# INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

## NPTEL

## NPTEL ONLINE CERTIFICATION COURSE

## Convective Heat over flat plate

## Lec-19
## Mass Transfer

**Dr. Arup Kumar Das**
**Department of Mechanical and Industrial Engineering**
**Indian Institute of Technology Roorkee**

Hello welcome in 19 lecture of convective heat transfer in this lecture we will discussing about mass transfer, is quite analogous to want ever we have discussed in the last few lectures which is heat transfer.

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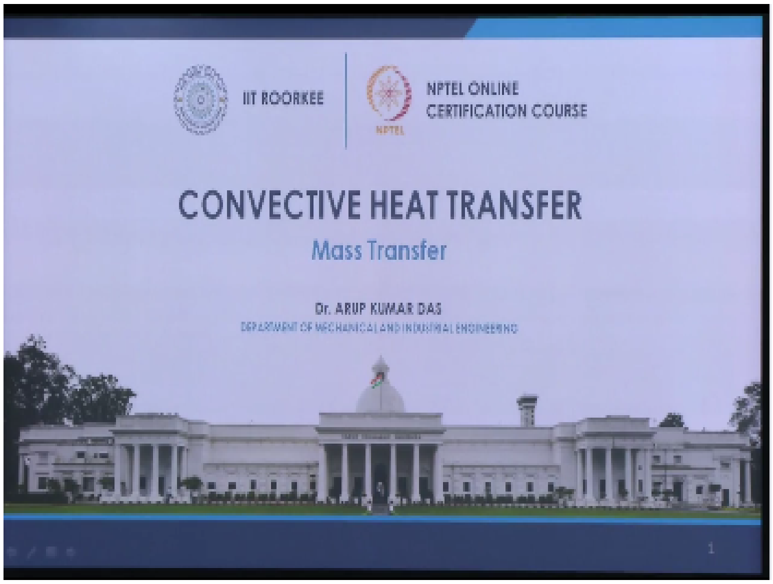

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CONVECTIVE HEAT TRANSFER

Mass Transfer

Dr. ARUP KUMAR DAS

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

So here we will try to see how convective heat transfer relationships can be used mass transfer okay, so our topic for this lecture is mass transfer so let me first tell you that what outline we are going to follow in this lecture in this lecture first we will be introducing what is mass transfer okay.

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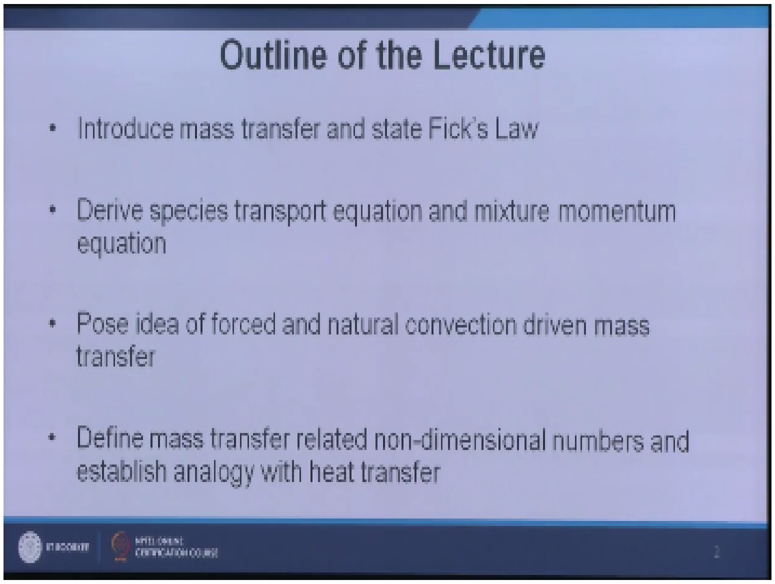

## Outline of the Lecture

- Introduce mass transfer and state Fick's Law
- Derive species transport equation and mixture momentum equation
- Pose idea of forced and natural convection driven mass transfer
- Define mass transfer related non-dimensional numbers and establish analogy with heat transfer

Then we will going for fick’s law derivation so we will deriving the fick’s law and we will state what fick’s law is having next we will deriving species transport equation and finally give you mixture movement equation okay so momentum equation for the overall mixture of different species then we will post the idea of forced and natural convection driven mass transfer so here we will be showing you similar to forced and natural convection how forced mass transfer and natural mass transfer can happen and at the end I will be showing you different mass transfer related non dimensional number and try to establish analogy with the heat transfer okay so let me go inside.

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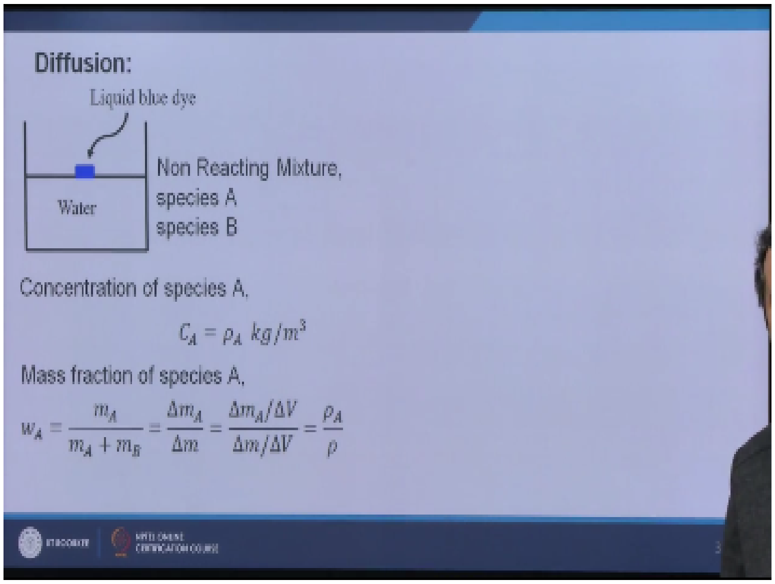

**Diffusion:**

Non Reacting Mixture,
species A
species B

Concentration of species A,

$$C_A = \rho_A \ kg/m^3$$

Mass fraction of species A,

$$w_A = \frac{m_A}{m_A + m_B} = \frac{\Delta m_A}{\Delta m} = \frac{\Delta m_A/\Delta V}{\Delta m/\Delta V} = \frac{\rho_A}{\rho}$$

To began with first I will be giving you idea of diffusion between two different components let us say we are having species A over here as water and spices B is some liquid blue dye just when ever this dye is entering in water you will find out that blue color dye is diffusing in water slowly okay thought they are none reacting mixture okay there can be reactions also but in this case though they are none reacting mixture and liquid blue dye those are non reacting mixture still we can find out diffusion helps in mixing okay.

So this type of situation if you if we start to study then we can immediately define the concentration of individual species for example let us say concentration of species A in this case is $C_A$ let us defined the concentration as C so concentration of species A is actually $C_A$ okay so this $C_A$ okay so this concentration is nothing but analogous of at a particular location if you try to find out the density of species A that will be equivalent to the concentration show we write down $C_A$ is nothing but $\rho_A$ and the concentration will be having same unit kg/ $m^3$ as density is having okay.

Next let us see that if we try to get the mass fraction of a species in the mixture okay so if we try to get the mass fraction let us defined mass fraction in terms of w so let us say mass transfer mass fraction of species A is nothing but $W_A$ so $W_A$ will be nothing but in A mass of A at a particular location divided by mass A + mass B so whole mass between two component mixture so if we do little bit of division in the denominator and numerator over here by $\Delta v$ we can convert this mass into corresponding ρ densities so density of species A divided by the mixture density ρ okay.

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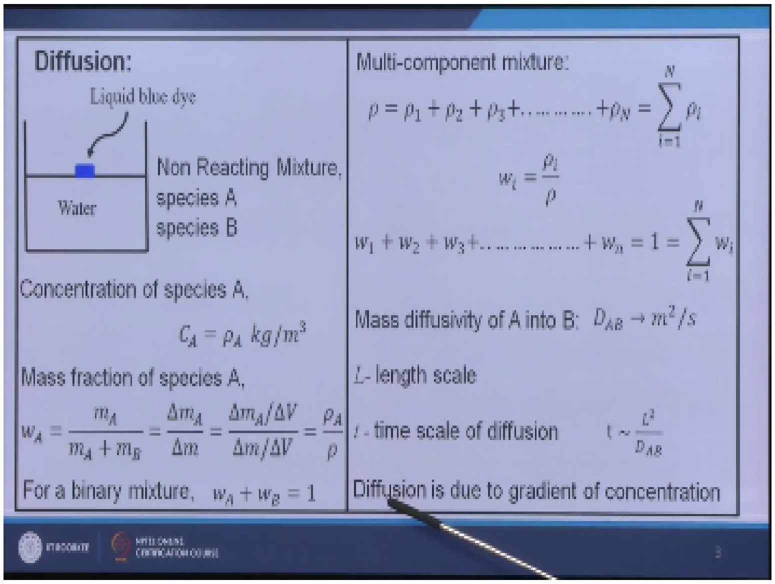

**Diffusion:**

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For a binary mixture, $w_A + w_B = 1$

Multi-component mixture:

$$\rho = \rho_1 + \rho_2 + \rho_3 + \ldots\ldots\ldots + \rho_N = \sum_{i=1}^{N} \rho_i$$

$$w_i = \frac{\rho_i}{\rho}$$

$$w_1 + w_2 + w_3 + \ldots\ldots\ldots\ldots + w_n = 1 = \sum_{i=1}^{N} w_i$$

Mass diffusivity of A into B: $D_{AB} \rightarrow m^2/s$

$L$- length scale

$t$ - time scale of diffusion $\quad t \sim \frac{L^2}{D_{AB}}$

Diffusion is due to gradient of concentration

So we get mass fraction in the form of density fractions $\rho_A / \rho$ so if we consider a binary mixture then definitely we will be find out that $\rho_A + \rho_A = 1$ so that means sorry $w_A + w_B = 1$ that means $\rho_A / \rho + \rho_B / \rho = 1$ okay where ρ is nothing but actually in $m_A + m_B / \Delta v$ that means your $\rho_A + \rho_B$ okay so ultimately this equation for a binary mixture is very valid and we can say there mass fraction summation is equivalent to 1 for binary mixture.

If we generalize this one for multi component mixture then we will be getting ρ is nothing but addition of all the component so $\rho_1$ to $\rho_n$ okay we can write in the form of summation also and if we try to defined mass fraction over here as wi so wi will be just ρi / ρ so just generalization of this side where ever we have discussed about binary mixture of liquid blue dye and water so this the generalization for multi component fluid so wi will be ρi/ρ okay and if you processed further for summation of w then you will be finding out summation of all the spices mass fraction is equivalent to 1 okay summation of all w's is equivalent to one okay just following the same analogy we can write down this one for multi component mixture okay.

Let us see now new thing as we have discussed about diffusion over here so one species will be diffusing into another one so there will be definitely some diffusely involved in this okay the rate at which the diffusion happens so we consider that one as it is mass diffusion not a heat diffusion so let us consider the mass diffusivity of A into B okay so let us write down that diffusivity as D so D AB means diffusion of A into B okay so if we try to find out what will be this factor so we call this one as mass diffusivity okay.

The unit of this one will be meter square / s okay so how much area it is diffusing / unit time that will be the mass diffusivity okay next let see if the land scale of the mixing process is l and the time scale of the diffusion is t then we can easily write down by matching the dimensions of the mass diffusivity and this l and t we can easily write down the t will be of the order of $l^2/ D_{AB}$ okay to $D_{AB}$ is nothing but mass diffusivity okay.

So here from we get the diffusion actually comes due to gradient of concentration if you are not having concentration gradient somewhere so let us say the liquid blue dye is having very high concentration but here it is low concentration that is why diffusion will be happening and helping in mixing, so you do not have any gradient of concentration then diffusion will not happen it is just similar to the heat transfer if you are not having the temperature difference heat transfer will not happen same case mass diffusion will not happen if gradient of concentration is no longer there okay.

So after this let us see a new law which we will be defining as fixed law and this is defined for binary mixture but before going there let me show you a small example over here let us say we are having silica.

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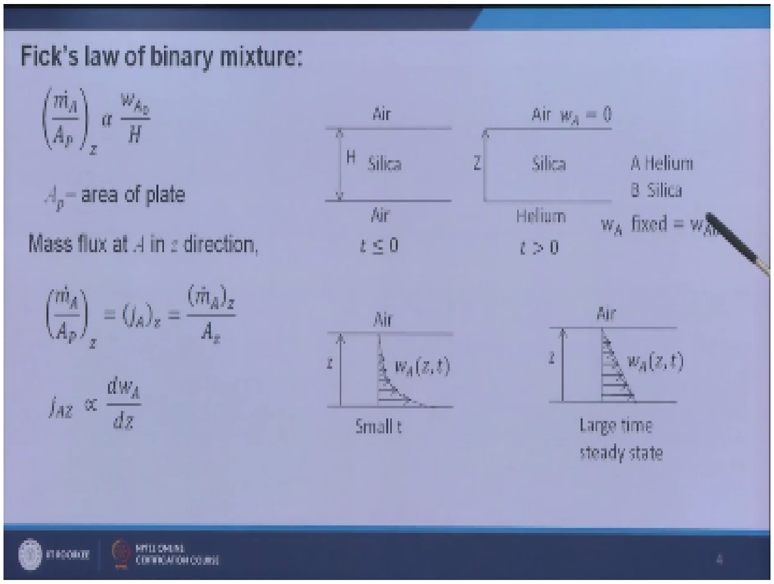

Fick's law of binary mixture:

$$\left(\frac{\dot{m}_A}{A_p}\right)_z \alpha \frac{w_{A_0}}{H}$$

$A_p$ = area of plate

Mass flux at $A$ in $z$ direction,

$$\left(\frac{\dot{m}_A}{A_p}\right)_z = (j_A)_z = \frac{(\dot{m}_A)_z}{A_z}$$

$$j_{AZ} \propto \frac{dw_A}{dz}$$

Over here in between 2 walls okay so this is a silica compact you can say okay having height H let us say now at the beginning let us say both the sides of this silica are having contact with air okay so it is having air so let us say at the beginning means whenever we have not started the time t less than 0 okay then let us say certainly you keep one side of this silica in contact with helium okay another side as this well remains in contact with air so in of in this silica one side you are having air another side you are having helium certainly your are keeping one side in contact with helium now silica will be observing helium  chemically okay.

So you can find out that slowly but steadily helium concentration we try to increase inside silica okay so after some time a little bit of time a small t you will find out the wall which is attached with the helium there we are having concentration of helium let us say the concentration of helium is actually wA so the high concentration of helium is over here okay and slowly you can find out it is diffusing so there is a gradient of concentration or there is a change of concentration towards the high towards the upper wall.

Where upper wall is in contact with air definitely no helium can be reached okay so if you wait for longer time then you can find out that it will be achieving some steady state and you will be find out a gradient of concentration of helium can be observed in this fashion where once again the bottom wall which is in contact with helium is having higher concentration and which is attached with the air side there we are having lower concentration okay now from here we can at least sat that this mass transfer rate inside this silica of helium will be actually very with respect to obviously z if z is more then you will be finding out or if z or H both are same over here.

Okay this is z direction and this I the magnitude of z for this present case which is H so you will finding out this mass transfer rate will be actually inversely proportional to this z direction or H spacing between the spices okay so if they are very close to each other then mass transfer rate will be enhancing and revising will happen when ever there away from each other okay at the same time if you see over here we are having some fixed helium  concentration at the beginning and here helium concentration was 0.

So this helium concentration wA0 if it increase then more and more helium can enter okay so that is form common sense that if the concentration is higher than obviously it will be giving more push for the helium to enter inside so as result we can say this m. or mass transfer for the helium is actually proportional to this w okay Wa0 right so and definitely this mass transfer rate will be increasing as the area of contact between the silica and helium increases is not because the contact area increases with more and more area is available for entry of helium.

So with this 3 things fix as proposed that m.A is actually propositional to wA0 whuich is the concentration of helium at the beginning m.A is inversely propositional to H okay which I have already said over here if the silica layer is becoming tine then mass transfer rate will be increasing as well as m.A is propositional to Ap which is nothing but the area through which the helium and silica escaping contact.

So if you add all this 3 terms of this equation then you will be getting m.A/ Ap is propositional to WA0 / h okay here as I have motioned Ap is plate area if we proceed further then we can defined this m.A/ Ap which is nothing but m.A is mass flux m.A mass transfer divide area which will become actually mass flux so this mass flux of A in z direction this becoming proportional to wA0 / H let us call that term mass flux as J so J is the mass flux let us say so JA in the direction of z is actually becoming proportional to WA0 / H okay.

Now as we have found out that this mass flux can be also retain in terms of the concentration so we can write down this mass flux is nothing but dwa/ dz okay dw/ dz is coming from here you see $AwA_0$ is actually the subtraction $W_{A0}$ – o so that is the gradient of the concentration which has actually caused this mass transfer, so we can write down this $W_{A0}$ as $dw_A$ so rate of change of concentration in-between the layers and this each can be written as dz okay. So change in location in the z-direction.

So we have written this z JAZ which is nothing but m.A/ Ap from here is equals to is proportional to $dw_A$ is coming from here and $D_z$ is coming from H, okay. So once we have this type of equation we can give one constant in the equation also let us give that constant as - $\rho D_{AB}$ okay, so this is the proportionality constant and this we call as fixed first law okay. So mass flux is actually equals to density x mass diffusivity x dw/dz, okay. This is the concentration gradient, okay.

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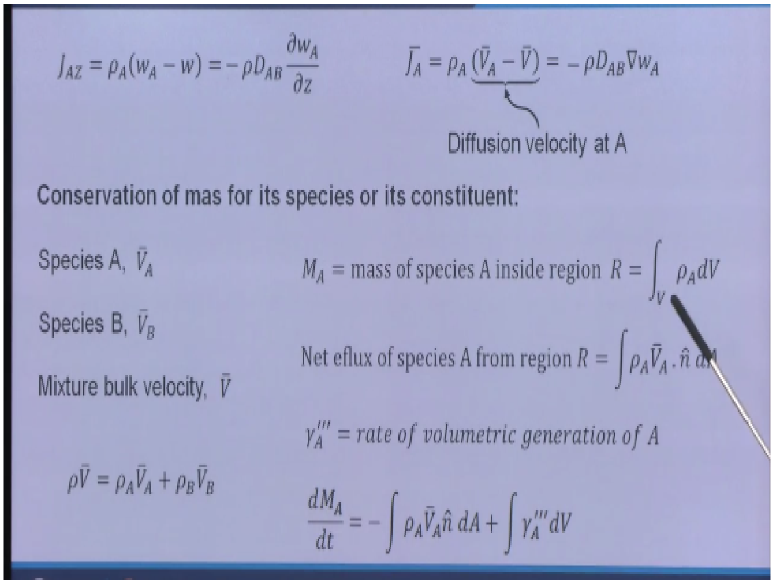

$$J_{AZ} = \rho_A(w_A - w) = -\rho D_{AB}\frac{\partial w_A}{\partial z} \qquad \bar{J}_A = \rho_A \underbrace{(\bar{V}_A - \bar{V})}_{\text{Diffusion velocity at A}} = -\rho D_{AB}\nabla w_A$$

Conservation of mas for its species or its constituent:

Species A, $\bar{V}_A$

Species B, $\bar{V}_B$

Mixture bulk velocity, $\bar{V}$

$$\rho\bar{V} = \rho_A\bar{V}_A + \rho_B\bar{V}_B$$

$M_A$ = mass of species A inside region $R = \int_V \rho_A dV$

Net eflux of species A from region $R = \int \rho_A \bar{V}_A . \hat{n}\, dA$

$\gamma_A''' = \textit{rate of volumetric generation of A}$

$$\frac{dM_A}{dt} = -\int \rho_A \bar{V}_A \hat{n}\, dA + \int \gamma_A''' dV$$

Next if we proceed future then this mass flux this mass flux can be written as $\rho W_{A}$ – W okay so if you take this component so here we can write down $J_A$ is nothing but $\rho_A$ x $\rho_A$ – V okay is equals to ρ x $D_{AB}$ x $\nabla_{WA}$ okay, so ρ x $D_{AB}$ x $\nabla_{WA}$ okay, so here this is nothing but your diffusion velocity okay this is nothing but your diffusion velocity at A okay. So once you have this one then we can easily go for conservation of mass.

This is mistake conversation of Mass observation of mass for its species or its constituents okay so that if we try to derive now let us consider that the species A is having velocity $V_A$ and species B is having velocity $V_B$ okay. And with this if you try to get the mixture velocity so mixture velocity you definitely can be written as V = $\rho_A$ $V_A$ + $\rho_B$ $V_B$ / ρ mixture density ρ okay.

So this equation we get from the mixture wall velocity V, if you proceed further and try to have a balance if mass transfer then let us see if mass of species A inside region R if you try to find out then what we need to do we need to integrate for the whole region, region R so volume of the region is V and we are actually finding out $\rho_A$ $D_V$ because we know $\rho_V$ is actually your mass.

So that we are doing for this arbitrary region R having volume V okay. So this is the mass of the species A inside region R, similarly if we try to find out net efflux, okay. Net efflux of species A from region R because there will be lots of surfaces so from there if we try to find out the net efflux whatever it is coming in or going out, then we can write down that is equals to integration of $\rho_A$ $V_A$. n^.

This n^ is nothing but surface normal of the region R, okay. So $\rho_A$ $V_A$ . n^ x $D_A$ okay. So this we have got as net inflow or outflow summation of net inflow or outflow or net efflux then let us say there is some sort of volumetric generation of species A also insider region R this may happen whenever you are having chemical reaction, okay. So this can be positive as well as negative but here let us consider that volumetric generation we are having.

If we are having volumetric absorption then in that case it will be negative okay. So this is $\gamma_A$ ‘” we are taking as volumetric generation okay. Then if we try to balance then we can write down this term that means this mass of the species is change of the mass of the species okay which will be nothing but $D_{MA}$ dt okay, so if you try to do $D_{MA}$ dt is equals to whatever flux we are getting okay plus you whatever generation you are having over the whole volume , okay.

So this conservation of the species we can write down easily by taking arbitrary region R okay.

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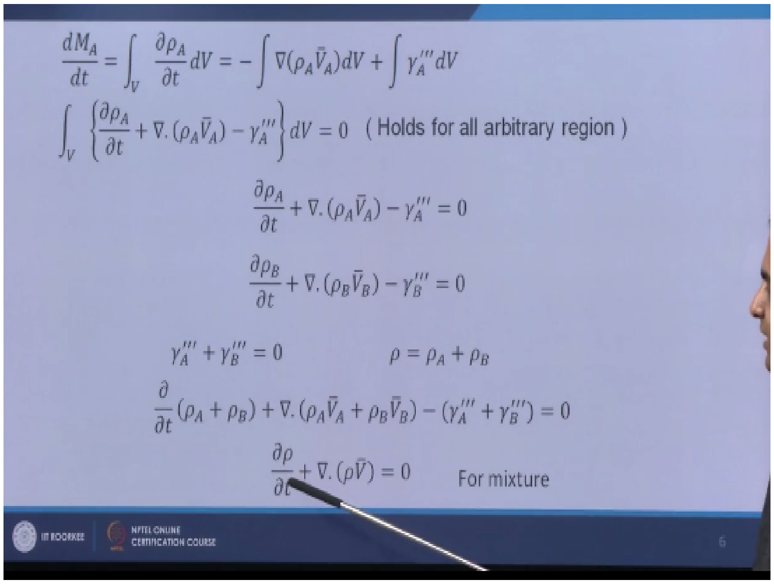

$$\frac{dM_A}{dt} = \int_V \frac{\partial \rho_A}{\partial t} dV = -\int \nabla(\rho_A \bar{V}_A) dV + \int \gamma_A''' dV$$

$$\int_V \left\{\frac{\partial \rho_A}{\partial t} + \nabla.(\rho_A \bar{V}_A) - \gamma_A'''\right\} dV = 0$$ ( Holds for all arbitrary region )

$$\frac{\partial \rho_A}{\partial t} + \nabla.(\rho_A \bar{V}_A) - \gamma_A''' = 0$$

$$\frac{\partial \rho_B}{\partial t} + \nabla.(\rho_B \bar{V}_B) - \gamma_B''' = 0$$

$$\gamma_A''' + \gamma_B''' = 0 \qquad \rho = \rho_A + \rho_B$$

$$\frac{\partial}{\partial t}(\rho_A + \rho_B) + \nabla.(\rho_A \bar{V}_A + \rho_B \bar{V}_B) - (\gamma_A''' + \gamma_B''') = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{V}) = 0$$ For mixture

Next once we have this arbitrary region R then you can put this value of $M_A$ as we have derived in the lass slide so d/dt($M_A$) become if we take the differentiation inside so it will be $\partial/\partial t$ ($\rho_A$) dv and that is having integration over the whole volume, okay. And in the right hand side we are having a flux and generation term respectively, okay. Then if you take all this term in one side and try to take derivative of the all terms then you get this fashion integration of V is into integration of over the whole domain over whole volume.

And then $\partial/\partial t$ ($\rho_A$) + $\nabla.\rho_A V_A - \gamma_A'''$ and that is integrated over the whole volume V okay. And now you see we have taken a arbitrary region R having volume V so this holds for any arbitrary region so we can say basically not this integration the term whatever we are having inside that is equals to 0 so let us take this term is equal to 0 and here we have written the term for species A as well as species say very much generalized we can take the similar equation for species V also, okay.

So we are having two species let us say species A and species B, the conservation equation goes like this, okay. Then let us take a two things over here first one is let us say it is only bi-component mixtures so whatever generation is there for species A and species B their summation will be equals to 0, why? Because in a overall system mass cannot be generated, so if this system is generating mass A is generating mass.

Then definitely V will be destroying mass, okay. So $\gamma_A''' + \gamma_B'''$ will be obviously equals to 0 and on the other hand just now we have shown that density ρ can be summation of ρ difference species ρ so $\rho = \rho_A + \rho_B$ let us consider at okay so with this if we add this two equations then we can get the first term becomes $\partial / \partial t\, (\rho_A + \rho_B)$ which can be written as $\partial / \partial t\, (\rho)$, second two terms summation becomes $\nabla( \rho_A V_{A\, +} \rho_B V_B)$ okay.

So this once again can be written as $\rho_v$ for the mixture as we have shown over here, $\rho_v = \rho_A V_A + \rho_B V_B$ okay which is the mixture wall velocity. So here this term this two terms will be giving me $\nabla(\rho.V)$ and the third two terms by virtue of this $\gamma_A{}^{‘”} + \gamma_B{}^{‘”}$ goes to 0, so for the mixture we get simple continuity equation in this fashion, okay. So considering mixture as homogenous this equation we get, okay.

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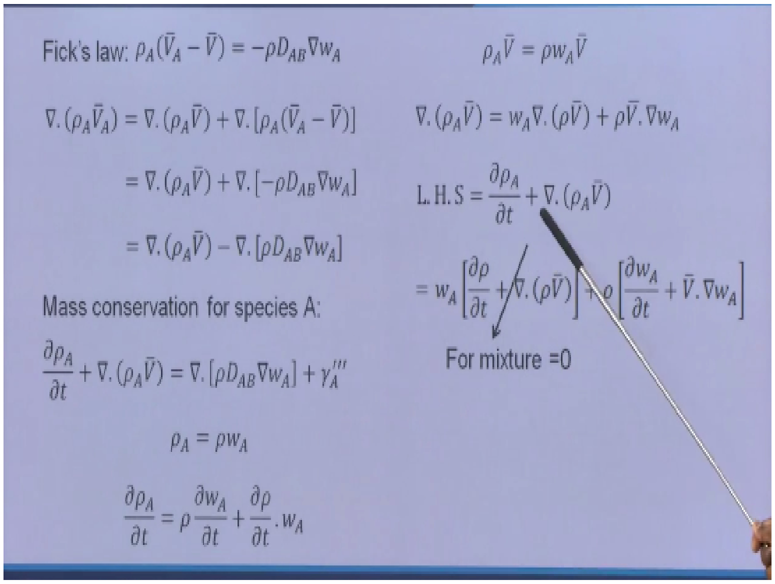

Fick's law: $\rho_A(\bar{V}_A - \bar{V}) = -\rho D_{AB} \nabla w_A$

$$\nabla.(\rho_A \bar{V}_A) = \nabla.(\rho_A \bar{V}) + \nabla.[\rho_A(\bar{V}_A - \bar{V})]$$
$$= \nabla.(\rho_A \bar{V}) + \nabla.[-\rho D_{AB} \nabla w_A]$$
$$= \nabla.(\rho_A \bar{V}) - \nabla.[\rho D_{AB} \nabla w_A]$$

Mass conservation for species A:

$$\frac{\partial \rho_A}{\partial t} + \nabla.(\rho_A \bar{V}) = \nabla.[\rho D_{AB} \nabla w_A] + \gamma_A'''$$

$$\rho_A = \rho w_A$$

$$\frac{\partial \rho_A}{\partial t} = \rho \frac{\partial w_A}{\partial t} + \frac{\partial \rho}{\partial t}.w_A$$

$$\rho_A \bar{V} = \rho w_A \bar{V}$$

$$\nabla.(\rho_A \bar{V}) = w_A \nabla.(\rho \bar{V}) + \rho \bar{V}.\nabla w_A$$

$$L.H.S = \frac{\partial \rho_A}{\partial t} + \nabla.(\rho_A \bar{V})$$

$$= w_A \left[\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{V})\right] + \rho \left[\frac{\partial w_A}{\partial t} + \bar{V}.\nabla w_A\right]$$

For mixture =0

Next let us go to Fick’s law this already we have shown that this is the derivation of Fick’s law we have obtained this was actually the mass flux J in the left hand side and here we are having diffusivity and then the gradient of the concentration, okay. So if we use this Fick's law part and try to have modification of this component $\nabla( \rho_A V_A)$ so let me show you how that is changing.

$\nabla( \rho_A V_A|)$ let us try to write down as $\nabla( \rho_A V) + \nabla( \rho_A V_A - V)$ so here you see this $\rho_A V$ is cancelling so this term we have added and subtracted in this. So if we do so here in this side we can actually replace it by Fick's law so you see this $\rho_A V_A – V$ over here it is coming so we can write down this is nothing but actually $\rho D_A V \times \nabla_A$ okay so that we have written over here $- \rho D_A V \times \nabla w_{A,}$ okay.

So if you do so then this second term of the species momentum equation we can write down in this fashion $\nabla \rho_A V – \nabla. \rho D_{AB} \times \nabla W_A$ Okay. Proceed further if we proceed further then you see the second term which was here this term if we replace by this two terms then this term can be taken in the right hand side so we get the mass conservation equation for species A in this fashion, okay. So this is the mass conservation equation for species A.

Now let us consider we know that $\rho_A$ is nothing but $\rho w_A$ this already in the first slide I have shown so if you try to use that and make derivative with respect to T then you get $\partial \rho_A/\partial t = \rho \partial W_A/\partial t + w_A$ you can keep fixed and then $\partial/\partial t (\rho)$ so this is actually chain rule we are using over here for derivative. So this we will be replacing over here in the mass conservation equation, okay.

So and for the second term you see for the second term we are having $\rho_A \times V$ so $\rho_A X v$ that once again can be written easily as, as we know $\rho_A$ is nothing but $\rho \times W_A$ species concentration so this we can write as $\rho_A X v = \rho W_A \times V$ okay and if we go for spatial derivative like this for the left hand side then we are getting $W_A \times \nabla. Pv + \rho v \times \nabla. W_A$ okay once again chain rule we have used, now let us put both this one and this $\partial/\partial t(\rho_A)$ over here in the mass conservation of species A.

And then in the left hand side atleast both the terms whenever we put then we can see we are getting $W_A (\partial \rho/\partial t) +\nabla. V$ okay. So this for mixture momentum equation just now we have showed this is equals to 0 okay. So this already we have showed over here this is equals to 0, so we can reduce that part of the equation this goes to 0, so in the left hand side we have only this term okay, which is nothing but this can be written as ρ x total derivative of w this is nothing but total derivative of w, okay. So once we incorporate this term over here in the mass conservation of species A.

Then finally we get $\rho$ x total derivative of w = $\nabla$. $\rho D_{AB}$ x $\nabla$. A+ $\gamma_A^{‘”}$okay which is the generation, okay. So this is the species conservation equation for FSA.

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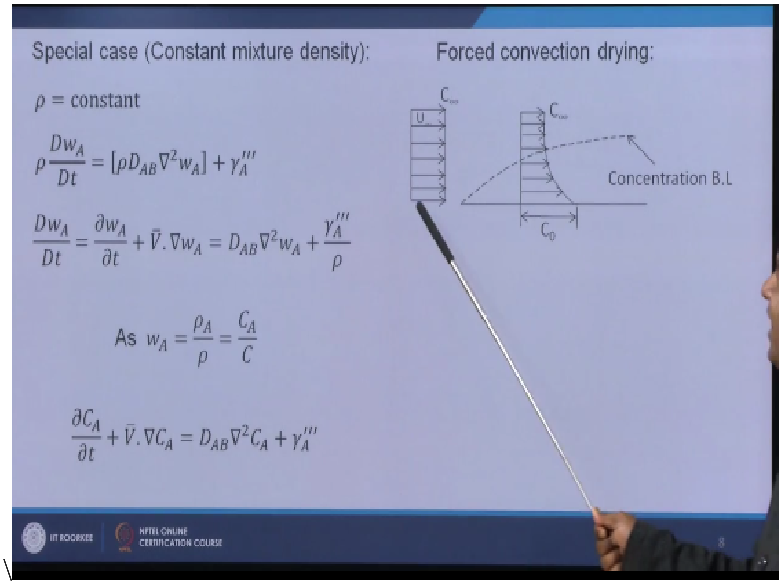

Special case (Constant mixture density):

$\rho$ = constant

$$\rho \frac{Dw_A}{Dt} = [\rho D_{AB} \nabla^2 w_A] + \gamma_A'''$$

$$\frac{Dw_A}{Dt} = \frac{\partial w_A}{\partial t} + \bar{V}.\nabla w_A = D_{AB} \nabla^2 w_A + \frac{\gamma_A'''}{\rho}$$

As $w_A = \frac{\rho_A}{\rho} = \frac{C_A}{C}$

$$\frac{\partial C_A}{\partial t} + \bar{V}.\nabla C_A = D_{AB} \nabla^2 C_A + \gamma_A'''$$

Forced convection drying:

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Next let me show you some special case if the constant density mixture we are having then what we can do, we can consider $\rho$ as constant so in this equation if we keep the $\rho$ constant so this can come out and $D_{AB}$ is always a constant so you can get the equations simplifies to $\rho$ and $D_{AB}$ came out, so this simplifies to this form, okay. And if you try to write down this total derivative in the form of $\partial/\partial t$ + V. $\nabla$.

Then the equation goes like this, okay. The equation goes like this and you divide by $\rho$ then the equation goes like this, okay. Proceeding further this $W_A$ is nothing but $C_A/C$ okay concentration of A by the overall concentration okay, so many a times this overall concentration will be equals to 1, okay for by component mixture atleast, okay. So here you will be finding out if we put this $W_A = C_A/ C$ over here in this equation.

So this whole equation is being converted top this form okay so $\partial/\partial t\ (CA) + V.\ \nabla C_A = D_{AB} \nabla^2 C_A + \gamma_A'''$ which is the generation part okay. So we have observed constant mixture density special case and then let us try to see during this type of diffusion how heat transfer is coming into picture so first I will be giving you one example where it is more or less similar to forced conviction. So let us say you are having a plate over this okay where the concentration of certain species is c okay, and here we are having some influent just similar to our flow over flat plate we are having some mean flow having concentration $C_\infty$, so we are having a concentration gradient over here, so just similar to your temperature profile if you try to plot over here the concentration profile so you will be getting at had with the wall of the plate you will be having high concentration whereas in the first team it is actually tending towards the $C_\infty$ the first team concentration, okay.

And similar to our thermal boundary layer we can also consider a concentration boundary layer which is nothing but where the concentration is reaching 99% of the first team concentration $C_\infty$ okay. So this type of concentration boundary layer can also be plotted analogous to your thermal boundary layer. Now let me show you how concentration equation can be plotted so you see if we consider that we are have informed this equation let us say consider we are having steady case.

So the first term drops down the second term is actually giving you for two dimensional situation $u\partial C/\partial x+v\partial V/\partial y$ okay, and if we consider that there is no reactions so this term will be going to 0 in the right hand side we are having $D_{AB}$ and then if we neglect just following our thermal boundary layer if we neglect $\partial^2 C/Vx^2$ okay, so this is x direction this is y direction so if we neglect that term then in the approximation of boundary layer this becomes my concentration equation along with the boundary conditions definitely at $y=0$, $C=C_0$

And at $y/\delta_0$ tends to $\infty$ that means whenever we are going away from the plate C becomes $C_\infty$ or the first team concentration. So this becomes my concentration equation and these are the boundary condition, okay this is similar to your forced convection situation.

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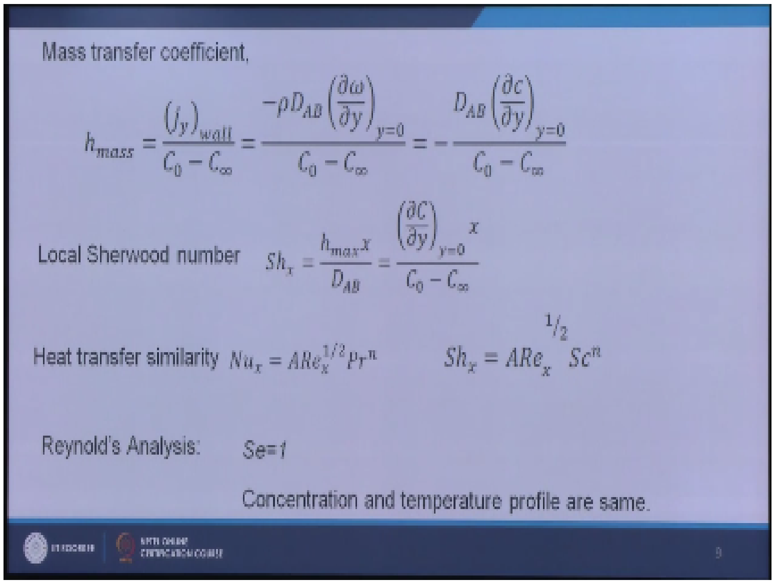

Mass transfer coefficient,

$$h_{mass} = \frac{(j_y)_{wall}}{C_0 - C_\infty} = \frac{-\rho D_{AB}\left(\frac{\partial \omega}{\partial y}\right)_{y=0}}{C_0 - C_\infty} = -\frac{D_{AB}\left(\frac{\partial c}{\partial y}\right)_{y=0}}{C_0 - C_\infty}$$

Local Sherwood number $Sh_x = \frac{h_{max}x}{D_{AB}} = \frac{\left(\frac{\partial C}{\partial y}\right)_{y=0} x}{C_0 - C_\infty}$

Heat transfer similarity $Nu_x = ARe_x^{1/2}Pr^n$ $\qquad Sh_x = ARe_x^{1/2}\ Sc^n$

Reynold's Analysis: $Se=1$

Concentration and temperature profile are same.

If you proceed further and try to find out the mass transfer coefficient just like or heat transfer coefficient, so h in the heat transfer coefficient it was $\dot{q}$ the temperature difference here it will be mass flux $J_y$ and at wall definitely okay, because we are interested in heat transfer, mass transfer coefficient at wall and this is nothing but concentration gradients $C_0$-$C_\infty$ okay. So if you use our fix law then this J can be written in this fashion and finally this, you see this ω can be converted to C as we have shown in the previous few slides the relationship between ω and C.

So ultimately the mass transfer coefficient comes out as this term okay, so minus diffusivity into ∂z/∂y at y=0 that means at wall divided by $C_0$-$C_\infty$ the concentration gradient, okay concentration difference okay. Next let us try to define local schedule number this is analogous to your Nusselt number so if you try to define Nusselt number that becomes hx/k here it is nothing but h mass transfer this is actually mistake so this is h mass transfer into x/$D_{AB}$ okay.

So if you do that then from if you put this equation over here then this schedule number becomes ∂C/∂y at y=0/x/$C_0$-$C_\infty$ okay. If we try to find out heat transfer similarity in heat transfer convective heat transfer we have shown Nusselt number will be nothing but $R^{1/2 \text{ and}}$ $Pe^n$ so the value of n depends on the limits of Pe, low Pe limit and high Pe limit.

Here also we find out schedule number is nothing but $AR^{1/2}$ and new number which is analogous to Pe comes over here as Smith number, okay this is Smith number later on I will be telling you what is Smith number we can write down this local schedule number in the function of Renolds number and Smith number, okay. So if you go for Renolds analysis then this Smith number can be written equals to 0 this is once again mistake this is Se Smith number okay, concentration and temperature profile then we will be looking analogous are same in case of Smith number equals to 1 okay.

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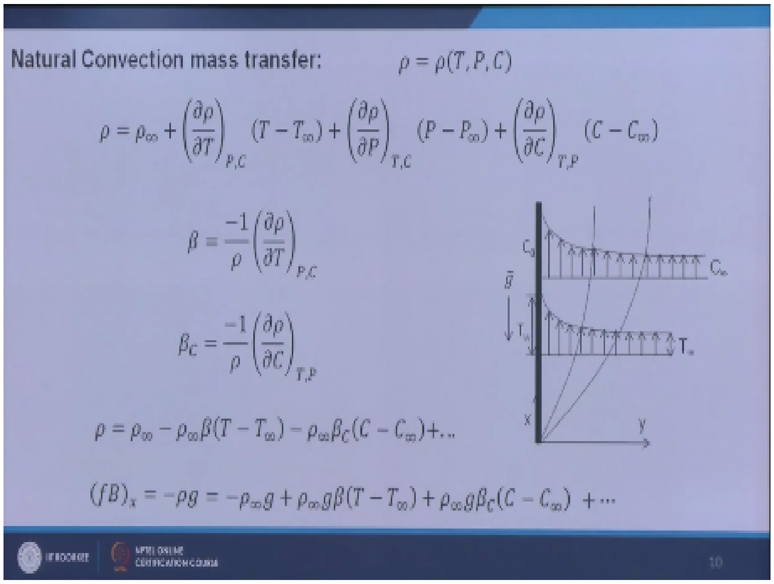

Natural Convection mass transfer: $\rho = \rho(T, P, C)$

$$\rho = \rho_\infty + \left(\frac{\partial \rho}{\partial T}\right)_{P,C}(T - T_\infty) + \left(\frac{\partial \rho}{\partial P}\right)_{T,C}(P - P_\infty) + \left(\frac{\partial \rho}{\partial C}\right)_{T,P}(C - C_\infty)$$

$$\beta \equiv \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{P,C}$$

$$\beta_C = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial C}\right)_{T,P}$$

$$\rho = \rho_\infty - \rho_\infty \beta (T - T_\infty) - \rho_\infty \beta_C (C - C_\infty) + \ldots$$

$$(fB)_x = -\rho g = -\rho_\infty g + \rho_\infty g \beta (T - T_\infty) + \rho_\infty g \beta_C (C - C_\infty) + \cdots$$

Then let us see the another dimension which is nothing but natural convection driven mass transfer, so let us have lead vertical like this now, we are having gravity acting this in the downward direction and here attached to the plate we are having high concentration $C_0$ and away from the plate we are having some you know ∞ symbol concentration $C_\infty$ okay, so if you see like this just like the temperature profile so here it is $T_w$ here it is $T_\infty$ so temperature profile will be like this our concentration profile will be also like this.

And along with the temperature thermal boundary layer we can also have the concentration boundary layer, so this is your thermal boundary layer and here we are having the concentration boundary layer. The concept remains same 99% of the full steam concentration, okay. So if you do like that then we have to also see that how momentum equation is going to change especially the buoyancy term because this is natural convection, so buoyancy term earlier what we have done using buoyancy approximation we have written $\rho$ equals to this first term only where $\partial\rho/\partial T$ came into picture, but here in this case we have to also see the $\partial\rho/\partial C$ term because concentration is also, concentration gradient is also coming into picture, okay. So this term we need to add okay, once we add this and let us write down earlier we know that $\beta$ is nothing but $-1/\rho\ \partial\rho/\partial T$ at constant pressure and concentration.

Now let us define $\beta_C$ okay, so $\beta_C$ is for the concentration is equals to $-1/\rho\ \partial\rho/\partial C$ at constant temperature and pressure, if you do so then this terms we can replace by $\beta\ \partial T$ so let us do that, so $\rho- \rho=\rho_\infty$- of $\rho\infty$ $\beta C$ over here okay, $\rho\infty$ into $\beta C$, $C – C_0$ okay , so this pressure $\partial \rho/ \partial P$ we are neglecting, so if you do so your buoyancy term will be having one extra term already we had only this is $\rho\infty$ into g and we had this term which actually given raise to rally number now we are having another extra term duct to the concentration decency - $C\infty$.

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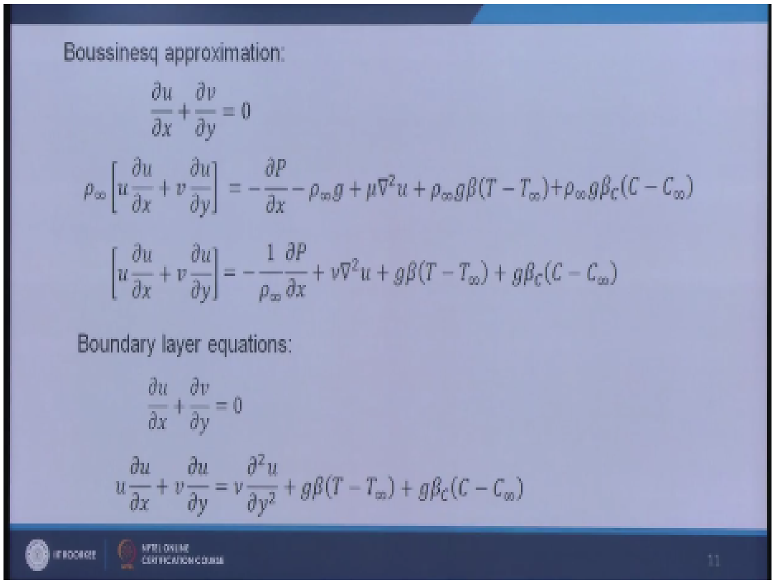

Boussinesq approximation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho_\infty\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial P}{\partial x}-\rho_\infty g+\mu\nabla^2 u+\rho_\infty g\beta(T-T_\infty)+\rho_\infty g\beta_C(C-C_\infty)$$

$$\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{1}{\rho_\infty}\frac{\partial P}{\partial x}+\nu\nabla^2 u+g\beta(T-T_\infty)+g\beta_C(C-C_\infty)$$

Boundary layer equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta_C(C-C_\infty)$$

Okay and if you proceed further to the momentum equation you see earlier the momentum equation was up to this where buoyancy term ahs taken this form okay, along with this one here the third term will be also present okay, and if you proceed further then you will be finding out this term will be prevail like this and subsequently here also in the boundary layer approximation along with this gβ ΔT and we will be having gβ C ΔC also okay. So these are the boundary layer equations where mass term for an heat transfer present simultaneously okay and continuity and momentum equation.

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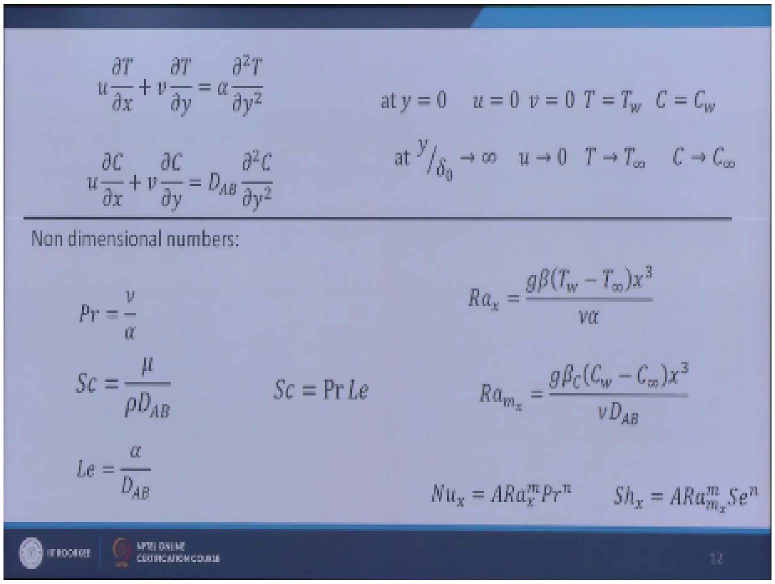

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

at $y=0$ $\quad u=0\quad v=0\quad T=T_w\quad C=C_w$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_{AB}\frac{\partial^2 C}{\partial y^2}$$

at $y/\delta_0\rightarrow\infty\quad u\rightarrow 0\quad T\rightarrow T_\infty\quad C\rightarrow C_\infty$

Non dimensional numbers:

$$Pr=\frac{\nu}{\alpha}$$

$$Ra_x=\frac{g\beta(T_w-T_\infty)x^3}{\nu\alpha}$$

$$Sc=\frac{\mu}{\rho D_{AB}}$$

$$Sc=\Pr Le$$

$$Ra_{m_x}=\frac{g\beta_C(C_w-C_\infty)x^3}{\nu D_{AB}}$$

$$Le=\frac{\alpha}{D_{AB}}$$

$$Nu_x=ARa_x^m Pr^n \qquad Sh_x=ARa_{m_x}^m Se^n$$

nd the corresponding obviously here energy equation will be remaining same convection and conduction to be an subsequently we have to also I have the transfer equation of spaces which I can write down like this earlier also I have shown this equations for forced convection case okay, so it will remain same and if you see the boundary condition this momentum equation and energy equation boundary condition already we have discussed so for the concentration will be having C equal to Cw at wall okay and C tends to C∞, away from the wall okay.

So once you do this thing then you can easily solve Nusselt convection thing, now let me show you what non - dimensional numbers are important for mass transfer already we know Pr which is nothing but v / α okay similarly to that we will be also having Sc number which is nothing but μ/ ρDAB where this Dab is nothing but the mass diffusivity which I have introduced in this lecture okay, then we will be having lease number lease number is nothing but α/ D AB α you know thermal diffusivity.

DAB is the mass diffusivity okay, so lease number give you the relationship between the thermal and the mass transfer effects okay, now if you try to manipulate this with number then you will be getting smith number is equals to nothing but Pr number into lowest number so smith number equals to Pr into Le okay on the other hand if you try to see the heat transfer extreme so rally number was actually gβ ΔTx3 / α, so here we are having mass transfer rally number Ram okay so that will be gβC so remember we have introduced βC and here in place of ∂T will be having.

∂C and this α will be changing into your diffusivity thermal diffusivity will be changing into mass diffusivity okay, so this new numbers will be handling and also we have see that the how Nu term analog oscillation Sh we have got for schedule number, so here Nu relationship was for Nu convection into was in terms of rally number here it will be mass terms for an rally Ram and Pr will be transforming to smith number, this is also Ac this is not AC okay, so all this analog stings we have seen in this lecture.

Let us summarize now so first what we have done in this lecture we have scattered the fix law, so we have seen fix law consult in this fashion.

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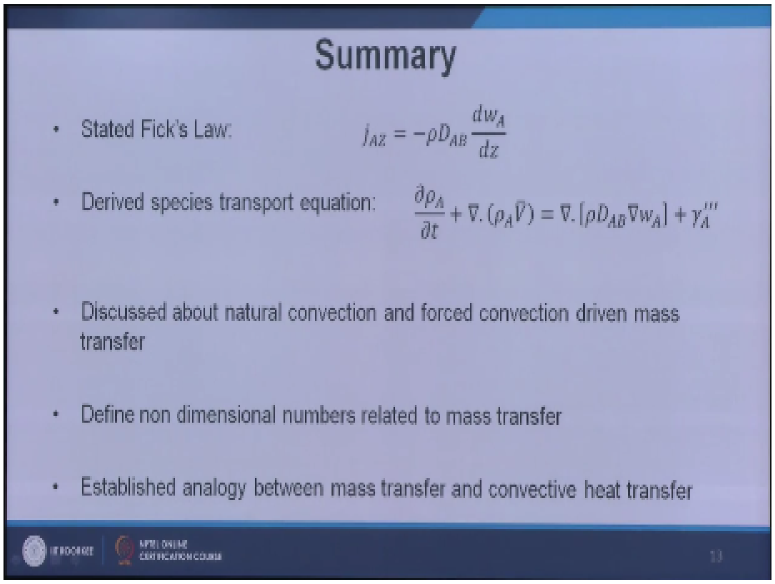

## Summary

- Stated Fick's Law: $j_{AZ} = -\rho D_{AB} \frac{dw_A}{dz}$
- Derived species transport equation: $\frac{\partial \rho_A}{\partial t} + \bar{\nabla}.(\rho_A \bar{V}) = \bar{\nabla}.[\rho D_{AB} \bar{\nabla} w_A] + \gamma_A'''$
- Discussed about natural convection and forced convection driven mass transfer
- Define non dimensional numbers related to mass transfer
- Established analogy between mass transfer and convective heat transfer

The we have showed you the transport equation for individual spaces and we have shown that transport equation of individual species goes like this where diffusivity is over here generation is over here, and this is the inertia term for the individual spaces okay, then we have discussed about natural convection and forced convection driven mass transfers, so this is two separating cases we have shown, we have also defined non-dimensional numbers related to mass transfer and finally established analogy between the mass transfer and convective heat transfer okay, so with this I will be ending my lecture.

But before leading let me test your understanding what we have understood in this lecture, where having three questions.

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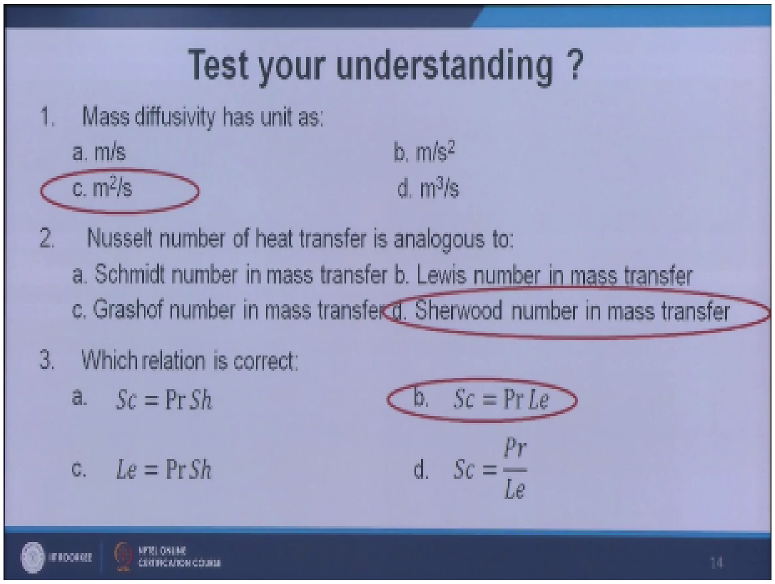

## Test your understanding ?

1. Mass diffusivity has unit as:

a. m/s b. $m/s^2$

[✓ c. $m^2/s$] d. $m^3/s$

2. Nusselt number of heat transfer is analogous to:

a. Schmidt number in mass transfer b. Lewis number in mass transfer

c. Grashof number in mass transfer [✓ d. Sherwood number in mass transfer]

3. Which relation is correct:

a. $Sc = Pr\,Sh$ [✓ b. $Sc = Pr\,Le$]

c. $Le = Pr\,Sh$ d. $Sc = \frac{Pr}{Le}$

So first question is mass diffusivity has in it as so we are having m/s, m/s2 and m3/s, so it is very simple in the first slide I have told you so easily correct terms that is m2/ s, second question Nusselt number of heat transfer is analog us to smith number in mass transfer Lewis number in mass transfer grass number or Sherwood number, so obviously we have understood we have discuss some time is Nusselt number is actually analog us to Sherwood number, so this is the correct answer last which relationship is correct okay , so Sc = pr into Sh Sherwood number smith number is equals to Pr into Le.

Le = Pr into Sh and smith number is = Pr / Le so obviously this also we have said that smith number is equals to Pr into Le this is the correct answer okay so with this I will be ending this lectures in our last lecture we will be discussing about some problems on convection inside a duct and as well as we will be discussing about mass transfer okay, so if you having any query please do not forget to keep posting in our discussion forum thank you.

**For Further Details Contact**

**Coordinator, Educational Technology Cell**
**Indian Institute of Technology Roorkee**
**Roorkee – 247667**
**E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in**
**Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in**

**Production Team**
**Sarath K. V**
**Jithin. K**
**Pankaj Saini**
**Arun. S**
**Mohan Raj. S**

**Camera, Graphics, Online Editing & Post production**
**Binoy. V. P**

**NPTEL Coordinator**
**Prof. B. K. Gandhi**

**An Educational Technology cell**
**IIT Roorkee Production**