

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

**NPTEL
NPTEL ONLINE CERTIFICATION COURSE**

Convective Heat Transfer

Lec – 15

Thermal Entrance Region: Uniform Wall Temperature

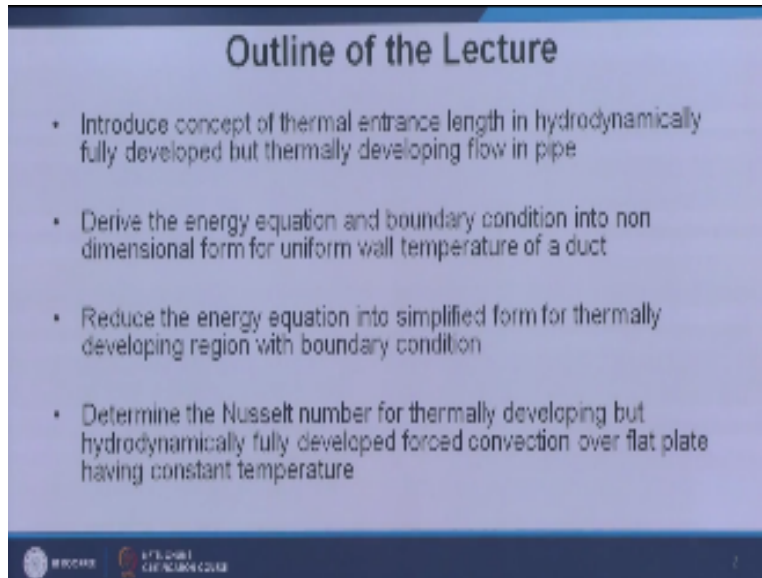
**Dr. Arup Kumar das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee**

Hello welcome in the 15th lecture of convective heat transfer course in this lecture we will be discussing about thermal entrance length in our last two lectures we have discussed about thermally fully developed region mainly but here we will be discussing at the beginning of the duct what happens that means when the temperature profile is developing to get a steady state profile okay.

So this thermal entrance region we have already defined in our 11th lecture so there we have shown that at the beginning how temperature profile actually gets developed in to a parabolic one okay. So here we will be showing that entrance length region and that two in this lecture we will be considering uniform wall temperature case okay. So that means the pipe is actually being heated up with some heating coil which is maintain in the wall temperature at constant okay.

If you have the other extant that means the heat flux is constant so what is our heating coils are supplying to the fluid inside the pipeline that is actually heat flux is constant that case you need to see in the next lecture okay. So as I have mentioned that we will be discussing about thermal entrance region over here, thermal entrance region with uniform wall temperature okay. So let me at the beginning tell you that what will be the outline what things will be covering in this lecture we will be introducing the concept of thermal entrance length.

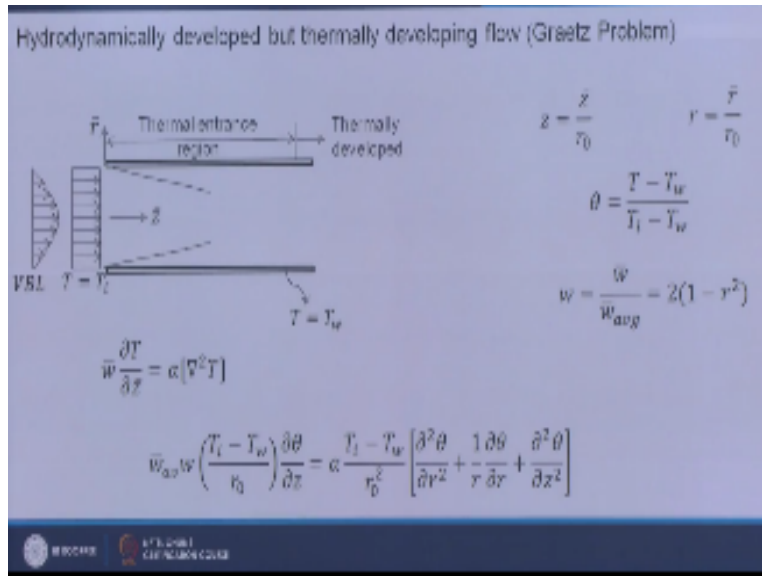
(Refer Slide Time: 01:57)



In hydro dynamically fully developed but thermally developed in flow in pipe. This assumption we are doing over here that we are considering the flow hydro dynamically has developed that means it has taken a parabolic velocity profile but temperature has not yet grown in to a steady state profile okay. So though it is hydro dynamically developed fully developed but thermally developing flow okay inside a pipe, then we will be deriving the energy equation and sub sequent boundary condition and we will giving a non dimensional form to this one for uniform wall temperature case okay.

Uniform wall temperature around a duct okay, we will try to reduce the energy equation in to simplified form which can be solved okay and that we will be doping for thermally developing region with constant wall temperature boundary condition okay. And finally we will try to determine by this equation energy equation will try to determine the Nusselt number for thermally developing but hydro dynamically fully developed force convection around a duct okay having constant temperature okay.

(Refer Slide Time: 03:19)



So let me start with this schematic so here we will be considering hydro dynamically developed but thermally developing flow, usually this type of problems are called Graetz flow okay Graetz problem we can call okay. So here you see this is our pipe line let say here we are having the center line okay, here we are having the center line and now somehow the velocity boundary layer has developed and had taken actually a parabolic profile okay.

And then you see over here we are considering that suddenly we are starting to heat over here the wall is getting a constant temperature $T = T_w$ okay, and here the fluid whatever was coming with the parabolic velocity profile have a constant temperature T_i okay. And this is your axial detection of the duct and this is your radial direction okay. The temperature profile will also become developed after the certain length and what is the consequence in the further down stimulation those things we have discussed in the last two lectures.

Here our major concentration will be over here and the beginning of the heating section this region is called actually thermal entrance region okay. You can see I have shown here the boundary layer thermal boundary layer being developed by the way what is thermal boundary layer this concepts we have already discussed in lecture 11 okay. So this boundary layer will be developing over here and whenever they will be margin at the center line beyond that we will be having thermally developed region okay.

So let me try to first see what non dimensional parameter we can use as here you can find out this thermal entrance region is small in length we can take the scale of that one is r_0 which is

nothing but the radius of the tube okay. So we are non dimensionalizing z in the axial direction by r_0 which is nothing but the radius of the tube, so $z = z_{\text{bar}} / r_0$ okay, obviously the non dimensionalized radius can be taken as r_{bar} / r_0 for non dimensionalized temperature we are considering θ , so θ is nothing but $t - t_w / t_i - t_w$ where t_i is the inflow temperature and t_w is the wall temperature constant wall temperature okay.

And as we have considered velocity boundary is fully developed that means taking a parabolic velocity profile we consider that w which is non dimensional velocity axial velocity is nothing but $w_{\text{bar}} / w_{\text{average}}$ is nothing but $2(1-r^2)$ a parabolic velocity profile okay. Now let me show you that what will be the equations so the convection will be having only one term okay so axial convection term because other two u and v those two things will be 0 because it is thermally hydrodynamically fully developed condition okay.

So u and v is 0 only w is having getting a parabolic velocity profile like this so only singly term in our convection will be remaining $w \partial t / \partial z$ okay and in the conduction side we will be having all terms but in this we are only considering that radial and axial terms are remaining and we are having azimuthal symmetry so θ directional terms will not be considering over here okay.

So if you expand this equation so you will be getting that if you expand then if you try to put all this non dimensional parameters and replace the dimensional terms then you will be getting the convection is becoming w_{average} in to w so this w_{bar} is giving w_{average} in to w and this t is actually releasing one $t_i - t_w$ and z is releasing r_0 and as a result $\partial t / \partial z$ is $\partial \theta / \partial z$ okay.

On the other hand in the right hand side this θ is actually being replaced t is being replaced by θ and one $t_i - t_w$ is being released and everywhere we are having second order term that means $r^2 z^2$, and here first order but multiplied with r so we can find out one r_0^2 can come outside because both z and r we have considered of the order r_0 okay, so ultimately this equation we are getting for the hydrodynamically developed but thermally developing flow okay.

(Refer Slide Time: 08:09)

$$\frac{Pe}{2} w \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}$$

$$Pe(1-r^2) \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}$$

at $r = 1$ $\theta = 0$ at $r = 0$ $\frac{\partial \theta}{\partial r} = 0$

at $z \rightarrow -\infty$ $\theta \rightarrow 1$ at $z \rightarrow \infty$ θ should be bounded

In thermal entrance region

$$1 - r \sim \delta$$

$$1 - r^2 = (1-r)(1+r) \sim \delta$$

$$z \sim 1$$

$\frac{\partial \theta}{\partial r} \sim \frac{1}{\delta}$	$\frac{\partial \theta}{\partial z} \sim 1$	Convection term $\sim \delta$
$\frac{\partial^2 \theta}{\partial r^2} \sim \frac{1}{\delta^2}$	$\frac{\partial^2 \theta}{\partial z^2} \sim 1$	Radial Conduction term $\sim \frac{1}{Pe \delta^2}$
		Axial conduction $\sim \frac{1}{Pe}$

So if you simplified it little bit then you can get that over here α and then $r_0 \times w$ average will be giving you pecllet number okay, so pecllet number by /2 and here we can get the rest terms $d \times \partial \theta / \partial z =$ radial conduction terms and then the axial conduction terms okay. Now if you put the value of the w which is nothing but $2 \times 1 - r^2$ then we will be getting equation of this form okay and subsequent boundary conditions definitely at $r = 1$ that means at the wall we are having $\theta = 0$ because t becomes t_w okay and at $r = 0$ obviously there will be no gradient of temperature so $\partial \theta / \partial r = 0$.

Now at z tens to $-\infty$ that means for before the pipe entry obviously θ will be one as at t becomes t_i okay and that z tens to ∞ that means if you go for downstream in the pipe line so there we are not knowing what is the value of θ because that will be depending on the length and so we can write down that this θ is nothing but actually will be bounded okay. Next as our interest is lying in this thermal entrance region so let us first do little bit of scale analysis so if you see this thermal entrance region we will be actually having $1-r$ okay because this $1-r$ is nothing but your thermal boundary layer thickness okay. So we can write down $1-r$ is the of the order of the boundary layer δ okay, so this boundary layer δ is of the order of $1-r$. now if you use this one then you can write down $1-r^2$ which is a dominant term in the left hand side convection side in the equation.

So you can give this becomes $1-r \times 1+r$, $1+r$ is obviously of the order of 1 so you can get this $1-r^2$ is of order θ okay and definitely z we have already considered of the z varies of the order of δ so

this z becomes the order of 1 okay. Now let us try to see that what are the orders of all this terms one by one so first if you see that θ / δ are terms so $\partial\theta / \partial r$ so as θ is of order one it varies between 0 and 1 so it can be of order one r will be of order δ , so $\partial\theta\partial$ term is the order of $1/\delta$.

Similarly $\partial\theta / \partial z$ here θ is of order one and z is of order 1 already we have mentioned so this becomes also of order one okay, if you go for the second order derivative it will become $1/\delta^2$ as $\partial\theta / \partial r$ is $1/\delta$, and $\partial^2\theta / \partial z^2$ obviously we will remain same because z is of order one okay. So we have got all the terms okay order of all the terms now if we try to see what is the convection order if you see the convection order so $\partial\theta / \partial z$ was actually of order one but $1-r^2$ is of the order of δ .

So we find out multiplication of this one is the of the order of δ okay. Let us see the radial conduction that means these two terms if you see this two terms obviously we are finding out that this pecllet number can be taken in the radial conduction side in the conduction side rather, so it is $1/\text{pecllet number}$ and then both the terms we are having $1/\delta^2$ for the first terms and for the second term it is nothing but $1/\delta \times r$ which is of the order of δ .

So it is actually $1/\delta^2$ term okay, no actually this term is actually becoming $1/\text{pecllet number}$ order and this terms is becoming $1/\text{pecllet number}$ in to $1/\delta^2$, so this terms will be actually dominating amongst this two term. So the magnitude or scale of the radial conduction will become $1/\text{pecllet}$ in to $1/\delta^2$ okay. Then axial conduction if you see axial conduction that already we have prove that this is of order one as $1/\text{pecllet number}$ came in this side so it becomes of $1/\text{pecllet numbers}$.

So, here from you can get that between this two conduction obviously this one is having higher magnitude so radial conduction dominates so what the axial conduction.

(Refer Slide Time: 12:52)

$$\delta \sim \frac{1}{Pe} \frac{1}{\delta^2} \text{ or } Pe \sim \frac{1}{\delta^3} \text{ or } \delta \sim \frac{1}{Pe^{1/3}}$$

Let us consider, $\eta = (1-r)Pe^{1/3} \sim 1$ $z \sim 1$

$$1-r = Pe^{-1/3} \eta \quad \text{or} \quad r = 1 - Pe^{-1/3} \eta$$

$$1-r^2 = (1+r)(1-r) = Pe^{-1/3} \eta \left(2 - Pe^{-1/3} \eta \right) = 2Pe^{-1/3} \eta - Pe^{-2/3} \eta^2$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \eta} \frac{\partial \eta}{\partial r} = -Pe^{1/3} \frac{\partial \theta}{\partial \eta} \quad \text{and} \quad \frac{\partial^2 \theta}{\partial r^2} = Pe^{2/3} \frac{\partial^2 \theta}{\partial \eta^2}$$

If you equate this convection and conduction side we get pecllet number is actually $1/\delta^3$ or we can write down δ is of the order of $1/Pe^{1/3}$ this keeps us some idea that what can be our similarity variable. So let us consider the similarity variable so the similarity variable η we are writing $1-r \times Pe^{1/3}$ okay. Now as we are considering $1/r \times Pe^{1/3}$ then definitely this η will become of the order of one okay so this is the duty of this similarity analysis so by considering that what is the order of δ or the boundary layer thickness we construct one variable call similarity variable it will becomes of the order of one okay.

And already we have shown that z is already of order one so we will get η and z coordinate now okay in place of z and r coordinate. Next let us see further that what will be the value of r from this one as we have defined detail in this fashion so little bit of side change we can have $r = 1 - Pe^{-1/3} \times \eta$ and subsequently $1-r^2$ which is nothing but $1+r$ to $1-r$ will be giving me $Pe^{-1/3} \times \eta \times 2$ that means $1+1- Pe^{-1/3} \times \eta$ and multiplication of this two terms will be giving me this one okay. $2 Pe^{-1/3} \eta - Pe^{-2/3} \times \eta^2$ okay.

Let us proceed further for the derivative of the θ terms okay so first $\partial \theta / \partial r$ so if you do that θ is the function of η , so $\partial \theta / \partial \eta \times \partial \eta / \partial r$ so $\partial \eta / \partial r$ can be found out easily by making derivative of this one, so this becomes nothing but $-Pe^{1/3}$ if you make the derivative of η with respect to R , so it becomes $-Pe^{1/3} \times \partial \theta / \partial \eta$ okay, and second derivative subsequently will be giving you $Pe^{2/3} \partial^2 \theta / \partial \eta^2$ okay, so once again you do the derivative with respect to this, once again chain rule will be giving you $Pe^{2/3}$ okay. Proceeding further if you put this derivatives as well as the value of $1-r^2$

in your equation that means in your equation means over here rather than you will finding out this turns out to be like this.

(Refer Slide Time: 15:38)

The slide contains the following mathematical content:

$$\left(4Pe^{-\frac{1}{3}\eta} - 2Pe^{-\frac{2}{3}\eta^2}\right) \frac{\partial \theta}{\partial z} = \frac{2}{Pe} \left[Pe^{\frac{2}{3}} \frac{\partial^2 \theta}{\partial \eta^2} - \frac{Pe^{\frac{1}{3}} \frac{\partial \theta}{\partial \eta}}{1 - Pe^{-\frac{1}{3}\eta}} \right] + \frac{2}{Pe} \frac{\partial^2 \theta}{\partial z^2}$$

$$\left(4\eta - Pe^{-\frac{1}{3}\eta^2}\right) \frac{\partial \theta}{\partial z} = \left[2 \frac{\partial^2 \theta}{\partial \eta^2} - \frac{2Pe^{-\frac{2}{3}} \frac{\partial \theta}{\partial \eta}}{1 - Pe^{-\frac{1}{3}\eta}} \right] + \frac{2}{Pe^{2/3}} \frac{\partial^2 \theta}{\partial z^2}$$

For $Pe \rightarrow \infty$

$$2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$$

at $\eta = 0$ $\theta = 0$ as $\eta \rightarrow \infty$ $\theta \rightarrow 1$

at $z = 0$ $\theta = 1$

$\eta^* = e^{\alpha_1} \eta$ $z^* = e^{\alpha_2} z$ $\theta^* = e^{\alpha_3} \theta$

Leveque solution

This is nothing but your $2x1-r^2$ okay, $\partial\theta/\partial z$ $2/Pe$ this is nothing but your $\partial^2\theta/\partial r^2$ and here you are having $1/r$ so this is $1/r$ term, $1/1-Pe^{1/3}\eta$ and this one is nothing but your $\partial\theta/\partial r$ okay, and the axial conduction term remains like this $2/Pe \partial^2\theta/\partial z^2$ little bit of simplification and side change will be giving you like this $(4\eta - Pe^{-1/3}\eta^2) \partial\theta/\partial z$ is equals to on the right hand side we are having $2\partial^2\theta/\partial \eta^2 - 2Pe^{-2/3}\partial\theta/\partial \eta / 1 - Pe^{-1/3}$ this is nothing but coming due to r , $+2/Pe^{2/3} \partial^2\theta/\partial z^2$ okay.

So you can see for large Pe what we can do this term, this term, this term and this term can be cancelled because all are carrying actually Peclet number to the power minus power, so you can find out only remaining term is nothing but $4\eta \partial\theta/\partial z = [2\partial^2\theta/\partial \eta^2]$ that means it is nothing but $2\eta \partial\theta/\partial z = \partial^2\theta/\partial \eta^2$ okay. Let us see the boundary conditions also, so we find out whenever η tends $\eta=0$ now $\eta=0$ means $r=1$ okay, so $\eta=0$ means $r=1$.

Because we have consider $\eta=1/r Pe^{1/3}$ okay, so we find out that at $\eta=0$, $\theta=0$ because $T=T_w$ okay. Then similarly, as we have considered $\eta=1-rPe^{1/3}$ though our boundary condition was at $r=0$ $\partial\theta/\partial \eta$, $\partial\theta/\partial r$ will be equals to 0 so from there we are getting that for large Pe , so Pe tends to ∞ means η tends to ∞ because η is nothing but $1-rPe^{1/3}$ so Pe becomes very big means η will be also very big.

There we are finding out η tends to 1 okay, so and in case of the inlet we are having at $z=0$, $\theta=1$ okay, this is nothing but $T_i - T_w / T_i - T_w$ okay. So we got the equation as well as the boundary conditions two boundary conditions for η and one boundary conditions for z okay, so this equation and sets of boundary conditions are actually called Leveque equation and the solution of this one has been proposed by Leveque.

So let us see how it can be solved at the beginning what we will be doing, we will trying to find out the stretching variables for that let us take $\eta^* = e^{\alpha_1} \eta$ we do not know what is the value of α_1 we need to find out. Similarly, $Z^* = e^{\alpha_2} Z$ and $\theta^* = e^{\alpha_3} \theta$, so by using this we will be trying to find out what are the values are α_2 , α_1 and α_3 respectively, okay. So let us put all these values in this equation and the boundary conditions.

(Refer Slide Time: 19:19)

The slide contains the following mathematical content:

$$e^{\alpha_2 - \alpha_1 - \alpha_3} 2\eta^* \frac{\partial \theta^*}{\partial z^*} = e^{2\alpha_1 - \alpha_3} \frac{\partial^2 \theta^*}{\partial \eta^{*2}}$$

B.C.

$$\eta^* = 0 \quad e^{-\alpha_3} \theta^* = 0 \rightarrow \theta^* = 0$$

$$\text{as } \eta^* \rightarrow \infty \quad e^{-\alpha_3} \theta^* \rightarrow 1$$

$$\text{at } z^* = 0 \quad e^{-\alpha_1} \theta^* = 1$$

So $\alpha_3 = 0$ from B.C

$$\alpha_2 - \alpha_1 = 2\alpha_1 \text{ from equation}$$

$$\text{or } \alpha_2 = 3\alpha_1$$

$$\eta^* = e^{\alpha_1} \eta \quad z^* = e^{3\alpha_1} z \quad \theta^* = \theta$$

$$\frac{\eta^*}{z^{*1/3}} = \frac{\eta}{z^{1/3}}$$

At the bottom of the slide, there are logos for 'IIT ROORKEE' and 'WHITE OAKS INTERNATIONAL COLLEGE'.

So first in the equation if you see there was η, θ and Z so subsequently we are having $e^{\alpha_2 - \alpha_1 - \alpha_3}$ okay, and in the right hand side for the conduction side we are having $\partial^2 \theta / \partial \eta^2$ so from there we are getting $e^{2\alpha_1 - \alpha_3}$ and all the θ and η is turning out to be θ^* and η^* , okay and if you see the

corresponding boundary conditions B.C. boundary conditions, so η tends to 0 means obviously η^* tends to 0.

In that case you find out θ was replaced to 0 now we are getting $e^{-\alpha_3} \theta^*=0$ okay, which gives nothing but for a finite value of α_3 this $\theta^*=0$ okay, and for the axis η^* tends to ∞ means obviously η^* tends to ∞ we are getting $e^{-\alpha_3} \theta^*$ tends to 1 okay, and for the inlet boundary condition $z^*=0$ we get $e^{-\alpha_3} \theta^*=1$, okay. Now from this two equations definitely we can understand that we need to make $\alpha_3=0$ okay.

Because it will simplified version if α_3 gets the value of 0 then θ^* becomes 1 and θ^* becomes 1 over here okay, in the inlet as well as in the axis okay. So let us now equate the co-efficient from the equations okay, from the convections, conduction equation so we get $\alpha_2-2\alpha_1-\alpha_3=2\alpha_1-\alpha_3$ and substitute the value of $\alpha_3=0$ so subsequently we get $\alpha_2-\alpha_1=2\alpha_1$ okay. So from here we can get that $\alpha_2=3\alpha_1$ okay.

So ultimately we can then write down η is nothing but $e^{\alpha_1}\eta$, $\eta^*=e^{\alpha_1}\eta$, $z^*=e^{3\alpha_1}z$ and finally $\theta^*=\theta$ as because $\alpha_3=0$ okay. So if you get so then it is very easy to find out what can be my similarity variable so we can take $\eta^*/z^{*1/3}=\eta/z^{1/3}$ okay. So this correlation, this relation can be found out from this two equations easily by eliminating e^{α_1} okay, so if you do so then we can get the similarity variable like this. Let us say similarity variable is ξ .

(Refer Slide Time: 22:12)

$$\xi = \frac{A\eta}{z^{1/3}} \quad \theta = \theta(\xi)$$

$$\frac{\partial \xi}{\partial \eta} = \frac{A}{z^{1/3}} \quad \frac{\partial \xi}{\partial z} = A\eta \left(-\frac{1}{3}\right) z^{-4/3} = -\frac{\xi}{3z}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\partial \theta}{\partial \xi} \frac{\partial \xi}{\partial \eta} = \theta'(\xi) \frac{A}{z^{1/3}} \quad \frac{\partial^2 \theta}{\partial \eta^2} = \theta''(\xi) \frac{A^2}{z^{2/3}} \quad \frac{\partial \theta}{\partial z} = \theta'(\xi) \frac{\partial \xi}{\partial z} = \theta'(\xi) \left(-\frac{\xi}{3z}\right)$$

Substituting $\frac{\partial^2 \theta}{\partial \eta^2}$ and $\frac{\partial \theta}{\partial z}$

$$2\eta \theta' \left(-\frac{\xi}{3z}\right) = \theta'' \frac{A^2}{z^{2/3}}$$

$$A^2 \theta'' = \frac{-2}{3} \frac{\eta}{z^{1/3}} \xi \theta'$$

So we can write down ξ is nothing but $\eta/z^{1/3}$ and let us take a constant in front of that which is A okay, so we need to evaluate the value of A and as we are having over here you see $\theta^*=\theta$ so from here we can take θ is nothing but a function of ξ okay. So let us use this one and try to first get what are the derivatives of ξ with respect to η and ξ , η and z , okay because η is function of η and ξ is function of η and z , okay.

So here $\partial\xi/\partial\eta=A/z^{1/3}$ because here we are having $A\eta/z^{1/3}$ okay, derivative with respect to z of ξ becomes actually this form so $A\eta$, η comes as constant and if we do the derivative of $z^{1/3}$ then we get $(-1/3)z^{-4/3}$ so it will be ultimately giving you $-\xi/3z$ okay. So both the derivatives we have obtained of ξ then let us try to get the values of the derivatives of the non-dimensionalized temperature θ .

So first let us see the value of $\partial\theta/\partial\eta$ so $\partial\theta/\partial\eta$ will be obviously as θ is a function of ξ now, so $\partial\theta/\partial\xi$. $\partial\xi/\partial\eta$ okay, so here you see we are writing $\partial\theta/\partial\xi$ as θ' and $\partial\xi/\partial\eta$ is nothing but $A/z^{1/3}$ so we have got the value of $\partial\theta/\partial\eta$ okay, it was there in the convection side, okay. Then double derivative if you see it was there in radiations, in the conduction side sorry, it was that in conduction side.

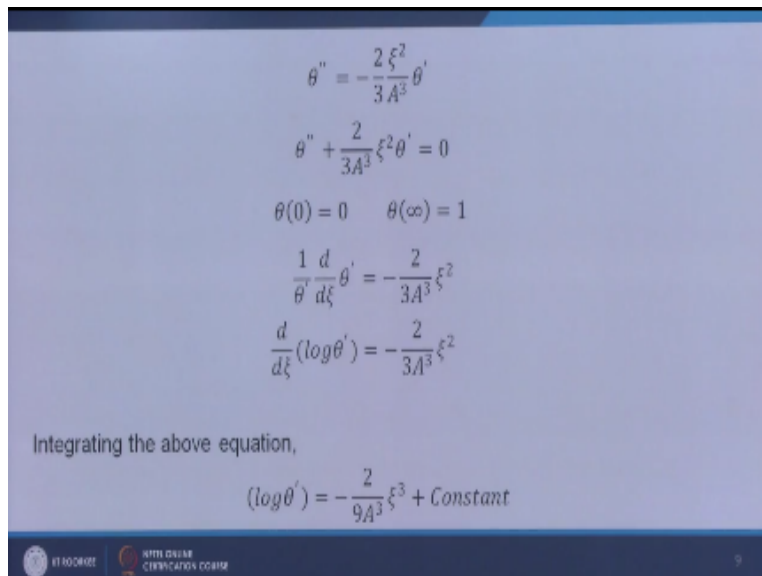
In same way if we do the double derivative because in our conduction side single derivative term we have neglected for higher Peclet number so we are only having ∂^2 double derivative of θ with respect to ξ that means $\partial^2\theta/\partial\eta^2$ we are having, okay. So double derivative of this one okay, that means once again if you the derivative with respect to η it gets $\theta''(A^2/z^{2/3})$ okay. On the other hand if you do the value of, if you find out the value of $\partial\theta/\partial z$ you will be getting it is nothing but $\theta'\partial\xi/\partial z$, okay.

So $\theta'\partial\xi/\partial z$ is nothing but $-\xi/3z$ so that we can plug in over here, so we have got both $\partial^2\theta/\partial\eta^2$ and $\partial\theta/\partial z$ so let us try to put that in the equation okay, so in the governing equation if you remember earlier it was something like this $2\eta\partial\theta/\partial z=\partial^2\theta/\partial\eta^2$ so here I have got both the derivative values so let us try to put that over here quickly. So we can get 2η then this is the value of $\partial\theta/\partial z$ and in the right hand side we are having actually $\partial^2\theta/\partial\eta^2$, okay little bit of side change and modification it gives me $A^2\theta'' = -2/3 \eta/z^{1/3} \xi\theta'$ here $\eta/z^{1/3}$ is nothing but ξ/A okay, ξ/A .

(Refer Slide Time: 26:02)

$$\theta'' = -\frac{2\xi^2}{3A^3}\theta'$$
$$\theta'' + \frac{2}{3A^3}\xi^2\theta' = 0$$
$$\theta(0) = 0 \quad \theta(\infty) = 1$$
$$\frac{1}{\theta'} \frac{d}{d\xi} \theta' = -\frac{2}{3A^3}\xi^2$$
$$\frac{d}{d\xi}(\log \theta') = -\frac{2}{3A^3}\xi^2$$

Integrating the above equation,

$$(\log \theta') = -\frac{2}{9A^3}\xi^3 + \text{Constant}$$


So we can ultimately get that $\theta'' = -2/3 \xi^2/A^3 \theta'$ okay, and simplified form of this one will be $\theta'' + 2/3A^3 \xi^2\theta' = 0$ with the corresponding boundary conditions this boundary conditions already we have seen $\theta=0$ means at the wall it is actually equals to 0 and θ tends to ∞ that means $\theta(\infty)=1$ it is at the axis, okay. Then let us see how this derivative, how this equation can be integrated so for that we are writing $1/\theta'$ and θ'' we are writing as $\partial/\partial\xi \theta'$ okay.

On the right hand side we are having a constant term not a constant term it is the function of ξ - $2/3A^3\xi^2$ okay. Now this can be integrated easily so this I am writing as $d/d\xi(\log \theta')$ and the right hand side we are having $-2/3A^3\xi^2$, okay. So if you integrate now one step then you will getting $(\log \theta') = -2/9\xi^3/A^3 +$ a constant, okay to evaluate the constant we will be using the boundary condition.

(Refer Slide Time: 27:23)

$$\text{let } A = \left(\frac{2}{9}\right)^{1/3} \quad \frac{d\theta}{d\xi} = B e^{-\xi^3}$$

$$\int_0^\xi \frac{d\theta}{d\xi} d\xi = B \int_0^\xi e^{-\xi^3} d\xi \longrightarrow \theta(\xi) - \theta(0) = B \int_0^\xi e^{-\xi^3} d\xi \longrightarrow \theta(\xi) = B \int_0^\xi e^{-\xi^3} d\xi$$

$$\text{From boundary condition: } \theta(\infty) = 1 = B \int_0^\infty e^{-\xi^3} d\xi$$

$$\theta(\xi) = \frac{\int_0^\xi e^{-\xi^3} d\xi}{\int_0^\infty e^{-\xi^3} d\xi}$$

$$\text{Taking, } \xi^3 = t \quad \xi = t^{1/3} \quad d\xi = \frac{1}{3} t^{-2/3} dt$$

So before that let us consider here you are having $2/9A^3$ so this let us make a unified constant so for that we are considering nothing but $A=(2/9)^{1/3}$ so if you make $A=(2/9)^{1/3}$ then A^3 will become $2/9$ as the result you will be finding out that this will become a constant, okay this will become a constant 1. So we get if we choose $A=(2/9)^{1/3}$ so this co-efficient of ξ^3 becomes 1 okay, so ultimately we will get that $\partial\theta/\partial\xi$ which is nothing but θ' okay, is equal to this constant we are considering actually $\log B$ okay, so if we consider $\log B$ then we are getting ultimately $\theta' = B e^{-\xi^3}$, here A and subsequently $(2/9)^{1/3}$ has actually given rise to 1, okay.

Then once we get this 1 one step further integration if we do from 0 to ξ we will be getting this is nothing but $\theta(\xi) - \theta(0)$ put in the upper and lower limits and the right hand side it is nothing but 0 to ξ it will be for $-\xi^3 d\xi$ with a B prefixed over there, okay. So if we put the boundary condition that $\theta(0) = 0$ at the wall so here you find out that $\theta(\xi) = B \int_0^\xi e^{-\xi^3} d\xi$, okay. So as we have obtained the profile temperature profile in terms of ξ but only unknown is B .

So let us try to plug in the other boundary condition that means what happens at $\xi = \infty$ so at $\theta(\infty)$ it is 1 which we have seen as boundary condition is actually equals to $B \int_0^\infty \xi e^{-\xi^3} d\xi$ will turn out to ∞ now, okay $e^{-\xi^3} d\xi$ okay. So here I get what is the value of B which is nothing but $1 / \int_0^\infty e^{-\xi^3} d\xi$, so once I plug in this value over here θ becomes $\int_0^\xi e^{-\xi^3} d\xi / \int_0^\infty e^{-\xi^3} d\xi$ okay, so this we have obtained the profile for θ , preceding further if we have evaluate this integration

let us consider $\xi^3 = t$ okay. So once you see $\xi^3 = t$ let us take what is $d\xi$ turns out to be $1/3 t^{-2/3} dt$ okay. So if you take this value of $d\xi$ over here.

(Refer Slide Time: 30:26)

$$\int_0^{\infty} e^{-\xi^3} d\xi = \int_0^{\infty} e^{-t} \frac{1}{3} t^{-2/3} dt = \frac{1}{3} \Gamma(1/3) = \Gamma(4/3) = 0.893$$

$$\theta(\xi) = \frac{1}{\Gamma(4/3)} \int_0^{\xi} e^{-\xi^3} d\xi$$

$$q = k \left(\frac{\partial T}{\partial r} \right)_{r=r_0} = \frac{k}{r_0} (T_i - T_w) \left(\frac{\partial \theta}{\partial r} \right)_{r=1}$$

$$h = \frac{q}{T_w - T_b} = \frac{q}{T_w - T_i}$$

as r is very small the volume integral of T_b will be T_i

$$h = \frac{-k}{r_0} \left(\frac{\partial \theta}{\partial r} \right)_{r=1}$$

We obtain this 0 to ∞ into and in place of $d\xi$ we can write down $1/3 t^{-2/3} dt$ okay. Now here if you see this $1/3$ if we take out, this is actually the expression of γ function $\int_0^{\infty} x t^{-2/3} dt$. This is actually a γ function for $1/3$.

So we write down $\gamma 1/3$, so this is coming from mathematics once again and the function is well knowing in mathematics okay, so we get over the value of this integration of 0 to ∞ it will be called $\int_0^{\infty} e^{-\xi^3} d\xi$ is actually $1/3 \gamma$ of $1/3$. now using the rule of the γ function this $1/3 \gamma$ of can be written has γ of $4/3$ okay, and the value of $\gamma 4/3$ if you see the γ tables in mathematics you will be finding out nothing but 0.893 okay.

So we obtain $\theta \xi$ which was earlier was like this okay, now this value is nothing but γ of $4/3$ or 0.893. So we can easily write down, it is nothing but $1/\gamma 4/3 \int_0^{\xi} e^{-\xi^3} d\xi$. So we have obtained the temperature profile in this fashion okay. Next let us try to see the heat flux as the subsequent heat transfer coefficient and nusselt number q the heat flux can be replaced into $\partial T/\partial r$ and $r = 0$ and if you convert r bar into r it becomes k/p to θ , it becomes $r_0 (T_i - T_w) \times \partial \theta / \partial r$ and $r = 1$.

So in this further if you evaluate what is value of heat transfer coefficient h which is nothing but $q/T_w - T_b$ okay, so this h if you try to put then the other assumption we can get as r is very small

the volume integral of T_b will be T_i . So in thermal terms whose length is very small, so if you do bulk in that small length you will be finding out very small amount of heat as been added. So actually the bulk temperature will not be changing much, bulk temperature will be considered to near T_i .

So here we are considering r is very small T_b is equivalent to T_i okay, so this comes from the very small thermal inter consideration okay, so if you do so then here you see this T_b can be replaced by T_i , so it is nothing but $h = T_b = T_i$. But in some case where thermal indene is finite it is very big one, in some case it might happen there it is along side is not possible to find out the value of T_b in that case.

But this alongside we are taking that T_b is more or less near to T_i for reducing our equation into a simplify form the assumption over here is very small r is very small zone in the term of length okay. With this we can write down that $q / T_w - T_i$ is actually h then you can write down h which is nothing but $-k / r_0$ into this derivative $\partial \theta / \partial r$ where $r = 1$ okay.

(Refer Slide Time: 34:03)

$$Nu_z = \frac{hD}{k} = \frac{h2r_0}{k} = -2 \left(\frac{\partial \theta}{\partial r} \right)_{r=1} = 2Pe^{1/3} \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\partial \theta}{\partial \xi} \frac{\partial \xi}{\partial \eta} = \frac{A}{z^{1/3}} \theta' \quad \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = \left(\frac{2}{9} \right)^{1/3} \frac{\theta'(0)}{z^{1/3}}$$

$$Nu_z = 2Pe^{1/3} \left(\frac{2}{9} \right)^{1/3} \frac{\theta'(0)}{z^{1/3}}$$

$$Nu_z = 2 \left(\frac{2}{9} \right)^{1/3} \frac{\theta'(0)}{(z/Pe)^{1/3}} = \frac{1.357}{(z/Pe)^{1/3}}$$

We know, $\theta(\xi) = \frac{1}{\Gamma(4/3)} \int_0^\xi e^{-\xi^3} d\xi$ $\theta'(0) = \frac{1}{\Gamma(4/3)}$

So let us try to find out the nusselt number further. So nusselt number you know which is nothing but hD/k or D is the diameter $h2r_0/k$, so ultimately nusselt number becomes $-2 \partial \theta / \partial r$ and $r = 1$ okay. So $\partial \theta / \partial r$ now if you try to convert this r to θ , which we have used for similarity variable then we will be getting $2Pe^{1/3} \times \partial \theta / \partial \eta$ and $\eta = 0$ going in this case okay, η was $1 - r$ okay.

So if you do further then you see this value $\partial \theta / \partial \eta$ we need to find out this is nothing $\partial \theta' / \partial \xi$ that is $\partial \xi / \partial \eta$ okay. This we have already seen while deriving the equation okay, subsequently $\partial \theta / \partial \eta$ where $\eta = 0$, it is becoming where the value of A we have put here and that we will consider A is nothing but $2/9^{1/3}$ that should be attached $0/z^{1/3}$ okay, so this value if you put over here for finding out the nusselt number.

The nusselt number becomes $2Pe^{1/3} \times 2/9^{1/3}$, we can get $\theta / 2^{1/3}$. So we have already seen that this can little bit modify and we know the value of $\theta' = 0$. So what we can do little bit of change and simplify over here nusselt number = $2 \times 2/9^{1/3}$, $\theta' / z/Pe^{1/3}$, here we are having $z^{1/3}$ this is coming up over here as $z/Pe^{1/3}$ okay. Now these factors and along with this 1.357 along with θ_0 as we know value of $\theta' = 0$ and θ' will be giving me actually $1/\gamma^{4/3}$.

So γ of $4/3$ is known to me that I have plug in here the whole constant is 1.357 so I get nusselt number in that thermal is nothing but $1.357/z/Pe^{1/3}$, thermal intendance z will be function.

(Refer Slide Time: 36:44)

Summary

- Governing equation for thermally developing but hydrodynamically fully developed forced convection over flat plate having constant temperature :

$$\text{For } Pe \rightarrow \infty \quad 2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$$

$$\text{at } \eta = 0 \quad \theta = 0 \quad \text{as } \eta \rightarrow \infty \quad \theta \rightarrow 1$$

$$\text{at } z = 0 \quad \theta = 1$$

} Leveque solution

- Nusselt number for thermally developing but hydrodynamically fully developed forced convection over flat plate having constant temperature :

$$Nu_z = \frac{1.357}{(z/Pe)^{1/3}}$$

So let us summarize in this lecture we understood governing equation of thermally developing but hydrodynamically fully developed forced convection over flat plate having constant temperature okay this is not correct convection inside the duct having constant temperature and there we have found out this is the equation $2\eta \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial \eta^2}$ for large number cases okay and the boundary conditions we have seen.

Two boundary conditions for η this is at the axis, this is at the wall and this is at the z boundary condition we required. This solution we have called has Leveque solution, and we have proceed further to find out what is nusselt number for thermally developing but hydrodynamically fully developed forced convection over flat plate having constant temperature. So this becomes nusselt number = $1.357/z/Pe^{1/3}$ okay. So this we have seen in the previous slide okay.

So just like your others lecture let us also test how you have understood, so we are having 3 questions over here. 1st one in thermally developing but hydrodynamically fully developed region. (Refer Slide Time: 38:03)

Test your understanding ?

1. In thermally developing but hydrodynamically fully developed region, radial conduction is of the order:

(a) $\frac{1}{Pe}$	(b) $\frac{1}{Pe \delta^2}$
(c) $\frac{1}{Pe \delta}$	(d) Pe

2. In thermally developing region with constant wall temperature Nusselt number:

(a) is constant	(b) depends on z
(c) depends on Pe	(d) both (b) and (c) are true

3. Velocity profile in thermally developing and hydrodynamically fully developed region is:

(a) Parabolic	(b) flat head
(c) linear	(d) No conclusion can be made

Radial conduction is of the order we are having 4 options $1/Pe$, $1/Pe \delta^2$, $1/Pe \delta$, Pe okay so obviously we have discussed over here the correct answer is $1/Pe \delta^2$. 2nd question is in thermally developing region with constant wall temperature nusselt number we have 4 options which statement is correct a is constant, depends on z , depends on Pe , both b and c are true, so this we have already seen the expression of nusselt number depends on both b and c are true.

So answer b is the correct one, last question is velocity profile in thermally developing and hydrodynamically fully developed region is 4 option we are having parabolic, flat head, linear a, non conclusion can be made. So the correct answer is parabolic so this is also simple question okay, so with this we end this lecture.

In our next lecture we will be discussing the same thing thermal entrance region but we will be taking the conditions as constant or uniform heat flux okay. So if you are having any query regarding this lecture or any other general query about convection heat transfer please keep on posting on our discussion forum thank you.

For Further Details Contact
Coordinator, Educational Technology Cell
Indian Institute of Technology Roorkee
Roorkee – 247667

E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in
Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team

Sarath K. V

Jithin. K

Pankaj Saini

Arun. S

Mohan Raj. S

Camera, Graphics, Online Editing & Post production

Binoy. V. P

NPTEL Coordinator

Prof. B. K. Gandhi

An Educational Technology cell

IIT Roorkee Production

© Copyright All Rights Reserved