

**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Convective Heat Transfer**

**Lec- 18**

**Heat Transfer with Phase Change**

**Dr. Arup Kumar Das**

**Department of Mechanical and Industrial Engineering**

**Indian Institute of Technology Roorkee**

Hello welcome in the 18<sup>th</sup> lecture of convective heat transfer in our previous lectures we have discussed about single phase convection here in this lecture we will be actually describing the heat transfer with change a phase okay, so our topic is heat transfer with phase change now first let me tell you what kinds will be covering in this lecture we will be understanding phase change modes.

(Refer Slide Time: 00:45)

**Outline of the Lecture**

- Understand phase change modes in context of convective heat transfer
- Introduce boiling and condensation and its different modes
- Understand velocity and temperature profile in film condensation
- Evaluate condensate rate and profile of film accumulation over the surface
- Mention governing equations for film boiling situations

EE 600333    HTEL ONLINE CHEMICAL ENGINEERING COLLEGE

In context of convective heat transfer what are the different phase change modes available where convection heat transfer is being observed okay, after that we will be introducing boiling and condensation with its different modes okay, we will be also understanding what is the velocity and temperature profile in film condensation and side by side will be also discussion that what are the equations required for boiling cases okay. You will be evaluating condensate rate and profile film accumulation over the surface okay over the cold surface and then we will be understanding.


The governing equation for film boiling as I have mentioned apart from film condensation we will be also learning film boiling situations okay.


(Refer Slide Time: 01:34)

### Convective mode in phase change (Gas-Liquid)

Two Categories

- ❑ Condensation (Vapor to liquid)
  - ❖ Cooling required
  - ❖ Droplet formation at low subcooling
  - ❖ Liquid film formation at high subcooling
- ❑ Boiling (Liquid to Vapor; Heating required)
  - ❖ Heating required
  - ❖ Bubble formation at low superheat
  - ❖ Gas film formation at high superheat





So let me start first let me showing what are the effective convective modes involving questions as we are having convection, so definitely solid we will not be coming into picture we have to consider only the liquid and gaseous phases and liquid and gaseous phase change process are actually involved with two categories condensation and boiling here I have shown the picture you can see in this picture the water is being boiled and it is forming some sort of vapor or water vapor which is actually striking this bottom side of this plate, oh what the plate we are having ice okay.

So here you can see this is actually forming vapor so liquid is actually forming vapor and on the other hand you are having the ice so plate is having very low temperature so this vapor is immediately condensing and here you can see the drop of liquid, so in this plate the vapor is being converted to liquid so this is nothing but condensation and here the liquid is being converted to vapor and which is coming out from this kettle mouth, so this is actually boiling. So this two figures actually are involved with convective modes so that is why here in this lecture we are going to discuss about condensation and boiling okay.

Melting solidification and sublimation those still we are not going to discuss over here, because those are related to solid where convection mode is not that much observed okay, so let me now tell that what are the characteristics of convection condensation and boiling, so for condensation we require cooling where as for boiling we require heating so here requires heating here requires

cooling okay next droplet formation at low sub cooling so for condensation you can find at low sub cooling, what is sub cooling is the temperature is lower than the saturation temperature that is called sub cooled condition.

And the degree of lowers is actually called sub cooling okay, so droplet formation we can observe at low sub cooling where as in case of boiling bubble formation can be observed at low super heat okay, so individual bubbler and individual and individual drop can be observed in cases of boiling and condensation of boiling and condensation respectively at low super heat and low sub cooling next liquid film formation at high sub cooling we can see if the sub cooling is very high we can find out if film of liquid is being is formed over here due to condensation in the same manner.

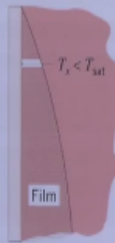
In case of high super heat gas film formation can be observed for boiling situation okay, so what happens that high degree of sub cooling or super heat and what happens at low degree of sub cooling and super heat those things we have discussed over here.

(Refer Slide Time: 04:39)

**Condensation:**

Heat transfer to a surface occurs by condensation when the surface temperature is less than the saturation temperature of an adjoining vapor.

- Film Condensation
  - Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.
  - Thermal resistance is reduced through use of short vertical surfaces and horizontal cylinders.
  - Characteristic of clean, uncontaminated surfaces



The diagram shows a vertical surface on the left. A red liquid film is shown flowing down the surface. A horizontal line indicates the surface temperature, labeled  $T_s < T_{sat}$ . The word 'Film' is written in a box at the bottom of the liquid layer.

STUDENTS | HOTEL ONLINE CERTIFICATION COURSE

Next let me start with the first mode which is nothing but condensation, so condensation even actually heat transferred with surface heat transfer to a surface occurs by condensation when the surface temperature is less than the saturation temperature this is very important surface temperature let see this is the surface temperature will be less than the saturation so  $T$  is actually less than the saturation temperature, okay saturation temperature is adjoining vapor adjoin vapor so if we are having some vapor over here so its corresponding pressures saturation temperature.

That temperature will be higher compared to the surface temperature then only condensation will happen okay. Next have I have mentioned we are having two modes of condensation where film is being produced where a particular droplets of liquid you can see, so those are actually film wise condensation and dropwise condensation so first let me discuss about film wise condensation, so film condensation is nothing but in this case entire surface is covered by condensate so if it is a surface entire surface you can see is covered by this a liquid condensate okay.

Which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface as it is totally covered by the liquid no phase vapor can come in contact to the code surface and further condense, so this gives a resistance okay, so this liquid film actually gives a resistance to heat transfer okay, next thermal resistance is reduced through use of hard

vertical surface is to break this condensate what you can so small, small plates you can use so this single plate you can break into small, small plates which will be breaking the liquid flow so thermal resistance is getting use through use of short vertical surfaces.

Or horizontal surfaces if you put horizontal surface then obviously liquid will not be draining and we will find out resistances actually, being increases okay. So resistance is increasing then characteristic of clean and uncontaminated surface, so in case of clean and uncontaminated surface drop wise condensation will be absent and we can find out majority of the surface is actually covered by film and we can have in condensation okay.

(Refer Slide Time: 07:01)



• Dropwise Condensation



- Surface is covered by drops ranging from a few micrometers to agglomerations visible to the naked eye.
- Thermal resistance is greatly reduced due to absence of a continuous film
- Surface coatings may be applied to inhibit *wetting* and stimulate dropwise condensation

Steam condensation on copper surfaces:

$$q = \bar{h}_{dc} A (T_{sat} - T_s)$$

$$\bar{h}_{dc} = 51,100 + 2044 T_{sat} \quad 22^\circ\text{C} < T_{sat} < 100^\circ\text{C}$$

$$\bar{h}_{dc} = 255,500 \quad T_{sat} > 100^\circ\text{C}$$



 IIT KHARAGPUR
  NPTEL ONLINE CERTIFICATION COURSE
 5

Next the other mode so dropwise condensation so here surface is covered by drops in place of liquid film whatever I have shown you in the previous slide, here we are having using liquid drops okay, so surface is covered by drops ranging from few micrometer to agglomeration visible to the naked eye, so there is small droplets also can be seeing which are actually you know very small in the range of micrometers and after agglomeration those micrometer droplets can become very big and even if it can be visible with our naked eye okay.

So this type of thing you can see that these droplets are visible to the naked eye, simultaneously you can see very small droplets are over there, at different places. Next, thermal resistance is greatly reduced due to the absence of a continuous film as vapor can come directly in contact with the surface. Okay, in this case, in between the droplets, so resistance is greatly reduced. Okay, then surface coating may be applied to inhibit wetting and stimulate droplet condensation so what you can do over the surface issues apply special type of coating.

Then you can find out that film wise condensation is actually discontinued and it will be forming some sort of dropwise condensation like this, if you play into relativity okay then steam condensation on copper surface if you see then that can be written in this fashion  $q = h_c (T_{sat} - T_s)$ ,  $h_c$  is the coefficient of dropwise condensation into  $h_c$  of the surface at which the condensation is happening – of  $T_{sat} - T_s$ ,  $T_s$  is the surface temperature  $T_{sat}$  is nothing but the saturation temperature for the vapor at that give impression okay, so this  $h_c$  is nothing but the heat transfer coefficient for the dropwise condensation there are several correlations for finding out this heat transfer coefficient I will tell you one correlation, so  $h_c$  this is can be written as  $51100 + 2044 (T_{sat} - T_s)^{0.75}$  into saturation temperature okay, if saturation temperature lies in between  $22^\circ$  to  $100^\circ$  okay.

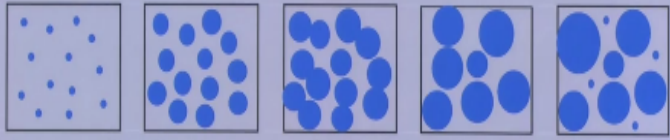
And if the saturation temperature is more than  $100^\circ$  then simply you can write down  $h_{dc} = 25500$  okay, so this type of correlations people used to find out the heat transfer coefficient for droplets condensation and ultimately obtained what is the heat flux for a situation where droplet condensation is happening okay.

(Refer Slide Time: 09:36)

**Stages in droplet evolution:**

Processes involved in dropwise condensation are:

1. Initial nucleation of drops on previously bare surface
2. Growth of drops
3. Coalescence of drops which grow large enough to touch each other
4. Additional nucleation of new drops in areas which become available due to coalescence (renucleation)
5. Loss of drops due to gravity (not shown in figures)



The diagram consists of five square panels illustrating the stages of droplet evolution. The first panel shows several small, isolated blue dots representing initial nucleation. The second panel shows the dots growing into larger blue circles, representing growth. The third panel shows the circles touching and merging into larger, irregular shapes, representing coalescence. The fourth panel shows the merged shapes with new small dots appearing in the gaps, representing renucleation. The fifth panel shows a few large circles and some smaller dots, representing the final state after some droplets have been lost.

Stage I: Growth      Stage II: Coalescence      Stage III: Renucleation

II IIT ROOKEER      NITEL ONLINE CERTIFICATION COLLEGE      6

Next let me show you what are the stages in droplet evolution, so processes involved in droplet condensations are initial equation of drop or previously bare surface so initial equation small, small equation sides you can see, then you can find out growth of drops this drops will be coming bigger by growth so you see all the nutrition site has actually grown up and then will be finding out coalescence of drops, so now we will see if the growth further then this to drops will be cooling over here and from you know produce drop over here, so coalescence of droplet which grow.

Large enough to touch each other so they are touching each other then we will be having additional nucleation of new drops so apart from this coolest drops there can be some additional nucleation also like this okay, additional nucleation of new drops in areas which become available due to coalescences after coalescences some area will be vacant so we can have nucleation also and side by side we can have loss of drop due to gravity if the surface is having inclination with horizontal.

So that I have not shown over here that droplet which is becoming bigger in size that can dis loss on the surface if the surface is having inclination with the horizontal okay, so by virtue gravity that can fall down, so all these are actually stages of your droplet condensation.

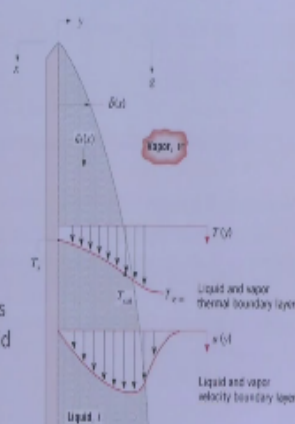


(Refer Slide Time: 10:59)

### Film Condensation on a Vertical Plate

Distinguishing Features

- Thickness ( $\delta$ ) and flow rate ( $m$ ) of condensate increase with increasing  $x$
- Generally, the vapor is superheated and may be part of a mixture that includes non-condensibles
- A shear stress at the liquid/vapor interface induces a velocity gradient in the vapor, as well as the liquid



The diagram illustrates the physical processes of film condensation on a vertical plate. It shows a vertical plate at temperature  $T_s$  with a coordinate  $x$  measured downwards from the top. A vapor layer of thickness  $\delta_{vl}$  is shown above a liquid film of thickness  $\delta_{ll}$ . The total condensate thickness is  $\delta_{tot}$ . The vapor temperature is  $T_\infty$  and the liquid temperature is  $T_l$ . The interface temperature is  $T_{sat}$ . The diagram also shows the liquid and vapor thermal boundary layers and the liquid and vapor velocity boundary layers.

IPROCESSOR MPPL/CAMER CHEE/CAMER COURSE

Next let us move back to the film condensation film wise condensation and let us consider that now we will be having a vertical plate, so situation will be changing if the plate is having inclination with the horizontal okay, so it is actually perfect vertical plate have  $90^\circ$  inclinations with the horizontal. So in this case what are the distinguish feature we can see and let say this is the plate having temperature  $T_s$  okay which is lower compared to the saturation temperature to vapor  $T_\infty$  okay.

Saturation temperature to vapor is let say  $T_{sat}$  is saturation temperature of the vapor then you can find out that whenever the vapor is coming in contact with the surface there will be condensation immediate condensation will be there, so the temperature profile will be actually increasing in this fashion as  $T_s$  requires to small and compared to the vapor temperature for necessary condensation, so this way the temperature profile will be reducing lower and the plate and higher at the vapor okay and as we are having film over here so this film due to the vertical ways of the plate the film will try to fall down by virtual of gravity.

So you will find out that there is a flow also entering in this, so the velocity can be velocity profile can be shown like that due to no sleep it will having 0 velocity adjacent to wall and far away from the wall where there is no film obviously the velocity will become 0, okay.

At the interface you can see parabolic film like this will be generated because film is grating from the top it will be accumulated at the bottom or sides, so you will be finding out thicker film at the bottom and thin film at the top, so nearer the film you will be finding out there is continuity between the liquid velocity and the gas velocity, okay. So it is very important to find out this thickness of the film which will be depending on the plate direction which is  $x$  direction over here.

So  $\delta$  is actually a function of  $x$ , so it is very important to find out the thickness  $\delta$  as a function of  $x$  as well as what is the film drainage rate okay, so flow rate of the condensate because it will draining with respect to time so we have to find out the flow rate, okay. Next generally the vapour is super heated and may be part of mixture that includes non-condensibles, so it can have that vapour is having mixture with some non-condensibles material also which will not be condense in whenever it is touching this surface.

Let us say that part that material is having saturation temperature lower than this surface temperature so that will be acting as non-condensibles gas okay, so it generally the vapour will be super heated having higher temperature compared to saturation temperature and may be part of mixture that includes non-condensibles. A shear stress and the liquid vapour interface includes a velocity gradient in vapour as well as liquid.

So if you find out and the interface there is some for a velocity gradient in the liquid side as well as in the gas side, okay so those also we need to consider in case of film condensation.

(Refer Slide Time: 14:12)

At  $x = 0 \rightarrow P = P_{v0}$

$u_v = V_v = w_v = 0 \text{ \& } T_v = T_\infty$

$$\frac{\partial P_v}{\partial x} = \rho_v g$$

$$P_v = P_{v0} + \rho_v g x = P_{v0} \left[ 1 + \frac{\rho_v g x}{P_{v0}} \right]$$

As  $P_v \approx P_{v0}$

$$T_{sat} = T_{sat}(P_v) = T_{sat}(P_{v0})$$

So let me first try to form the governing equations, so at  $x=0$  that means at the leading edge you can consider that the pressure is nothing but your vapour pressure, okay so  $P_{v0}$  okay, because there is no film okay, or we can say they are the film is being initiated okay, and we can also see over their all velocities are 0 okay, because there is no film at all and temperature is nothing but your vapour temperature  $T_\infty$  okay, so  $T_v$  is  $T_\infty$ .

So at  $x=0$  we can get pressure all velocities and temperature as there is no film at the leading edge okay. Then if you see the equations over there so we will be finding out that from there the gradient starts so we can write down  $\partial x / \partial P_v = \rho_v g$  so this comes out from the momentum equation at the leading edge, okay. So here if you equate by the way this  $P=P_v$  we have used over here, now if you integrate this one then you will be finding out  $P_v$  is equal to some constant.

Now if we apply this boundary conditions that constant will become  $P_{v0} + \rho_v g x$ , okay now if you take  $P_{v0}$  come on it becomes  $1 + \rho_v g x / P_{v0}$ . Now near the leading edge  $x$  is very small so what we can do, we can neglect this part and we can write down  $P_v = P_{v0}$  near the leading edge, okay and as a result the saturation temperature, saturation temperature which is actually a function of vapour pressure.

So  $P_v$  saturation temperature is a function of vapour pressure and just now we have shown this is nothing but vapour pressure at the leading edge  $P_{v0}$  okay, so this saturation temperature actually becomes a function  $P_{v0}$  at the, which is the pressure at the leading edge, okay. So we have got some idea of  $T_{sat}$  which will be a function of  $P_{v0}$  okay.

(Refer Slide Time: 16:12)

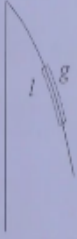
Mass flow rate of liquid film,  $\dot{m} = (\rho AV_n)_l = (\rho AV_n)_v$  where,  $V_n = \bar{V} \hat{n}$

Energy balance,  $\dot{m} \left( h_l + \frac{V_l^2}{2} \right) = \dot{m} \left( h_v + \frac{V_v^2}{2} \right) + \dot{Q} - \dot{W}$

$$\dot{Q} = - \left( k \frac{\partial T}{\partial n} A \right)_l + \left( k \frac{\partial T}{\partial n} A \right)_v \quad \forall \quad \dot{W} = 0$$

As,  $k_v \ll k_l$  Therefore,  $\left( k \frac{\partial T}{\partial n} A \right)_v = 0 \rightarrow \dot{Q} = - \left( k \frac{\partial T}{\partial n} A \right)_l$

$$(\rho A \bar{V} \hat{n})_{int} \left[ h_f + \frac{V_f^2}{2} \right] = (\rho A \bar{V} \hat{n})_{int} \left[ h_g + \frac{V_g^2}{2} \right] - k_l \left( \frac{\partial T}{\partial n} \right)_{int} A$$

$$k \left( \frac{\partial T}{\partial n} \right)_{int} = \rho (\bar{V} \hat{n})_{int} h_{fg}$$


9

Next let us try to find out what happens at the liquid film, so first let me show you let us say this is the plate vertical plate and here we are having the film, okay parabolic film let us say it is forming like this  $T_i$  current at the bottom side and  $T_r$  at the top side. Now let us take a small strip around the interface where in the left hand side we are having liquid and right hand side we are having gas, okay.

So if you do so first let us try to do the mass balance so if you find out the mass balance so  $\dot{m}$  okay will be actually  $\rho AV_n$  so perpendicular directional velocity  $V_n$  of this strip so  $(\rho AV_n)_l$  so this will be liquid will be entering through this lend this section okay, and then is equals to  $(\rho AV_n)_v$  in the vapour section, so for this small strip if you do the mass balance considering that this is very lengthy film in the other direction, so this two surfaces we are actually neglecting so we are having  $\dot{m} = (\rho AV_n)_l = (\rho AV_n)_v$  okay. Remember this  $V_n$  is nothing but  $V \hat{n}$ ,  $\hat{n}$  is the perpendicular direction of the surface, okay tiny surface whatever we have taken.

So if we take this mass flow rate and then let us go for the energy balance, if we go for the energy balance then we can find out that whatever we are getting from this side so which is nothing but  $\dot{m}$  mass flow rate into  $h_l + V_l^2/2$  so  $h_l$  is nothing but the enthalpy of the liquid side  $V_l$  is the velocity of the liquid which is entering in the domain okay, by 2 is equals to  $\dot{m}$  same mass flow rate obviously these we have same so that is why we have taken both side  $\dot{m}$ .

$\dot{m}(h_v)$  which is the enthalpy of the vapour side  $+V_v^2/2$ ,  $V_v$  is nothing but your vapour velocity, okay. On the other hand side we are writing  $\dot{Q}$  which is the heat transfer occurring in this small step  $-\dot{W}$  which is nothing but work transfer okay, so coming from now first law of thermodynamics, okay. Now you see here  $\dot{W}$  will be obviously 0 because there is no feasible work done over here so  $\dot{W}$  let us put 0 and for the  $\dot{Q}$  there will be conduction, okay.

Because this is very small strip convection cells will not be there so this is conduction, so if you go for the conduction then you can find out  $\dot{Q} = -k \partial T / \partial n A$  for the liquid side is equals to the that means here as this strip is across the interface some part will be for the liquid, some part will be for the gas, so this is conduction in the liquid side and this is the conduction in the vapour side, okay.

Now if you see actually this  $k_v$  thermal conductivity of vapour is very, very small compared to  $k_l$  so if you incorporate this then we find out  $\dot{Q}$  is nothing but  $-k \partial T / \partial n A$  for the liquid side, so conduction in the vapour side can be neglected in comparison to the conduction of the liquid side, okay. So let us consider  $\dot{Q}$  is equal to this term conduction in the liquid side only, so if you put this over here in this equation and  $\dot{W} = 0$  so we get this equation  $\rho A V \hat{n}$  at the interface, okay.

So let us consider that this mass  $\dot{m}$  is nothing but this at the interface okay,  $\rho A V \hat{n}$   $V_n$  is nothing but  $V \hat{n}$  okay, then  $h_f + V_f^2/2$  okay, is equals to once again that same  $\dot{m}$  over here okay, same  $\dot{m}[h_g + V_g^2/2]$  and in case of  $\dot{Q}$  we have written only the conduction for the liquid side, okay. So with this here you see the left hand this two terms we can actually simplify considering  $V_l$  and  $V_g$  they are very small compared to the enthalpies and  $h_f - h_g$  can be written as  $h_{fg}$  okay, so ultimately we get the energy equation across this strip is equals to  $k \partial T / \partial n$  at interface is equals to  $(\rho V \hat{n})_{int} h_{fg}$  okay. So we have taken several assumptions over here please remember  $k_v$  is less than  $k_l$  and here we have consider the velocity  $h$  is very small, okay so this we got as our energy equation. So let me show you now the equations for the following liquid film.

(Refer Slide Time: 20:46)



For falling liquid film:

Continuity: 
$$\frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} = 0$$

x-momentum: 
$$\rho \left( u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \rho g + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

y-momentum: 
$$\frac{\partial P}{\partial y} = 0$$

$$\frac{\partial P}{\partial x} = \frac{\partial P_y}{\partial x} = \rho_v g$$

Energy: 
$$\rho C_p \left( u \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$



10

So this is the continuity equation  $\partial u/\partial x + \partial V/\partial y = 0$  then we are having the x momentum equation like this okay, where we will be having the buoyancy term over here  $\rho g$  y momentum equation is  $\partial P/\partial y = 0$  and the subsequently from here we get  $\partial P/\partial x = -\partial P_y/\partial x = \rho_v g$  okay, and then subsequently energy equation is in the left hand side we are having convection, right hand side we are the conduction okay.










(Refer Slide Time: 21:16)

**Nusselt's analysis:**

- Laminar
- Steady
- Constant property
- Negligible stream-wise diffusion of momentum
- Negligible stream-wise diffusion of heat
- Negligible inertia and convection term

Using these assumptions  $x$ -momentum and energy equation can be written as,

$$0 = (\rho - \rho_v)g + \mu \frac{\partial^2 u}{\partial y^2} \qquad 0 = \frac{\partial^2 T}{\partial y^2}$$

So this is the falling film analysis but let us now do some simplification of all these equations so that we can solve and in order to do so let us take Nusselt's analysis. Roma scientist Nusselt has derived this analysis. What he is proposing, he is proposing let us take this film as laminar, let us take this film as steady so all temporal terms we can actually neglect and then we are having constant property so all the properties like  $\rho, \mu$  can come out okay, and then negligible streamwise diffusion in the momentum term.

So in this momentum term if you see we are having streamwise and cross-streamwise diffusion so streamwise term will be cancelling. Similarly we are having negligible streamwise diffusion of heat so in the conduction term we can also neglect the streamwise diffusion. Negligible inertia and convection, so inertial side and convection side totally we can neglect, okay.

So with these assumptions which have been proposed by Nusselt we are actually reducing our momentum equation and energy equation in this form. So you see all the  $\partial^2/\partial x^2$  term has been cancelled from the momentum equation, inertia term has been cancelled, steadiness to obtain steadiness  $\partial/\partial t$  term has been cancelled same thing has been done from this side also, okay. So all  $\partial^2/\partial x^2$  term can be cancelled and convection term has been cancelled, okay.  
(Refer Slide Time: 22:37)

Boundary conditions:

$$y = 0 \rightarrow u = 0 \text{ and } T = T_w$$

$$y = \delta(x) \rightarrow \frac{\partial u}{\partial y} = 0 \text{ and } T = T_{sat}$$

From momentum equation,  $\frac{\partial^2 u}{\partial y^2} = -\frac{(\rho - \rho_v)g}{\mu}$

From energy equation,  $\frac{\partial^2 T}{\partial y^2} = 0$

After integrating under given set of BC's,

Liquid film velocity,  $u = -\frac{(\rho - \rho_v)g}{\mu} \left( \frac{y^2}{2} - \delta y \right)$

Temperature profile,  $\frac{T_{sat} - T}{T_{sat} - T_w} = 1 - \frac{y}{\delta}$

Now if you try to solve this equation using this boundary conditions at  $y=0$  that means at the plate we will be having velocity equals to 0 and  $T = \text{wall temperature}$  which is nothing but  $T_s$  okay, and at  $y=\delta$  that means at the boundary layer okay, so here at the film edge if you write down then there will be having  $\partial u/\partial x=0$  here you can see velocity exists what the gradient of velocity we can say at the film edge is 0.

So  $\partial u/\partial y=0$  and  $T$  becomes saturation temperature because this is the end of the film, okay. So if we take this boundary conditions first let us take the momentum equation to solve this momentum equation so this was my reduced momentum equation from here using Nusselt analysis, so that momentum equation if you integrate and energy equation is like this so if we integrate this two equations we get first the  $u$  pattern from after integrating of this one like this and using this two boundary conditions obviously we get the liquid film velocity as this very simple integration we are having and finally we will be getting this type of equation, okay.

Here you can see this  $u$  is actually a parabolic function of  $y$  so that is why we have got this type of equation, this type of velocity profile okay, and on the other hand side temperature profile using this two temperature boundary conditions and this equation we get  $T_{sat} - T / T_{sat} - T_w = 1 - y/\delta$  okay. So it will be sort of linear profile we have shown like that okay continuously we will be showing the profile okay, so we have got velocity profile and the temperature profile in the content in this fashion.

(Refer Slide Time: 24:16)



Mass flow rate of liquid film:  $\dot{m} = \int_0^{\delta} \rho u dy = \int_0^{\delta} \rho \frac{(\rho_v - \rho)g}{\mu} \left( \frac{y^2}{2} - \delta y \right) dy$

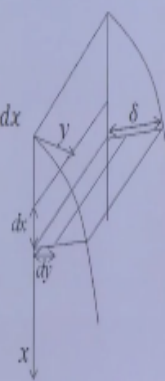
After integrating,  $\dot{m} = \rho \frac{(\rho - \rho_v)g}{\mu} \frac{\delta^3}{3}$

Energy balance through liquid film,  $d\dot{m}h_{fg} = k \frac{\partial T}{\partial n} dA$ ;  $dA = dx$

$$\frac{\partial T}{\partial n} \approx \frac{\partial T}{\partial y} \approx \frac{T_{sat} - T_w}{\delta} \approx \frac{\Delta T}{\delta}$$

Differentiating mass flow rate with  $x$ ,  $\frac{d\dot{m}}{dx} = \rho \frac{(\rho - \rho_v)g}{\mu} \delta^2 \frac{d\delta}{dx}$

Substituting  $\frac{d\dot{m}}{dx}$ ,  $k \frac{\Delta T}{\delta} dx = \rho \frac{(\rho - \rho_v)g}{\mu} \delta^2 d\delta h_{fg}$



Next let us try to see what is the mass flow rate of liquid film, so mass flow rate  $\dot{m}$  that we have already seen that will become  $\rho u dy$  and unit will become 0 to  $\delta$ , as we already obtained the profile for  $u$ , so let us put that one over here so this one is becoming one as simple integration  $dy$  and if we proceed further we will be getting  $\dot{m} = \rho \times \rho - \rho_v \times g \delta^3 / \mu \times 3$  okay.

Then if you go for the energy balance this equation we have already derived by taking the small slit around the film the remainder for some definite street you can write down this equation, here if you see this street is very thin, then you can write down  $dA = dx$  okay, because in the direction it will be very small. So once you have this equation then from here you can write down  $\partial T / \partial n$  that is  $= \partial T / \partial y$  because in the  $x$  direction we are considering more gradient.

That is almost a vertical streak okay, here we can write down nothing but  $\partial T / \partial x$  okay. So if you put all these things over here then you will get  $\partial \dot{m} / \partial x =$  this fashion okay, so here you see once again we have brought some differential which you can integrate, so let us try to integrate respect to  $x$  before that  $\partial \dot{m} / \partial x$  is here using this equation. So once you put this equation it becomes  $\partial T / \partial x dx = \rho \times \rho - \rho_v \times g / \mu$  okay which can be now easily integrated.

(Refer Slide Time: 26:15)

Rearranging and integrating  $\delta$  from 0 to  $x$ :  $\int \delta^3 d\delta = \int \frac{\mu k \Delta T}{\rho(\rho - \rho_v) g h_{fg}} dx$

After integrating,  $\frac{\delta^4}{4} = \frac{\mu k \Delta T}{\rho(\rho - \rho_v) g h_{fg}} x + A$

At,  $x = 0 \rightarrow \delta = 0 \rightarrow A = 0$

Therefore,  $\delta = \left[ \frac{4 \mu k \Delta T}{\rho(\rho - \rho_v) g h_{fg}} x \right]^{1/4}$

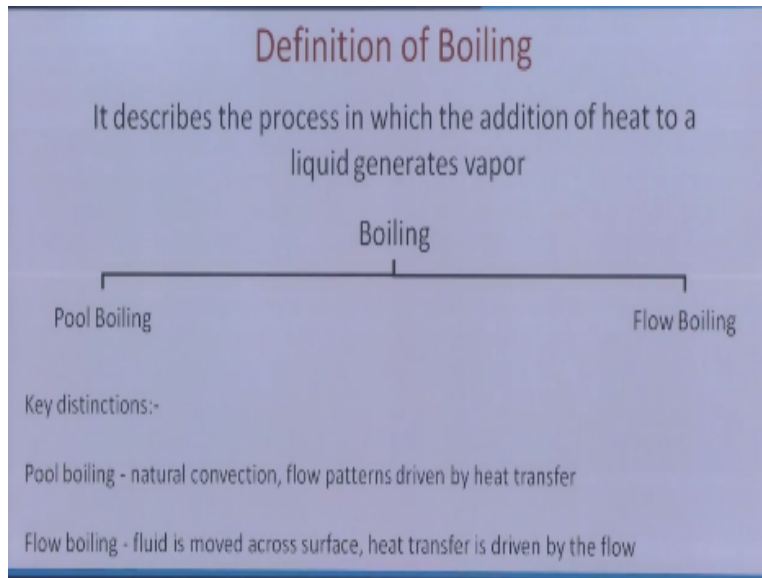
Rearranging,  $\frac{\delta}{x} = \left[ \frac{4 \theta \alpha}{(\rho - \rho_v) g x^3} \times \frac{C_p \Delta T}{h_{fg}} \right]^{1/4}$

Further,  $\frac{\delta}{x} = \left[ \frac{4}{Ra_x} Ja \right]^{1/4} = \left[ \frac{4}{Ra_{fx}} \right]^{1/4}$  where,  $Ra_{fx} = \frac{Ra_x}{Ja}$

Once you integrate you will be finding out and put the limit to 0 to  $x$ , you will be finding out this becomes  $\frac{\delta^4}{4} = \frac{\mu k \Delta T}{\rho(\rho - \rho_v) g h_{fg}} x + A$  okay, and evaluation of  $A$  is very easy  $x = 0 \rightarrow \delta = 0 \rightarrow A = 0$ . So finally we will get the  $\frac{\delta}{x}$  in the form of  $\left[ \frac{4 \theta \alpha}{(\rho - \rho_v) g x^3} \times \frac{C_p \Delta T}{h_{fg}} \right]^{1/4}$ . Let us try to reduce this by finding out some non conventional number, as you see we have clubbed these two into parts okay, and both these parts actually giving this me valid number.

That has given me a rally number, here this is nothing but jack of number this is giving me  $\frac{\delta}{x} = \left[ \frac{4}{Ra_x} Ja \right]^{1/4}$ . So ultimately we can write down this film number is nothing but  $Ra$ , so this becomes  $\frac{\delta}{x} = \left[ \frac{4}{Ra_{fx}} \right]^{1/4}$ .

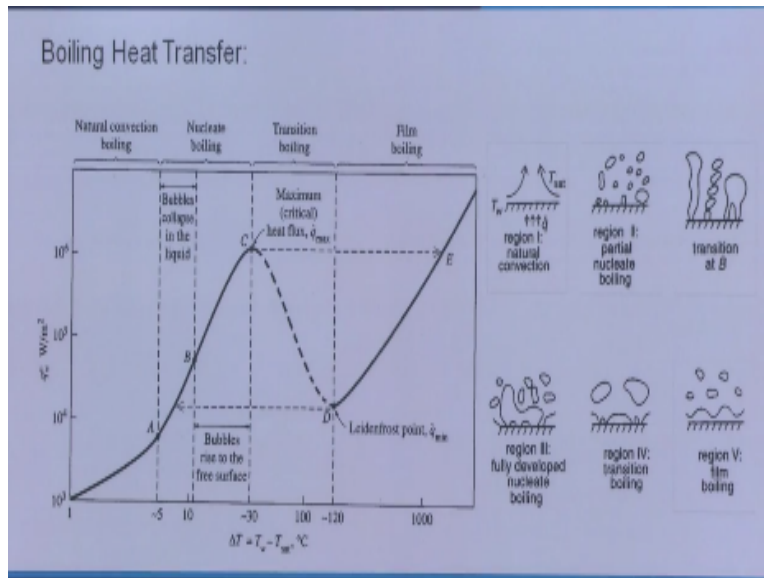
(Refer Slide Time: 27:32)



Next let us move to other portion after this the evaluation of the boundary layer convection side let us move to the boiling side. So in case of boiling we know that would be actually the process in which addition of heat to a liquid generates vapor. So just opposite of conduction, so in this case boiling two modes, pool boiling, so it is nothing but near a pool we will be having some hot surface from fire where the boiling will be appearing okay.

And another one is flow boiling where there will pre dominate motion it will be actually boiling heating around okay, so here we are having pre dominate motion and there is no dominate motion but the motion will be generated convection currents okay. So key distinction we have already mentioned, natural convection takes part in the pool boiling okay and in case of flow boiling this actually the fluid is moved across surface, heat transfer is driven by the flow, so this you can say convection given and this is natural convection given okay.

(Refer Slide Time: 28:46)



Next this very elemental I would like to show different boiling zones, the beginning we will be having the natural convection where the temperature rising up to the saturation form at the most temperatures and here the volume will be in the heat flaks, wall temperature will be higher. After that you will be having small bubble formation which is nothing but at low super heat and this is actually nuclide boiling.

Then after that there will be sudden jump in temperature in case of the reflex boiling and you will found out that this  $\rho$  is nothing but transition boiling and after this sudden you will be finding out the whole surface or by a liquid film which is nothing but it is film boiling and if you reverse this from the jumping point which is nothing but the level strophe point okay. So if you lower that this one temperature or lower the heat flex it will become like this and then it will jump to the nuclide boiling subsequently go for the natural convection.

So let me show you different modes, so in the natural convection we can find out more bubble is involved, so only the natural convection is pre dominate okay for example horizontal surface. Here in the nuclide boiling the small bubbles you can see so in this role small bubbles you can see. At transition bubbles will be margin vertical or horizontally and then it was margined vertically and here also I have shown the margin in the horizontal identification also.

This will be becoming bigger in size to occupy the whole surface okay and ultimately we will be having differently, so in this region we will be having film over the surface right. so these are the different zones we are having.

(Refer Slide Time: 30:57)

Film boiling along vertical plate:

$$\rho \left( u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} - \rho g + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$P = P_v \quad \mu = \mu_v$$

For liquid:  $0 = -\frac{\partial P_l}{\partial x} - \rho_l g$  as  $u_l = 0$  and  $V_l = 0$

$$P_l = P_{lo} - \rho_l g x \quad \frac{\partial P}{\partial x} = \frac{\partial P_l}{\partial x} = -\rho_l g$$

$$\rho \left( u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} \right) = (\rho_l - \rho)g + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$y = \delta(x) \rightarrow \mu_v \left( \frac{\partial u}{\partial y} \right)_v = \mu_l \left( \frac{\partial u}{\partial y} \right)_l \quad \rightarrow u \approx 0$$

Now let me show you the equation for film boiling, so it is opposite of film conduction, so this is the temperature which is having surface higher than the saturation in the liquid, so from here you see we will be having different saturation compare to vapour formation when it comes to the surface formation but the vapour by the virtue of the it will be trying to move up. So you can find out the vapour film is higher comparing to the lower side.

So the equation corresponding to the boundary conditions subsequently to the solution will also be similar, let me show the equations quickly, so these are the momentum equation as we have shown over here in the case of convection also  $\mu = \mu_v$  okay because the vapour is attached to the surface. So the liquid these are the equation and regarding velocity this is the equation, the low velocity liquid dominate.

So we will get  $P_l = P_o - \rho g x$  it is deviating is  $\rho g$  and it is momentum equation will be something like this  $\partial p / \partial x - \rho g$  is converted into this one okay and then this will be the interface the stress boundary at  $y = \delta / \partial y$  at vapor region and this will be the boundary region at the film age. So with this we can solve the similar way this equation and get film boiling okay. So with this let me end this lecture so in this lecture we have described the

(Refer Slide Time: 33:21)

## Summary

- In this lecture we have described the basic differences between boiling and condensation along with its different modes
- Discussed mechanisms of droplet growth and shrinkage during condensation on a surface
- Balanced mass, momentum and energy equations around the film during condensation
- Predicted the liquid film accumulation rate along with its profile along the vertical surface
- Described governing equations for film boiling in a vertical plate

The basic differences between boiling and condensation along with its different modes, then we have discussed mechanism of droplet growth and shrinkage during condensation on a surface. We have balanced mass, momentum and energy around the film during condensation and we have predicted the liquid film accumulation rate along with its profile along the vertical surface and finally we have described equations for film boiling in a vertical plate okay. So let me test your understanding at the end of this lecture, having three questions once again

(Refer Slide Time: 33:58)

## Test your understanding ?

1. Necessary condition for condensation
  - a.  $T_w < T_{sat}$
  - b.  $T_w > T_{sat}$
  - c.  $T_w = 0\text{ K}$
  - d.  $T_w = T_{triple\ point}$
2. Necessary assumption for Nusselt analysis
  - a. Laminar
  - b. Neglect inertia and convection
  - c. Neglect buoyancy
  - d. Neglect crosswise diffusion of momentum
3. Condensate rate is proportional to
  - a.  $1/\delta$
  - b.  $\delta^2$
  - c.  $\delta$
  - d.  $\delta^3$

1<sup>st</sup> question necessary condition for consideration we are having 4 conditions wall temperature is lesser than the saturation temperature, wall temperature is greater than the saturation temperature,  $T_w = 0\text{ K}$ ,  $T_w = T_{\text{triple point}}$ . It is very simple one the answer is  $T_w < T_{\text{sat}}$ . Necessary assumption for nusselt analysis the options laminar, neglect inertia and convection, neglect buoyancy, neglect crosswise diffusion of momentum okay so you can see laminar and neglect inertia and convection needs to be neglected.

Last question is condensate rate is proportional to  $1/\partial t a, \partial^2 t a, \partial^3 t a$ , so the answer is  $\partial^3 t a$  okay. So with this we will be ending this lecture, in our next lecture we will be discussing about mass transfer. So if you are having any query regarding this lecture or any other general query about convection heat transfer please keep on posting on our discussion forum thank you.

**For Further Details Contact**

**Coordinator, Educational Technology Cell**

**Indian Institute of Technology Roorkee**

**Roorkee – 247667**

**E Mail: [etcell.iitrke@gmail.com](mailto:etcell.iitrke@gmail.com), [etcell@iitr.ernet.in](mailto:etcell@iitr.ernet.in)**

**Website: [www.iitr.ac.in/centers/ETC](http://www.iitr.ac.in/centers/ETC), [www.nptel.ac.in](http://www.nptel.ac.in)**

**Production Team**

**Sarath K. V**

**Jithin. K**

**Pankaj Saini**

**Arun. S**

**Mohan Raj. S**

**Camera, Graphics, Online Editing & Post production**

**Binoy. V. P**

**NPTEL Coordinator**

**Prof. B. K. Gandhi**

**An Educational Technology cell**

**IIT Roorkee Production**

**© Copyright All Rights Reserved**