side and by taking the limit we have got this equation, okay.

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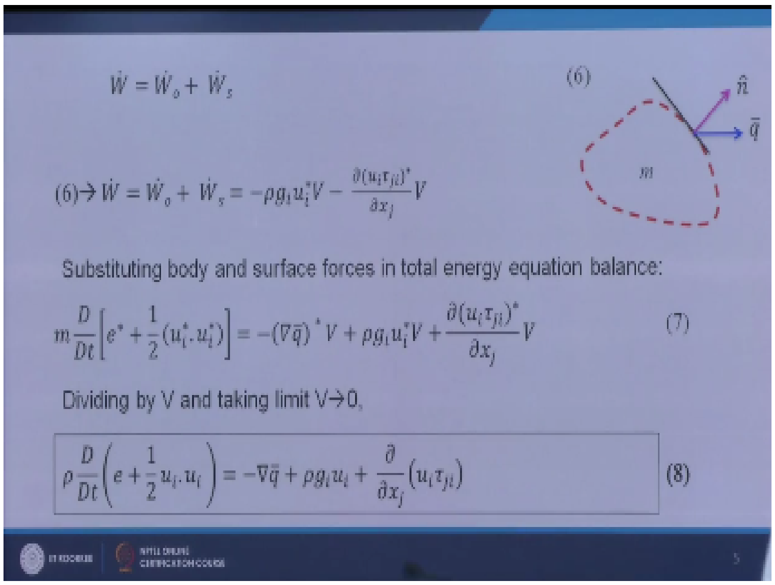


Now so this equation is very important this last equation is very important this is actually balance of total energy equation okay, let hand side we are having rate of change of energy in storage and in the right hand side we are having account of energy transit.

Next let us see that what are the balance of mechanical energies because we know that if we have to find out the thermal energy balance equation this will be simply total energy balance equation minus the mechanical energy balance equations, okay so we have already found out their total energy balance equation let us find out the mechanical energy balance energy in order to get the thermal energy balance equations.

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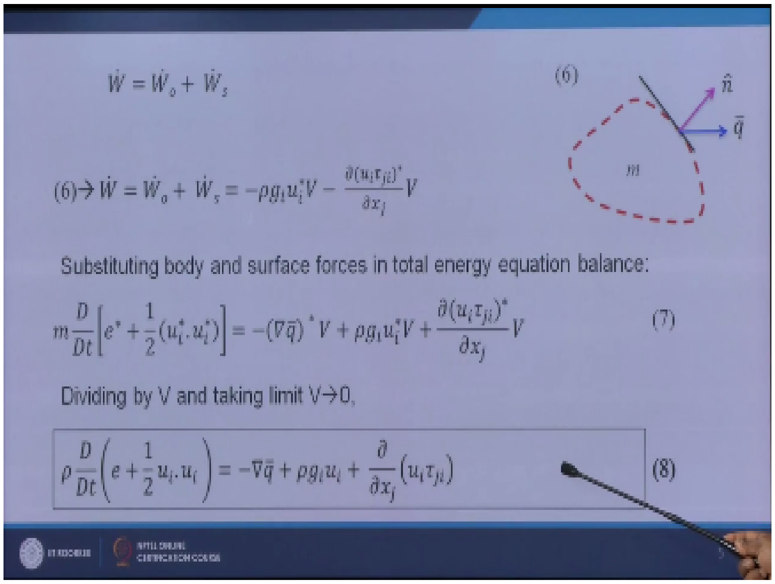


So for mechanical energy balance equation from our mechanics we know that Cauchy's equation of motion gives us this type of equation, so in the left hand side we are having ρ into $Dt(u_i)$ okay, which is equivalent to $u_i$ inertia okay. Then in the right hand side we are having the body force related term okay, $\rho g_i$, $g_i$ is the acceleration in a particular $i^{th}$ division plus this is due to surface states okay, so then $x_j(\partial_{ji})$ right.

So this is your Cauchy's equation of motion all of we know this one. Next if we just multiply this equation by $u_i$ okay, then you will be finding out this can be written as $\rho.D/Dt(u_i.u_i/2)$ why because if you just multiply $u_i$ over here this will be becoming $u_i.DDt(u_i)$ which can be written as half of $DDt(u_i^2)$ so here I have taken 1/2 inside so it can be written as $\rho DDt(u_i^2/2)$ okay.

In the right hand side this will be simple very simple $\rho.g_i$ and I have multiplied by $u_i$ so $\rho.g_i(u_i)$ and then this term will be ui. $\partial/\partial x_j(\partial_{ji})$ which can be written as this two term we are actually using over here DDx(u.v)=u(DDx(v)+v.DDx(u)), so here you see $\partial/\partial xj(u_i\tau_{ji})$ can be written as $\tau j_{ji}$ $\partial/\partial x_j(u_i)+u_i(\partial/\partial x_j(\tau_{ji}))$ okay.

So if you just alter the sides then you will be getting $u_i(\partial/\partial x_j(\tau_{ji})$ will become $\partial/\partial x_j(u_i\tau_{ji})$-$\tau_{ji}(\partial/\partial x_j(u_i))$ okay. so this simplified form we have got from the balance of mechanical energy, now if we subtract this mechanical energy balance equation from the previous total energy balance equation, okay so that means if we subtract from this one okay.

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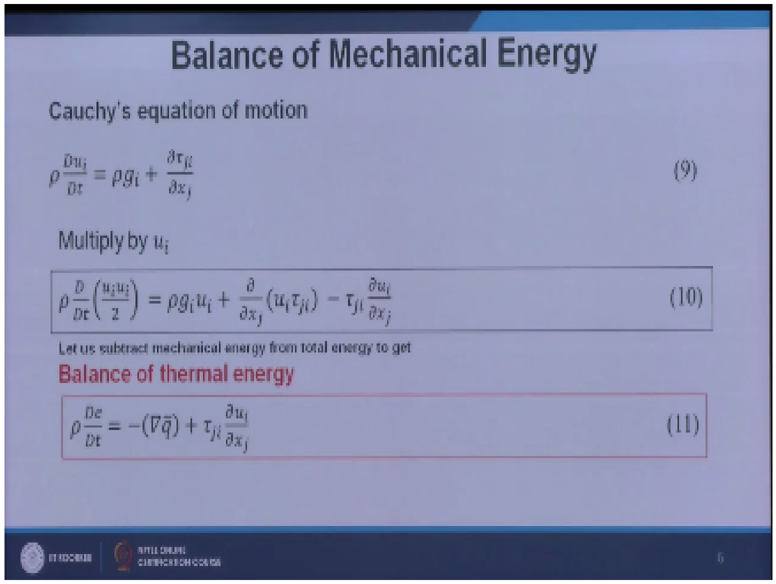


The total mechanical energy balance equation this one okay, then we will be finding out our balance of thermal energy okay. so after subtraction you will be finding out some terms will be dropping out from both the sides especially this term will be equivalent to your previous this term so this will be cancelling out so eventually you will be finding out that we are having in the left hand side $\rho DDt(e)$ and in the right hand side we will be having $-\nabla q+\tau_{ji}(\partial/\partial x_j(u_i)$ because this term will be cancelling with this term, right.

So you can find out finally the thermal energy balance equation comes as this one $\rho De/Dt=-(\nabla q)$ $+\tau_{ji}.$ $/\partial x_{ji}(u_i)$ okay, this is very, very important equation so this equation we will be further simplifying in terms of you know getting simplified forms so let us see first this last term, in the last term you see we are having $\tau_{ji}$ okay, so let us see what is this $\tau_{ji}$ stress tensor.

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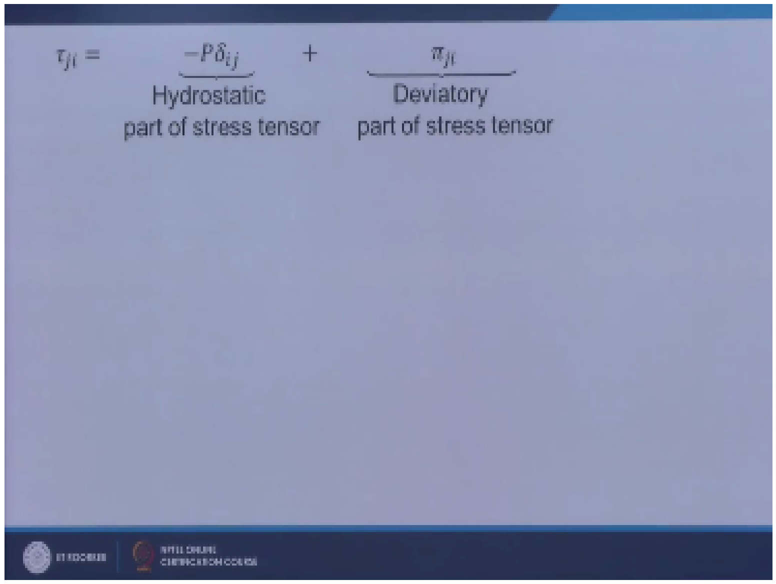


So if we see the stress tensor from our fluid mechanics knowledge we can write down that stress tensor is nothing but having two part hydrostatic part as well as deviatory part okay, so hydrostatic part of the stress tensor is -P $\delta_{ij}$ and deviatory part I can write down as $\pi_{ji}$ you see this was $\tau_{ji}$ this is also $\pi_{ji}$ so let me give you one example of this type what is this hydrostatic part and what is this deviatory part. Let us take the stress tensor is having this fashion.

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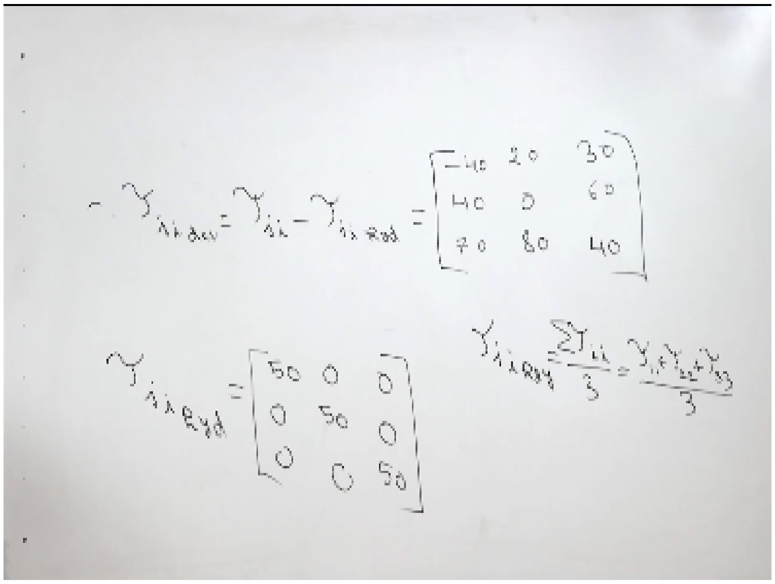

So $\tau_{ji}$ is having nine components let me give you a example with nine different values which will be earlier to understand so for simplicity let us take 10,20,30,40,50,60,70,80,90 okay, so in 3/3 matrix for 3 different direction XY and Z direction we will be getting τ ji like this okay let us say this is some obituary stress tensor. Now for the first part hydro static part we will be considering only the diagonal terms okay.

So hydro static part we can write down so τ ji hydro static part okay this can be written as this is actually having only the diagonal terms rest terms are 0 so this can be written as this 00 then 0 then 0 then 0 then 00only the diagonal terms will be remaining. Now what is this diagonal terms diagonal terms will be here from your original state tense or if you see the diagonal terms at those and divide it by 3.

So here what we get 50 + 10 = 60 + 90 = 150, so 150 / 3 if you do it is becoming 50 okay so here all the terms will be equivalent to 50 right. So we have got hydro static part is having only the diagonal elevates or we can say wherever I = j okay in this τ ji there only the hydro static part exists okay, so this τ ji hydro static we can write down as ∂ii for all I/ 3 that means ∂ 11 + ∂ 22 + ∂ 33 / 3 so if you do so here you will be getting 50, 50, 50.
Now to get the deviatory part you have to subtract this hydro static part from you stress tensor okay so we will be finding out that our deviatory part becomes τ ji - τ ji hydro static okay, so hydro static part we have only got the 50 terms in the diagonal so here we will be getting rest terms as usual so 20 30 here we had 40 this terms will be changing 60 70 80 and the diagonal terms will be changing.

Now at the first place we had 10 so 10 – 50 it will be becoming -40 this terms will becoming 50 – 50 so that means 0 and this term will become 90 – 5- which means 40, so this becomes might deviatory part okay. So in this fashion the deviatory part can be obtained so here also we are having this two components hydro static part of the stress tensors.

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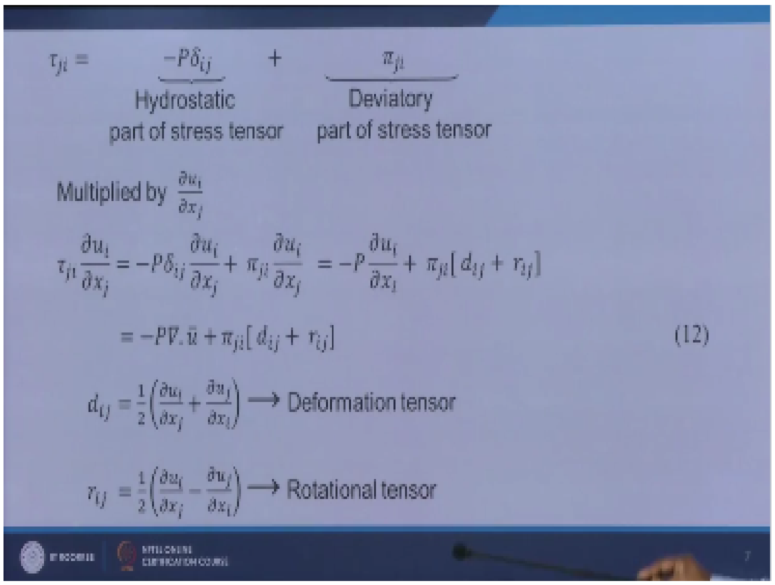


As well as deviatory part of the stress tensors. Now let me show you further so if you multiply this τ ji with ∇ ∇ xg (ui) why I want to do so.

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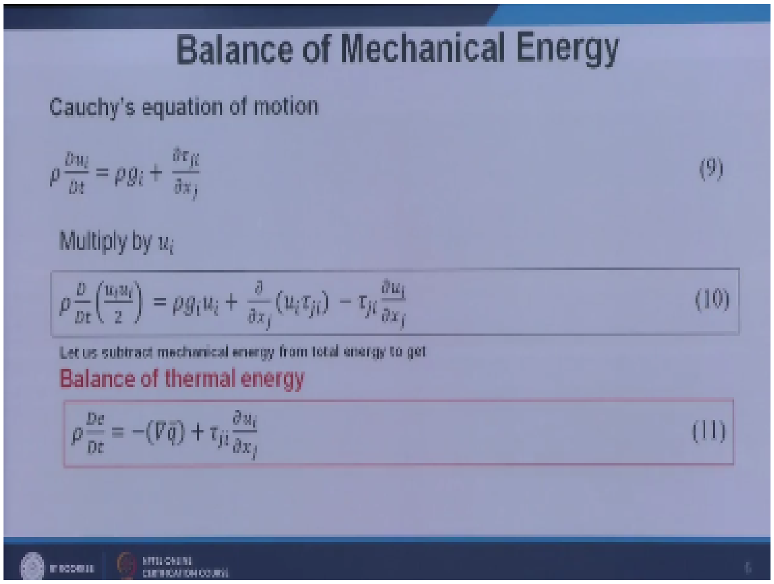


Because my last part of the balance of thermal energy I had τ ji x ∇ ∇ xg (ui)so let us see what is this term. So here you see if I multiply this then I will get ∇ ∇ xg (ui) which is the last terms of my thermal net g balance equation is becoming the hydro static part multiplied by ∇ ∇ xg (ui) and deviatory part multiply by ∇ ∇ xg (ui) okay.

Now for further simplification what we can do this ∇ ∇ xg (ui) for the deviatory part attach to the deviatory part I can write down in two terms which is nothing but dij  rij here I am saying this dij and rij is are deformation tensors and rotational tensors so how I have written this dij rij dij is actually half ∇ ∇ xg (ui) + ∇ ∇ xg (uj) and here the rotational tensors are having equal terms but in between we are just the change of sign.

So if you just add this two so you will be finding lout the last terms will be cancelling and half of this plus half of this will become ∇ ∇ xg (ui) which is nothing but this part over here okay. So this ∇ ∇ xg (ui) we have written dij + rij deformation tensor plus rotational tensor okay. Next let us see further now the rotational tensor you whatever I have shown you here is actually anti symmetric.

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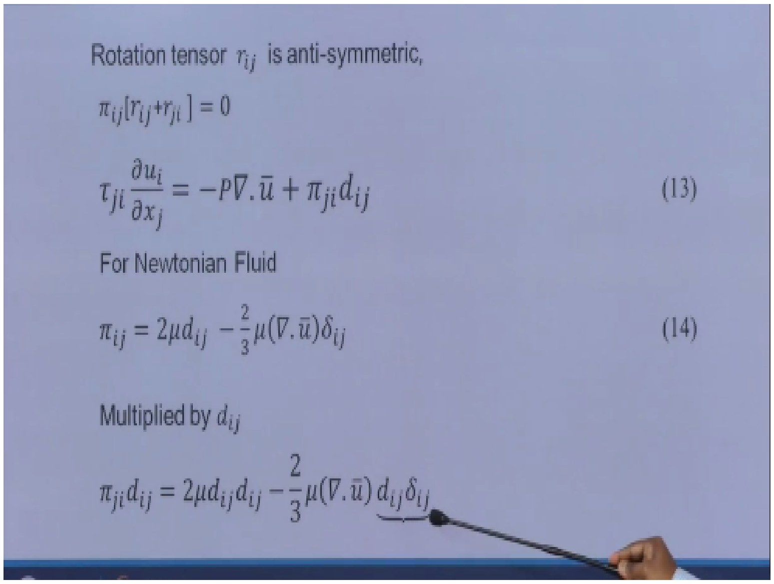


So we can write down ij( rij+rji) =0. So I can drop rij term from the equation, so we get rest thing same and only in this place where we had earlier ji x ∂ ∂xj am writing ji x dji okay. One reason of not writing the next one is that the rotational tensor is anti symmetric. Right in similar fashion now let us go for some common fluid.

So we know that fluids can be different types and the most common one is neptunium fluid, so this fluid this πji diffused tensor can be written as 2μdij- 2/3μ (Δ.u)δij. So here you have two different things see this for this thing we are having viscosity coming into consideration. Now if you multiply this one with dij because dij πij we are getting over here.

So then we will be finding out this become 2μdij once again we have multiplied - 2/3μ (Δ.u)δij so this is actually chronicle Δ,. Now if we proceed further as per the Δ operator or the chronicle delta properties we know that dij x Δij so this is actually chronicle delta. Now if we proceed further as per the delta operator and chronicle operator properties we know that dij x Δij

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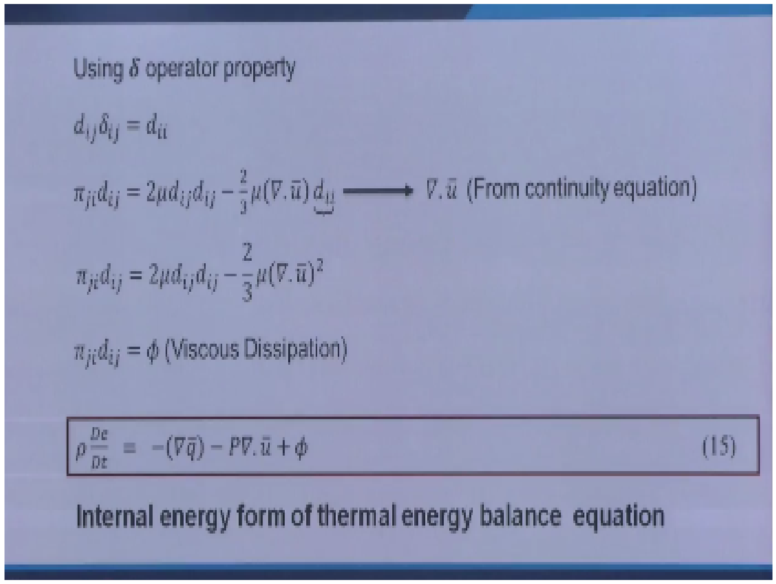


Will become dii okay because it will be becoming 0 for this Δij when ij they are not equal this will become 0. So this πij will be changing to 2μdij- 2/3μ (Δ.u)dii this chronicle delta property. From now fluid continuity we can write down that this dii =Δ.μ okay. So after simplification we get the πij 2μdij- 2/3μ $(\Delta.u)^2$.

Okay but now let us define the whole term πji whatever we have got over here as a term viscous dissipation ϕ. So we are calling this one has viscous dissipation ϕ okay. Now if we just we all the term back in our thermal energy balances equation you see left hand side remain same so your energy in storage rate of energy in storage, and right hand side in the place of stress part ji x ∂ x jui am writing –pΔ.u .

This is coming from the hydrostatic part and then + ϕ viscous dissipation coming from the deviated part. So this form we call as internal energy form of thermal energy balance equation okay, here this is internal energy next let us see what are the other types of form so here let us tries to replace.

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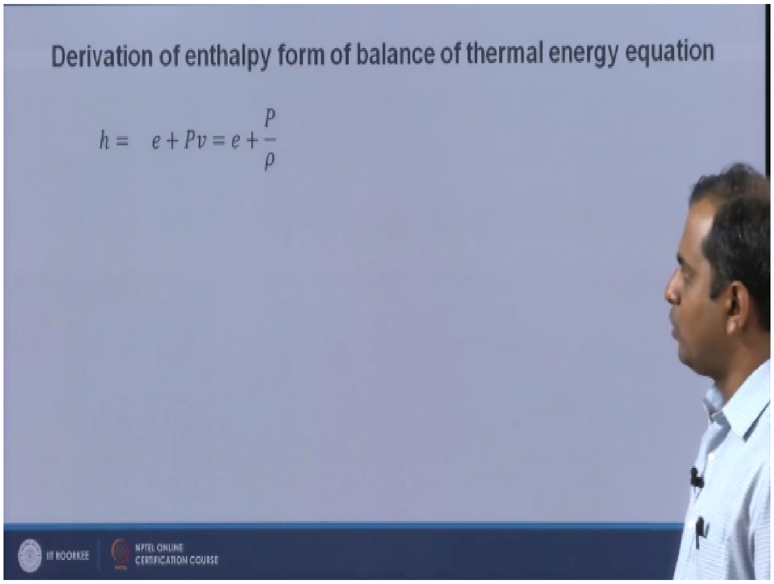


E/H which is nothing but enthalpy we are trying to see enthalpy form of the energy balance equation over here. Now all of you know that h from the thermodynamic is the enthalpy from the thermodynamics is can be written as e+p/v e is the internal energy ok b can be written as written by 1/enthalpy volume can be written as the 1/ -h h becomes h +p/ρ now let us do the derivate tips so you do derisive of e and derivative of and h _h and the p/ρ.

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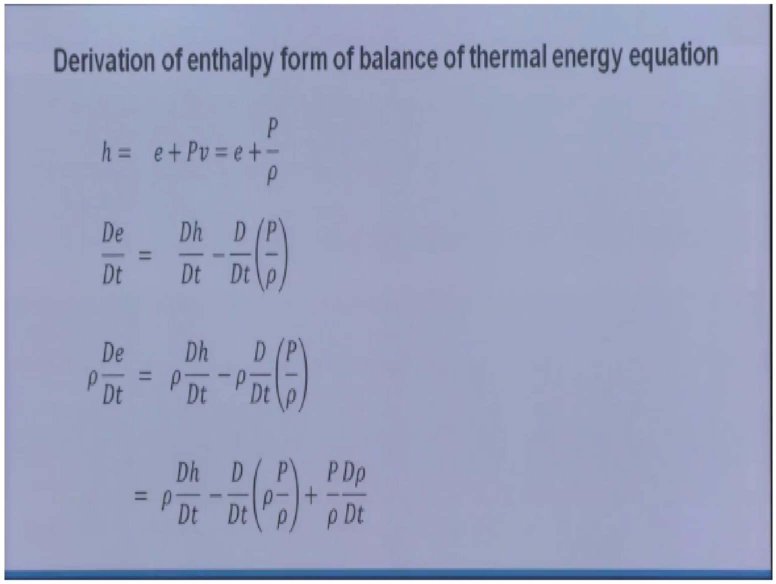


Okay this is the multiply by over here then we can get the ρ is coming of all the terms and this term that can determine as directive of P E 2 ρ/ρ +p/ρ in to okay now 20 vp we now that 1/ρ *Do/This terms becomes _of p*.okay .

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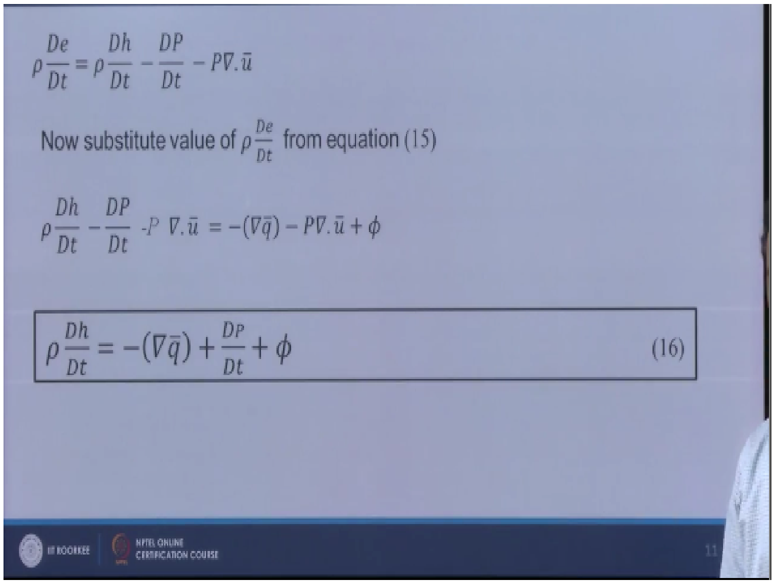


We have got –of pf. And over here and okay so we have actually found out in the terns of and over here so some costive would that internal energy we are the equation we finding out that in place of de in to and the dot n that terms in the internal energy equation in the both side the sane time in the out so finally we get the dh /do is equal to this will come in this side so +dp/do we disused and the distribution will be removed in many here and over there is axially balance in the and so let  us summarize today's lecture .

So what we have done is describe the battle the balance total energy okay in the close system this is the equation finally we have got derived balance thermal energy and the space tenser and the final form thus we presented disused proptinal for a titanium fluid we have simplified that disused in the times chemical proptinal and the looks that like okay we have casted the thermal equation and in t6he internal energy for as well as we have showmen that enthalpy form okay let us taste whatever you have under stood in the this lecture let us test you in the understanding so here I have actually.

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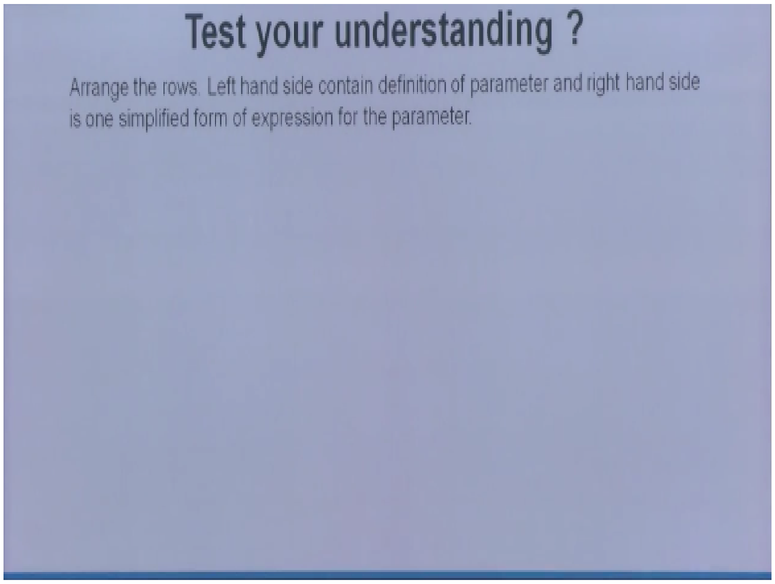


In the giving the two rows the total side of the row we will find in the some parameter which I have already disused in the lecture and in the right hand side getting the simplify parameters you have the match one by one in that so let us see the option what we have having so the option are like this left hand side.

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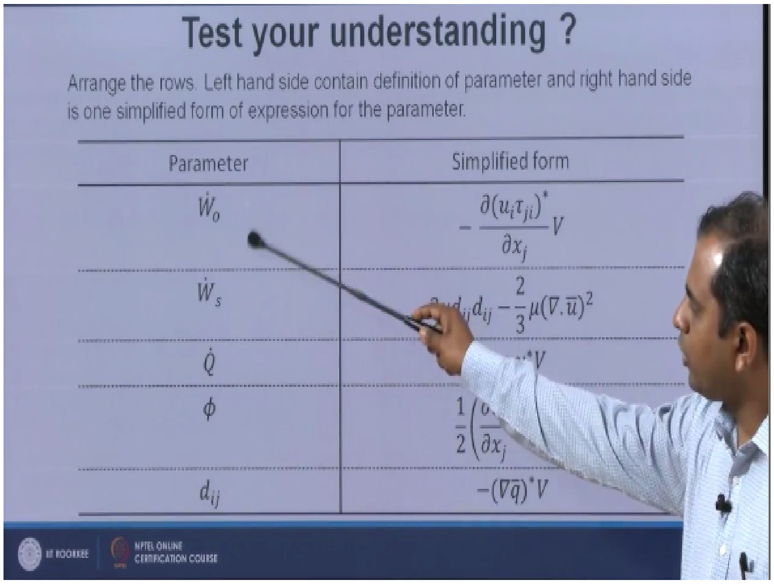


And I thing that you know the meaning of this and this left hand side term in the right hands time the double side we have to match up so please take your time to see week this one and arrange so I think the final answer becoming like this and all of you are correct and the right answer and this will come on the reduction and you showed in the w. s will coming out this one okay q . is this one finally five is coming out this were her over here dig is coming here out like this okay.

So I want to thank you for listening this lecture and please visit your lecture next lecture they are will be discussing about different forms of thermal energy valentine equation and if you having please post us in the discussion thank you.

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