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Convective Heat over flat plate

Lec-16

Thermal Entrance Region: Uniform Heat Flux

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Hello welcome in 16 lecture of my course convective heat transfer here in this lecture we will be discussing about thermal entrance region once again but in this case we will be considering the pipe is having constant heat flux boundary condition in our last lecture we have considered about constant wall temperature condition here we will be considering the pipe is receiving constant heat flux so as I have mentioned that we will be discussing about thermal entrance region with uniform heat flux, so let me first show you that what outline will be following in my lecture so first we will be deriving the energy equation.

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Outline of the Lecture

- Derive the energy equation and boundary condition into non dimensional form for uniform wall heat flux around a duct
- Reduce the energy equation into simplified form for thermally developing region with boundary condition
- Determine the Nusselt number for thermally developing but hydrodynamically fully developed forced convection inside a tube having constant applied heat flux

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And boundary conditions okay in the form of non dimensional numbers okay for uniform wall heat flux around a duct okay then we will be reducing those energy equations into simplified form



okay for thermally developing that we say in trans length along with the necessary boundary conditions which will be evolving from the constant heat flux boundary condition okay then we will determine the Nusselt number for thermally developing but hydro dynamically fully developed forced convection inside a tube having constant applied heat flux will be find out the Nusselt number okay.

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$$w = \frac{\bar{w}}{\bar{w}_{av}} = 2(1 - r^2) \quad r = \frac{\bar{r}}{r_o} \quad z = \frac{\bar{z}}{r_o} \quad q = \text{Const} \quad q = k \left(\frac{\partial T}{\partial \bar{r}} \right)_{\bar{r}=r_o}$$

Let, $\theta = \frac{T - T_i}{\frac{q r_o}{k}} \quad T = T_i + \frac{q r_o}{k} \theta$

$$\frac{\partial T}{\partial \bar{r}} = \frac{q r_o}{k} \frac{\partial \theta}{\partial \bar{r}} \quad \frac{\partial \theta}{\partial \bar{r}} = \frac{q r_o}{k} \frac{1}{r_o} \frac{\partial \theta}{\partial r} \quad \text{or,} \quad \left(\frac{\partial \theta}{\partial r} \right)_{r=r_o} = 1$$

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So let us go inside this first I will be showing stigmatically what situation we are going to study in this so as I have mentioned we will be having a tube so this is a tube having center line this one are coordinate is radial coordinate is in this side okay and this is axial coordinate in this side okay and here we are considering thermally fully developed profile, so that means parabola boundary condition and there we are considering that in the tube we are giving constant heat flux.

Let us say the fluid whatever is coming that is having some uniform temperature T_i okay, so with this as we are having hydro dynamically fully developed flow so our w profile will be nothing but a parabola profile and once we non dimensionalized is w bar with respect to the average w so we will be getting w is nothing but $2(1 - r^2)$ okay where r is nothing but r bar / r_o okay, next let us see for the wall heat flux which is a constant q we can write down q is nothing $k \partial t / \partial r$ bar at r bar = 0, so which is coming from the wall okay.

Boundary of the duct or tube okay so next we will be using this boundary condition for getting the value for temperature non dimensionalization this already we have shown in several cases so let us that same thing $\theta = T - T_i$, T_i is the inflow temperature for the fluid which is constant okay and there divided by $q r_0 / k$ so $q r_0 / k$ you can get from here so $q r_0$ will be multiplied by k so it will be transforming into $\partial \theta / \partial r$ okay.

So if we take this type of non dimensionalization for temperature then we can write down temperature in the form θ in this fashion okay so $T = T_i + q r_0 / k \times \theta$ let us find out the derivatives of temperature with respect to r first we because we will be requiring $\partial T / \partial r$ here so $\partial T / \partial r$ becomes q now here q is constant okay and T_i is also constant so this term goes to 0, so $q r_0 / k$ that can be taken out so this $q r_0 / k$ I have taken out and then we are having $\partial \theta / \partial r \times \partial r / \partial r$ okay.

So what we are finding out over here $\partial r / \partial r$ is actually $1 / r_0$ so ultimately it becomes $q / k \partial \theta / \partial r$ okay so from there we get from this and this we get $\partial \theta / \partial r$ at $r = 1 = 1$ okay.

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$$\frac{\partial T}{\partial \bar{z}} = \frac{q}{k} \frac{\partial \theta}{\partial z} \quad \frac{\partial^2 T}{\partial \bar{z}^2} = \frac{q}{k r_0} \frac{\partial^2 \theta}{\partial z^2} \quad \frac{\partial^2 T}{\partial \bar{r}^2} = \frac{q}{k r_0} \frac{\partial^2 \theta}{\partial r^2}$$

$$\bar{w} \frac{\partial T}{\partial \bar{z}} = \alpha \left[\frac{\partial^2 T}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial T}{\partial \bar{r}} + \frac{\partial^2 T}{\partial \bar{z}^2} \right]$$

$$\bar{w}_{av} w \frac{q}{k} \frac{\partial \theta}{\partial z} = \frac{\alpha q}{k r_0} \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right]$$

$$\left(\frac{\bar{w}_{av} r_0}{\alpha} \right) w \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}$$

$$\frac{Pe}{2} 2(1 - r^2) \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}$$

Next if we try to find out the other derivatives and like this $\partial T / \partial r$ we will be finding out $\partial T / \partial z$ so if we do so it becomes $q / k \partial \theta / \partial z$ okay because we have considered $T = T_i + q / k r_0 \times \theta$ so we will be finding out that it is becoming and we have considered z is $= z \text{ bar} / r_0$ okay because it is thermally developing zone so in trans lens so the axial this stands will be of the similar order of the radius okay.

So we have seen that if we do the derivative with respect to $z \text{ bar}$ so it will become $q / k \times \partial \theta / \partial z$ okay so this is $\partial \theta / \partial z$ and then once we will be having $\partial z / \partial z \text{ bar}$ so $\partial z \text{ bar}$ gives $1 / r_0$ and r_0 and here q_0 that r_0 will be canceling out and subsequently we will be getting $q / k \partial \theta / \partial z$ okay so if we do second time derivative we will be getting similarly $q / k r_0 \times \partial^2 \theta / \partial z^2$ okay and if we do second time derivative of T with respect to $r \text{ bar}$ so then we are getting $q / k r_0 \partial^2 \theta / \partial r^2$.

So all this benefits of the temperature let us put in the energy equation so energy equation is like this we have seen as it is thermally fully developed so u component and v component of the convection is not coming only the axial component w component is coming into picture and in case of conduction we are having the radial conduction and axial conduction azimuthally conduction we have not considered over here okay azimuthally symmetry as been assumed okay then let us see if we put all this non dimensionalization.

That means convert T to θ $z \text{ bar}$ to z and $r \text{ bar}$ to r then we get over here once again w can be converted to $w \text{ bar}$ can be converted to w by multiplying $w \text{ bar}$ average then we can find out that the equation actually simplifies to $w \text{ average} \times w \text{ bar average} \times w, q / k$ into $\partial \theta / \partial z$ in the left hand

side for convection in the right hand for conduction $\alpha / k \times q / r_0$ okay and then we are having the radial conduction non dimensionalized radial conduction + axial conduction term okay further simplification cancelation of terms from both side we can get w average r_0 / α which is nothing but $k / \partial cp$ okay $k / \partial cp \alpha$ is over here once again.

So we can get over here some non dimensionalized number which is nothing but actually Pe number. Pe number if we define based on the diameter so Pe number / 2 will be coming over here okay and as w is actually a parabolic velocity profile so that can written as $2 \times 1-r^2$ okay.

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$$(1 - r^2) \frac{\partial \theta}{\partial z} = \frac{1}{Pe} \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right]$$

B.C. at $r = 0 \quad \frac{\partial \theta}{\partial r} = 0$

$r = 1 \quad \frac{\partial \theta}{\partial r} = 1$

as $z \rightarrow -\infty \quad \theta \rightarrow 0$

as $z \rightarrow \infty \quad \theta$ is finite

For thermally developing zone:

$z \sim 1 \quad 1 - r \sim \delta_0 \quad \theta \sim \delta_0 \quad \text{as } \frac{\partial \theta}{\partial r} = 1 \quad \text{at } r = 1$

$(1 - r^2) = (1 - r)(1 + r) \sim \delta_0 \quad \text{as } 1 + r \sim 1$

So ultimately we get very simplified equation like this $1 - r^2 \partial \theta / \partial z = 1 / Pe +$ radial conduction + axial conduction okay.

Let us also see the boundary condition so obviously at the wall $r = 1$ at the axis $r = 0$ our $\partial \theta / \partial r$ will be 0 that means there will be no gradient of temperature across the axis in the radial direction okay so this gives us the symmetry boundary condition sort of and then $r = 1$ is nothing but your wall boundary condition so in the wall boundary condition we are having actually $\partial \theta / \partial r = 1$ which is nothing but constant heat flux boundary condition okay we have chosen θ such that this boundary condition actually reduces to a simplified form $\partial \theta / \partial r = 1$ okay.

And then for the axial directions you are having z tends to $-\infty$ and z tends to ∞ that means z tends to $-\infty$ means it is far before the entry of the pipeline and z tends to ∞ means in the

downstream or the pipe line so in case of z tends to $-\infty$ we will have $\theta \rightarrow 0$ because it becomes $T_i - T_i / T_i - T_w$ so it becomes 0 and in case of z tends to ∞ through it is supplying heat where the heating coils as a function of you know constant heat flux but temperature definitely we will not be going to infinite it will be always the finite so let us have θ is finite boundary condition at z tends to infinity okay.

So with this equation and boundary conditions two boundary conditions of r and two boundary conditions of z we can describe the thermally developing region inside a pipe line okay which in which we have considered that hydraulically full developed fluid is going on okay, next let us try to see some scale analysis so for the thermally developing zone we need to see that what is first the scale of your axial direction and radial direction.

As it is thermally developing zone so the zone will be very small obviously that will be of the order of r_0 okay so z will be coming of the order of 1 because z is nothing but $z \text{ bar} / r_0$ okay and in case of radial direction $1 - r$ is nothing but $r_0 - r \text{ bar} / r \text{ bar}$ so that will be coming in the form the boundary layer thickness δ_0 okay so this two scale once we decided then we can find out quickly the other scales for example from the boundary conditions here you see $\partial \theta / \partial r$ needs to be of the order of 1.

So as we have decided r is of the order δ_0 so obviously θ needs to be of the order of δ_0 okay because this $\partial \theta / \partial r$ needs to be order 1 okay so if r is of the order of δ_0 obviously needs to be order if δ_0 okay so we have already found out z r and θ order so let us find out the rest terms of the equation so first let us start with $1 - r^2$ so $1 - r^2$ will be $1 - r + 1 / r$ $1 + r$ okay so here we can get it will be also of the order of δ .

Why because $1 + r$ is actually of the order of 1 okay so $1 + 1$ it would be order of 2 so basically this $1 - r^2$ will be of the order of δ_0 here this $1 - r$ is actually determining what is the order okay next let us see that what is the order of convection.

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$$\text{Convection} \sim w \frac{\partial \theta}{\partial z} \sim \delta_o \frac{\delta_o}{1} \sim \delta_o^2$$

$$\text{Radial conduction} \sim \frac{1}{Pe} \frac{\delta_o}{\delta_o^2}$$

$$\text{Axial conduction} \sim \frac{1}{Pe} \frac{\delta_o}{1} \text{ (small) as } \delta_o \ll 1$$

$$\delta_o^2 \sim \frac{1}{Pe} \frac{\delta_o}{\delta_o^2} \quad \text{or, } \delta_o \sim (Pe)^{-1/3}$$

So first in this equation we have the convection so convection was $w \times \partial \theta / \partial z$ okay as we have seen that part of convection this w is nothing but $2 \times 1 - r^2$ so $1 - r^2$ is order of δ_o so this w can be written in the order of $\partial \theta$ and $\partial \theta / \partial z$, z is of order 1 and θ is of order $\partial \theta$ just now we have said so we get the convection is of the order of $\partial \theta^2$ okay then radial conduction if you think about so in this equation if you see the radial conduction is having $1/Pe$ number and then θ order and r order square okay so here also r order square θ is order over here okay.

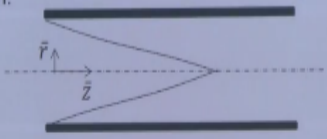
So we will be finding out radial conduction becomes $1/Pe \times \partial / \partial \theta^2$ okay so radial, conduction order we have found out let us see the axial conduction in case of axial conduction it is nothing but $1/Pe$ and then let us go back to the equation.

$1/Pe$ then order of θ and order of z , order of z is one that's whole square will be also of order of 1 and θ will be of order $\partial \theta$, so ultimately we get $1/Pe$ number. $1/Pe$ number $\times \delta_o / 1$ okay now you see this order is actually smaller compared to this one, okay. Because δ is very small okay so we can get the conduction order is actually this one which is the radial conduction, axial conduction can be neglected in compare to the radial conduction for small δ obviously, okay. So what we can take let us equate the convection order and radial conduction order.

So δ_o^2 which was the convection order and δ_o / Pe number δ_o / δ_o^2 which was the radial conduction order, if you equate from here we can get δ_o is of the order if $Pe^{-1/3}$ okay so we have got the bound layer thickness order, okay. So it will be helping us to construct the similarity variable.

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Thermal entrance region:



$$z \sim 1 \quad 1 - r \sim Pe^{-1/3} \quad r = 1 - Pe^{-1/3} \eta$$

$$\eta = (1 - r) Pe^{1/3}$$

$$\frac{\partial \eta}{\partial r} = -Pe^{1/3}$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial r} = -Pe^{1/3} \frac{\partial \theta}{\partial \eta}$$

$$\frac{\partial^2 \theta}{\partial r^2} = Pe^{2/3} \frac{\partial^2 \theta}{\partial \eta^2}$$

So let us try to have similarity variable in case of our thermal entrance terms region okay, so here once again schematically I have shown the thermal entrance region so upto this we will be having the thermal entrance region, so in that as we have already taken JD 's is of the order of 1 and earlier we have taken $1-r$ is of the order of $\delta 0$ and in the last slide by equating the convection and conduction order we have shown that $\delta 0$ is actually $Pe^{-1/3}$ so $(1-r)$ can be written as $Pe^{-1/3}$, okay.

And let us take the similarity variable η in this fashion, η is nothing but $Pe^{-1/3} \times (1-r)$ okay, so r becomes $1 - Pe^{-1/3} \times \eta$ so η is our similarity variable, so if we take so here I have clearly showed what is the variable η so if we take so then quickly we can try to find out the derivatives of θ which will be useful for finding out the derivatives of temperature, okay theta. So let us find out first θ δr .

So $\delta \eta / \delta r$ simply it becomes $-Pe^{1/3}$ okay and then subsequently if we try to find out $\delta \theta / \delta r$, $\delta \theta / \delta r$ will be nothing but $\delta \theta / \delta \eta \times \delta \eta / \delta r$, $\delta \eta / \delta r$ just now we have found out has $-Pe^{1/3}$ so it becomes $\delta \theta / \delta r$ becomes $-Pe^{1/3} \times \delta \theta / \delta \eta$, okay. Then let us do one small derivative if you do one small derivative $\delta^2 \theta / \delta r^2$ it becomes once again $Pe^{2/3} \times \delta^2 \theta / \delta \eta^2$ we need to plug in this $\delta \eta / \delta r$ once more over here from this $\delta \theta / \delta r$ to $\delta^2 \theta / \delta r^2$, okay. So both the radial conduction parts we have obtained.

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$$(1 - r^2) = (1 - r)(1 + r) = Pe^{-1/3} \eta (2 - Pe^{-1/3} \eta)$$

$$(1 - r^2) \frac{\partial \theta}{\partial z} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

$$Pe^{-1/3} \eta (2 - Pe^{-1/3} \eta) \frac{\partial \theta}{\partial z} = \frac{1}{Pe} \left(Pe^{2/3} \frac{\partial^2 \theta}{\partial \eta^2} - \frac{Pe^{1/3}}{1 - Pe^{-1/3} \eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

$$(2\eta - Pe^{-1/3} \eta^2) \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2} - \frac{Pe^{-1/3}}{1 - Pe^{-1/3} \eta} \frac{\partial \theta}{\partial \eta} + Pe^{-2/3} \frac{\partial^2 \theta}{\partial z^2}$$

Taking limit $Pe \rightarrow \infty$ $2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$

at $r = 1$ $\frac{\partial \theta}{\partial r} = 1$ Let us take $\theta = Pe^{-1/3} \theta_1$

Next let us find out the value of $(1-r)^2$ which is there in the convection side due to the parabolic velocity profiles, so it becomes actually $(1 - r) \times (1 + r)$ which is nothing but $Pe^{1/3} \times \eta \times 2 - Pe^{-1/3} \times \eta$, this 2 is nothing but $(1+)$ $(1-)$ $Pe^{-1/3} \times \eta$ which is $1 + r$ and this is actually your $1 - r$ okay. So after that if we try to put everything in this equation then we simply get $Pe^{-1/3} \eta$, okay.

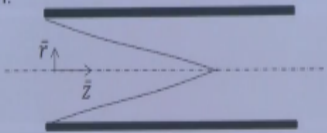
So into $(2 - Pe^{-1/3} \eta)$ this is nothing but your $1 - r^2$ we have derived over here, into $\delta \theta / \delta z = (1/ Pe)$ and then in the right hand side we have already evaluated $\delta^2 \theta / \delta r^2$ and $\delta \theta / \delta r$ let us put those so $\delta^2 \theta / \delta r^2$ is nothing but $Pe^{2/3} \times \delta^2 \theta / \delta \eta^2$ and r we have written as here r we have written as $1/ (1 - Pe^{-1/3} \eta)$ okay. And $\delta \theta / \delta r$ is nothing but $Pe^{1/3} \times \delta \theta / \delta \eta$, okay.

And last term remains as it is over here because Z is of order 1 okay so if you simplify this equation little bit then we get this type of equation and we find out that in three terms here in axial conduction in this $\delta \theta / \delta \eta$ term and a part of this $\delta \theta / \delta z$ term we are having Pe power

okay. Now for large Pe number limit okay, so taking Pe very large what we can do, those terms we can cancel out and make our equation simplified, so in case of large Pe number we can drop down this terms and we can write down $2 \eta \delta \theta / \delta z$ in the left hand side coming from the convection and in the right hand side $\delta^2 \theta / \delta \eta^2$ coming from the radial conduction only. So this becomes very simplified equation for large Pe number cases, okay. And let us see also the corresponding boundary conditions, so here first boundary condition we had earlier in terms of r it was earlier at $r = 1$ which is at the wall $\delta \theta / \delta r = 1$ which was actually constant hit flows boundary condition, so from their first we need to convert to your η boundary condition.

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Thermal entrance region:



$$z \sim 1 \quad 1 - r \sim Pe^{-1/3} \quad r = 1 - Pe^{-1/3} \eta$$

$$\eta = (1 - r) Pe^{1/3}$$

$$\frac{\partial \eta}{\partial r} = -Pe^{1/3}$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial r} = -Pe^{1/3} \frac{\partial \theta}{\partial \eta}$$

$$\frac{\partial^2 \theta}{\partial r^2} = Pe^{2/3} \frac{\partial^2 \theta}{\partial \eta^2}$$

Because we have already seen that $\eta = (1-r) Pe^{1/3}$ so here if we put the value of $r = 1$ then we get $\eta = 0$ okay, so at $\eta = 0$.

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$$(1 - r^2) = (1 - r)(1 + r) = Pe^{-1/3} \eta (2 - Pe^{-1/3} \eta)$$

$$(1 - r^2) \frac{\partial \theta}{\partial z} = \frac{1}{Pe} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

$$Pe^{-1/3} \eta (2 - Pe^{-1/3} \eta) \frac{\partial \theta}{\partial z} = \frac{1}{Pe} \left(Pe^{2/3} \frac{\partial^2 \theta}{\partial \eta^2} - \frac{Pe^{1/3}}{1 - Pe^{-1/3} \eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

$$(2\eta - Pe^{-1/3} \eta^2) \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2} - \frac{Pe^{-1/3}}{1 - Pe^{-1/3} \eta} \frac{\partial \theta}{\partial \eta} + Pe^{-2/3} \frac{\partial^2 \theta}{\partial z^2}$$

Taking limit $Pe \rightarrow \infty$ $2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$

at $r = 1$ $\frac{\partial \theta}{\partial r} = 1$ Let us take $\theta = Pe^{-1/3} \theta_1$

The boundary condition comes out to be the boundary condition comes out to be though θ will not be changing but due to this r to η one $Pe^{1/3}$ will be coming out so the boundary condition will transform into $Pe^{1/3} \times \partial \theta / \partial \eta = 1$, okay. So this becomes the equation and this is the boundary condition, now you see the boundary condition is $Pe^{1/3} \times \partial \theta / \partial \eta = 1$ is complicated one.

Let us try to replace θ by some simplified one so that this boundary condition can be written in some better format, okay. So let us take $\theta = Pe^{-1/3} \times \theta_1$, this will not change the equation because in equation both the sides we are having θ so this $Pe^{-1/3}$ can be cancelled from both the sides, but it will be definitely helping in this boundary condition because in boundary condition we are having $Pe^{1/3} \times \partial \theta / \partial \eta = 1$.

So once we put this $\theta_1 = \theta / Pe^{-1/3}$ those $Pe^{1/3}$ and $Pe^{-1/3}$ from here we will be cancelling out and it will become simplified, okay. So let me show you after putting this $\theta = Pe^{-1/3}$ with θ_1 what form of boundary condition we get. So this is the equation, now you see.

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$$2\eta \frac{\partial \theta_1}{\partial z} = \frac{\partial^2 \theta_1}{\partial \eta^2} \quad \text{at } \eta = 0 \quad \frac{\partial \theta_1}{\partial \eta} = -1$$

Other boundary conditions: $\text{as } r \rightarrow \delta \quad \theta \rightarrow 0 \quad \text{or, as } \eta \rightarrow \infty \quad \theta_1 \rightarrow 0$
 $\text{as } z = 0 \quad \theta = 0 \quad \text{or, at } z = 0 \quad \theta_1 = 0$

$$\eta^* = e^{\alpha_1 \eta} \quad z^* = e^{\alpha_2 z} \quad \theta_1^* = e^{\alpha_3 \theta_1}$$

$$e^{\alpha_2 - \alpha_1 - \alpha_3} \eta^* \frac{\partial \theta_1^*}{\partial z^*} = e^{2\alpha_1 - \alpha_3} \frac{\partial \theta_1^*}{\partial \eta^{*2}}$$

$$\text{at } \eta^* = 0 \quad e^{\alpha_1 - \alpha_3} \frac{\partial \theta_1^*}{\partial z^*} = -1 \quad \alpha_1 = \alpha_3$$

$$\text{at } \eta^* \rightarrow \infty \quad \theta_1^* \rightarrow 0$$

$$\text{at } z^* = 0 \quad \theta_1^* = 0$$

θ has been converted to θ_1 and $Pe^{-1/3}$ has been cancelled from both sides okay but boundary condition it is very surprisingly changing to at $\eta = 0$ which is nothing but $r = 1$, it is changing to very simplified form $\partial \theta / \partial \eta = -1$, earlier it was $Pe^{1/3} \times \partial \theta / \partial \eta = -1$ okay, so this simplified equation and boundary condition we get, apart from that we are having some other boundary conditions also.

Let us see, so first that $r \rightarrow \delta, \theta \rightarrow 0$ or so this was a boundary condition earlier so from here we get if η has $\eta \rightarrow 0, \theta_1 \rightarrow 0$ okay, and at the inlet as $z \rightarrow 0, \theta \rightarrow 0$ okay, so this is being converted to at $z \rightarrow 0, \theta_1 \rightarrow 0$. So now we have got θ once equation and 1, 2 and 3 boundary conditions for θ_1 , two for η direction and 1 for z direction, okay. Let us now try to get the similarity variable once more in-between η and z .

So first let us try start quantities $\eta^* = e^{\alpha_1 \eta}, z^* = e^{\alpha_2 z}$ and $\theta_1^* = e^{\alpha_3 \theta_1}$, okay and first we need to find out all this derivatives we have to replace all this θ_1 to θ_1^* 's in equation as well as boundary conditions for that let us put this start quantities in the equation first so here you can get in the left hand side as we are having $\eta \theta_1$ and z we are getting $e^{\alpha_2 - \alpha_1 - \alpha_3}$ in the convection side in the conduction side we are having δ^2 it is a mistake over here it will be $\delta^2, \delta^2 \theta_1 / \delta \eta^2$ so it becomes actually $\delta^2 \theta_1^* / \delta \eta^{*2}$ and it releases $e^{2\alpha_1 - \alpha_3}$ okay. So then the boundary condition if you see the very important boundary condition it becomes $\eta^* = 0$ it will be $e^{\alpha_1 - \alpha_3} \times \delta \theta_1^* / \delta z^* = -1$, okay.

Now we are having this is the equation and this is the boundary condition from here easily we can tell that okay, if we take $\alpha_1 = \alpha_3$ then this boundary condition becomes very simplified, okay. And other boundary conditions at $\eta \rightarrow \infty$ and $z = 0$ remains as usual similar $\theta_1^* \rightarrow 0$ and $\theta_1^* = 0$ okay, so one relationship between the constants α_1 , α_2 and α_3 already we have obtained from the boundary condition $\alpha_1 = \alpha_3$.

Now let us see the equation coefficients, if you equate the power of the e to the power, if you equate the coefficients of this left hand side and right hand side terms and get a relationship between α 's then you can write down from there.

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$$\alpha_2 - \alpha_1 - \alpha_3 = 2\alpha_1 - \alpha_3 \quad \alpha_2 - 2\alpha_1 = \alpha_1 \quad \alpha_2 = 3\alpha_1$$

$$\eta^* = e^{\alpha_1 \eta} \quad z^* = e^{3\alpha_1 z} \quad \theta_1^* = e^{\alpha_1 \theta}$$

$$\frac{\eta^*}{z^{1/3}} = \frac{\eta}{z^{1/3}} \quad \frac{\theta_1^*}{z^{1/3}} = \frac{\theta_1}{z^{1/3}} = f\left(\frac{\eta}{z^{1/3}}\right)$$

$$\text{Let } \xi = \frac{A\eta}{z^{1/3}} \quad \theta_1 = B z^{1/3} f(\xi) \quad \text{where } \theta = B \left(\frac{z}{Pe}\right)^{1/3} f(\xi)$$

$$\frac{\partial \xi}{\partial \eta} = \frac{A}{z^{1/3}} \quad \frac{\partial \xi}{\partial z} = -\frac{\xi}{3z}$$

$\alpha_2 - \alpha_1 - \alpha_3$ which was the power of e in the convection side = $2\alpha_1 - \alpha_3$ which is the power of e in the conduction side, okay. So from here we can get if you further simplify $\alpha_2 = 2\alpha_1 - \alpha_3 = \alpha_1$, here α_3 has been cancelled from both the sides okay and if you further proceed you will get $\alpha_2 = 3\alpha_1$, so we have got two relationships one is $\alpha_2 = 3\alpha_1$ and another one is $\alpha_1 = \alpha_3$, using this let us try to eliminate α_2 and α_3 and write down the star equations in terms of α_1 only, so we get $\eta^* = e^{\alpha_1 \eta}$ $z^* = e^{3\alpha_1 z}$ because α_2 is actually $3\alpha_1$ and $\theta_1^* = e^{\alpha_1 \theta}$ because $\alpha_3 = \alpha_1$ okay. Now if we put that from here if we try to get the similarity parameter ξ so this two equations will be helping me so let us see from this two equation we can get $\eta^*/z^{1/3}$ is actually equals to $\eta/z^{1/3}$ okay.

And from this two we can easily get that $\theta_1^*/z^{1/3} = \theta_1/z^{1/3}$ okay and this can be also written as we are seeing that the order is more or less same it can be also written as this will be a function

of $\eta/z^{1/3}$ so ultimately let us have the similarity variable ξ actually $= \eta/z^{1/3}$ multiplied by a constant A which needs to be determined, okay. So we have actually written the value similarity variable side by side let us also take θ^1 is nothing but B into Z the power 1/3 f(ξ) okay. Its comes from here okay so $B z^{1/3} f(\xi)$ where ξ is this one okay. So after defining this ξ and θ_1 in form of in form of ξ let us first try to see what is the derivative of ξ with respect to ξ and z okay. these are very simple make the derivative of these respective ξ first comes to the z constants and this will be by considered a ξ constant okay.

Once you do this verity you will be getting. Using this to let us find out the values of temperature derivatives. So first $\partial\theta_1$ by $\partial\eta$ so if you do a chain rule then you will be finding out the standing out to be it is standing out these AB f' okay so AB f' to the turning out to be. Here we have using these $\partial\xi$ by $\partial\eta$ which we have to found to over here and $\partial\theta_1$ by $\partial\eta$ will be making derivative of the one should be B will be coming and A will be coming from here as the result it is AB f' okay. if you proceed further second derivate of θ with respect to η will become $A^2 B$ is z it the power 1/3 f " okay.

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$$\alpha_2 - \alpha_1 - \alpha_3 = 2\alpha_1 - \alpha_3 \quad \alpha_2 - 2\alpha_1 = \alpha_1 \quad \alpha_2 = 3\alpha_1$$

$$\eta^* = e^{\alpha_1 \eta} \quad z^* = e^{3\alpha_1 z} \quad \theta_1^* = e^{\alpha_1 \theta}$$

$$\frac{\eta^*}{z^{*1/3}} = \frac{\eta}{z^{1/3}} \quad \frac{\theta_1^*}{z^{*1/3}} = \frac{\theta_1}{z^{1/3}} = f\left(\frac{\eta}{z^{1/3}}\right)$$

$$\text{Let } \xi = \frac{A\eta}{z^{1/3}} \quad \theta_1 = B z^{1/3} f(\xi) \quad \text{where, } \theta = B \left(\frac{z}{Pe}\right)^{1/3} f(\xi)$$

$$\frac{\partial \xi}{\partial \eta} = \frac{A}{z^{1/3}} \quad \frac{\partial \xi}{\partial z} = -\frac{\xi}{3z}$$

$$\frac{\partial \theta_1}{\partial \eta} = \frac{\partial \theta_1}{\partial \xi} \times \frac{\partial \xi}{\partial \eta} = \frac{A}{z^{1/3}} \theta_1' = \frac{A}{z^{1/3}} B z^{1/3} f'(\xi) = A B f'(\xi)$$

And if you find out what is the derivative of θ_1 with respect to z from its depends on z over here then will be getting the requires a it requires derivation of 2 multiplies so that if you do that one by two to the one constant and the being the other derivative and then typing the second one constant who doing the derivative of the first one who is get the simplified form like this. This is nothing but B by 3 into z to the power $2/3$ into $f - \xi f'$. So we have obtained both the temperature derivatives of θ_1 okay? Temperature derivatives of θ_1 a with respect to and z respectively.

Second derivative and first derivative of z . these the energy equation. So let us try to put all these values in the energy equation. So have I have told 2η and this is nothing but your $\partial\theta_1 / \partial z$ okay. So $\partial\theta_1 / \partial z$ okay? So these are all put over here and I mean right hand side we are having $\partial^2\theta_1$ then $\partial\eta$ let as put it over here okay. Now we are using in this equation we can see that B can be canceled. So ultimately you get equation having A^2 only okay? And a and we see little bit of a modification of this η by z to be power $1/3$ which is the ξ we can get a simplified form like this okay?. if you if you change the sides of this one then it will be getting f'' plus 2ξ by $3A^3$ into $\xi f' - f$ is equal to 0 okay.

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$$\frac{\partial^2 \theta_1}{\partial \eta^2} = \frac{A^2 B}{z^{1/3}} f''(\xi)$$

$$\frac{\partial \theta_1}{\partial z} = B \cdot \frac{1}{3} z^{-2/3} f(\xi) + B z^{1/3} f'(\xi) \left(\frac{-\xi}{3z}\right) = \frac{B}{3z^{2/3}} [f - \xi f']$$

$$2\eta \frac{B}{3z^{2/3}} [f - \xi f'] = \frac{A^2 B}{z^{1/3}} f''$$

$$\frac{2\eta}{3z^{1/3}} [f - \xi f'] = A^2 f'' \quad \rightarrow \quad \frac{2\xi}{3A} [f - \xi f'] = A^2 f''$$

$$f'' + \frac{2\xi}{3A^3} [\xi f' - f] = 0$$

at $\xi = 0 \quad AB f' = -1$

as $\xi \rightarrow \infty \quad f \rightarrow 0$

Now you see a boundary conditions of sequentially changing to at ξ is equal to 0 $AB f'$ to is nothing but $\partial\theta_1 / \partial \eta$. So we have actually seen that here $AB f'$ is nothing but $\partial\theta_1 / \partial \eta$ which is actually equal to which is actually equal to -1 so $AB f'$ is becoming -1 okay. And a

obviously the other boundary condition that θ tends to $-f$ and ξ tends to ∞ f is becoming 0 okay. So this the equation and we are having a two boundaries conditions over here okay.

If you proceed for the and try to find out what is the value of A to make this equation simplified in loop we can choose the value of a in this fashion $3A^3$ is equals to one if you choose when you see $3A^3$ is equal to 1 if you choose then the equation will become very simple okay?. So you can rock down this $3A^3$. So in that case a in that case a sorry apart from this one we can also take the is AB is equal to one then the boundary condition will be also very simple.

At ξ equal to AB f' is equal to -1 okay? So we have got $3A^3$ equals to one and AB equal to one. If you choose like this then from here we can get individual values of A and B like this. A is $2/3$ to the power $1/3$ and B is $3/2$ to the power $1/3$ okay. Now let us planning all those thing or equation so our our similarity variables becomes η by $3z/2$ to the power $1/3$ and θ_1 becomes $3z/2$ to the power $1/3$ f okay. So earlier here we had A and B respectively.

Once you get the value of A and B we can get the actual value of ξ and θ_1 over here okay?. And in case of equation the equation becomes now very simplified in loop have we put $3A^3$ is equal to 1 and the boundary condition also very simple as we have put AB is equal to 1 so this my equation and will having two boundary condition in this fashion okay. now we are a finding out f this is not a very simple equation as we have the second equation as we are having second order first order and 0 is order.

So it is a, we need some numerical simulation okay? You have to find out if numerical simulation. And if you try to get back the value of θ I have we have taken the equation over here from θ_1 we can go back to θ as we have taken θ is nothing but a $3/2$ number to the power $1/3$. $-1/3$ into θ_1 . So it is the go back here so θ becomes $3z$ by $2Pe$ to the power $1/3$ into f . so once you get the value f by the numerical simulation of that then you can get the temperature distribution one dimensional tempered the distribution θ in this fashion okay.

Next let us try to get the value of nascent number so in order to use the fist will become h transfer to the co efficient h . h is nothing but q by $T_w - T_b$ okay? if you proceeds for the so you will be getting $h r_0$ which is which can be you know the nascent number to into $h r_0$ by k is nascent number where you can find out this is nothing but $q r_0$ by k by $T_w - T_b$ and $T_w - T_b$ can be reduce $(T_w - T_i) - (T_b - T_i)$ so now this can be return as a θ_w and this can be return as θ_b because $T_w - T_i$ by $q r_0$ by k is nothing but θ_w and $T_b - T_i$ by $q r_0$ by k is θ_b okay.

So we get r_0 by k is nothing but $1/\theta_w - \theta_b$ okay? Proceeding for the nascent number will be nothing but q into this factor so it becomes $2/\theta_w - \theta_b$ okay. Now a θ_b above θ_b this will be nothing but $w \theta dr$ integration from 0 to r and then $w dr$ integration from 0 to r . as it is thermally fully develop so w will become actually 2 into $1-r^2$ so 2^2 can be canceled from denominator and numerator okay.

And as we see we see θ_w okay the θ_w as we have shown in the previous one θ variable so θ was $3z$ by $2Pe$ to the power $1/3$ and then $f(\xi)$ okay. now as we are at all ξ obviously ξ would be 0. So we get θ_w is this fashion. So here you see we have θ_w here we have got θ_b which is 0 so we can easily find out the nascent number. So nascent number becomes θ_w is nothing but this one so this quantity we have return $-\theta_b$ which is 0 so this is actually returns 2 by θ_w okay.

Now once you get what is the value of this a f_0 okay so after simulation on the equation and the boundaries condition we have shown on a equation and the boundary conditions we have shown and the equation so then will be getting the values comes out to the 1.639 okay? If the nascent number becomes 1.639 by z by Pe to the power $1/3$. So here also we can see nascent number function of z and the pelican number over here okay.

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Summary

- Governing equation for thermally developing but hydrodynamically fully developed forced convection inside a tube having constant applied heat flux:

$$2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$$
- at $r = 1$ $\frac{\partial \theta}{\partial r} = 1$ as $r \rightarrow \delta$ $\theta \rightarrow 0$ as $z = 0$ $\theta = 0$

$$f'' + \xi[\xi f' - f] = 0 \quad \text{with } f'(0) = -1 \text{ \& } f(\infty) = 0$$
- Nusselt number for thermally developing but hydrodynamically fully developed forced convection inside a tube having constant applied heat flux:

$$Nu_z = \frac{1.639}{\left(\frac{z}{Pe}\right)^{1/3}}$$

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So we this I end this lecture let us summarize what we have learnt so first we have seen the governing equation for thermally developing but hydrodynamically fully developed

forced convection inside a tube having constant applied heat flux. So this is very important constant heat flux so the equation we can reduce to this from $2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$ is equal to $\frac{\partial^2 \theta}{\partial \eta^2}$ okay. Corresponding boundary conditions so this is the boundary condition at over all okay.

$\frac{\partial \theta}{\partial r}$ is equal to 1 and these are the one boundary conditions at the axis okay. At this is the boundary condition at the entry. Once you reduce to subsequence stages so the by are the simulative variable and $\theta_2 \theta_1$ finally we get $f'' + \xi[\xi f' - f] = 0$ with the boundary condition $f'(0) = -1$ and $f(\infty) = 0$. So we have also proceeded further to show that relationship between nusselt numbers is respect to z .

So nusselt number for thermally developing but hydrodynamiclly fully developed forced convection inside a tube having constant applied heat flux comes out to be nusselt number is equal to $1.639 z$ by pelican number to the power $1/3$ okay which is dependent on z okay. so this you have learnt in this lecture so I will after this one let us see that how for you have understood in this lecture so let me test to the understanding where three questions are usual.


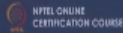
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Summary

- Governing equation for thermally developing but hydrodynamically fully developed forced convection inside a tube having constant applied heat flux:

$$2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$$
- at $r = 1$ $\frac{\partial \theta}{\partial r} = 1$ as $r \rightarrow \delta$ $\theta \rightarrow 0$ as $z = 0$ $\theta = 0$
- $f'' + \xi[\xi f' - f] = 0$ with $f'(0) = -1$ & $f(\infty) = 0$
- Nusselt number for thermally developing but hydrodynamically fully developed forced convection inside a tube having constant applied heat flux:

$$Nu_z = \frac{1.639}{\left(\frac{z}{Pe}\right)^{1/3}}$$

So first one is in thermally developing but hydro dynamically fully developed fully region inside a duct having constant heat flux, radial conduction is of the order four options are there I think already understood which one is the correct answer let me telling once again $1/Pe$ number $1/Pe$ $1/\partial^2$ here we are having $1/Pe$ $1/\partial$ and finally Pe . So obviously you are the correct answer that is $1/Pe$ is the correct order okay.

So this we have discussed also in this lecture. Next one in thermally developing region Nusselt number depends on both Pe and z for. In this lecture we have discussed about constant wall temperature constant heat flux none of these case a will be the to giving you the dependant on the nusselt number and Pe number and z and fourth option is both a and b are true. If you to this lecture and we see the previous lecture also then probably you have understood correct one is both a and b are true.

Actually in thermal in terms region the nusselt number will depend on both Pe and z. so that is why it will not be depending on the wall temperature or constant heat flux that will not be mattering. It will be mattering on the thermal in terms easy or not okay. so it will be having z dependence has well as in the pe number dependence. Last question is like this in thermal developing but hydro dynamically fully developed region inside a duct having constant heat flux $1-r$ is of the order this is very simple $1-r$ obviously we have used for our similarity variable.

So $1-r$ is the order of four options here we having Pe number to the power $-1/3$ Pe number to the power $1/3$ Pe and $1/Pe$ number. Very simple globally have correct answer. So correct answer is the first one Pe number to the power $-1/3$ okay so with this side a end this lecture a in the next lecture will be discussing a special case which called a rally we cannot convection which we can observe incase of flow inside a duct inside to parallel plates actually okay.

The dimensional parallel plates there will be understanding what type of convection said there are end going for the stability analysis of that one okay. if you have any query regarding this lecture and any other general query about convective transfer please keep on posting discussion for on thank you.

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