INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Convective Heat over flat plate

Lec-14

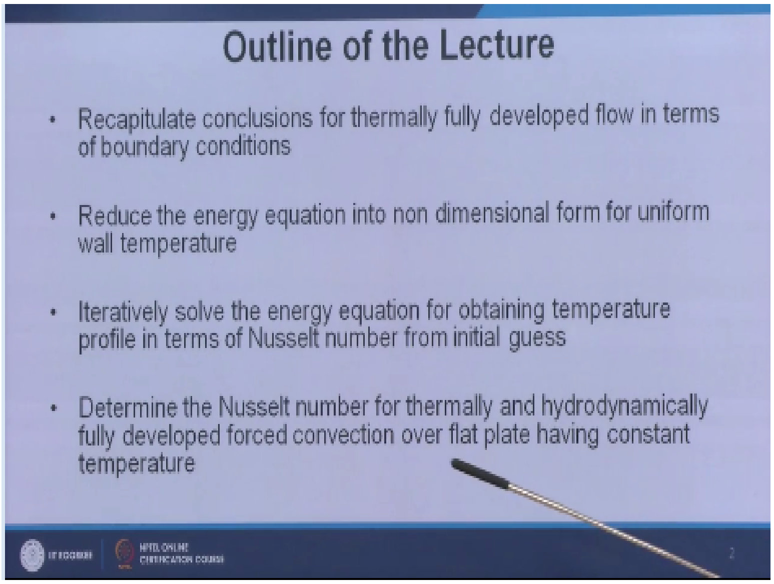
Thermally and Hydrodynamically Developed Flow: Uniform Wall Temperature

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee

Hello welcome in the 14 lecture of convective heat transfer in this lecture we will discussing about once again thermally and hydro dynamically fully developed flow, but in the last lecture as we have taken the heat flux of the wall of the tube was constant here in this lecture we will considering that the tube is having constant wall temperature okay so as I have said that our topic of this lecture will be thermally and hydro dynamically developed flow with uniform wall temperature okay.

So if you have to visit what happens in case of uniform heat flux please see our last lecture okay so let me first tell you that what main points will be covering over here as it is thermally fully developed flow first we will be recapitulating.

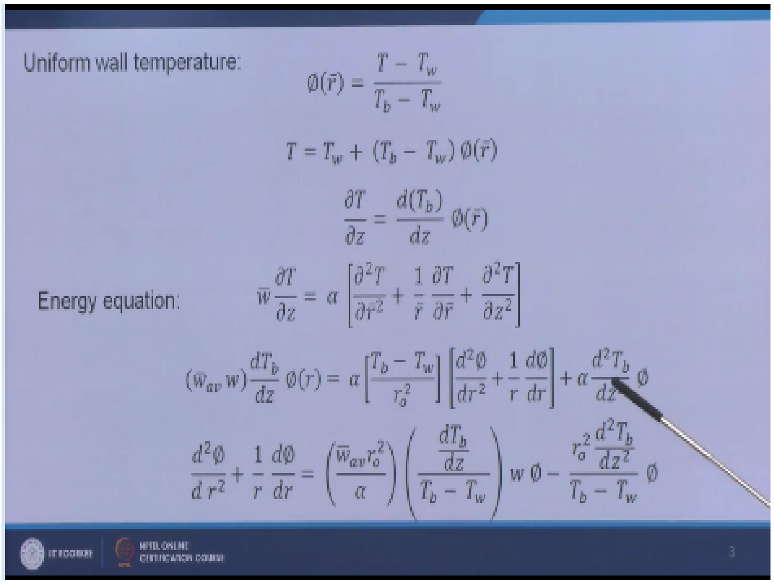
(Refer Slide Time: 01:10)



The conclusions whatever we have derived in our last lecture for thermally fully developed flows okay and here we will be specifically seeing that what are the boundary conditions for fully developed flow we are getting okay next we will be applying this uniform wall temperature case okay here only we will be deviating from the last lecture we will applying uniform wall temperature and derived what is the energy equation in non dimensional form okay, then we will show you this energy equation is not easy to solve in analytical methodology we have to go for some iterative solver.

So here I will be showing you how that numerical scheme can be taken and iterative solver how we can sue for solving this energy equation whatever we have derived and from there finally we will be getting that what is the temperature profile and Nusselt number okay, and at the end we will be showing you that what is the Nusselt number for thermally and hydro dynamically developed forced convection over a tube having constant temperature okay, so this 4 things we will be seeing in this lecture.

(Refer Slide Time: 02:40)



Next let us start from this thermally fully developed zone so for that if you remember in our last lecture we have considered the temperature can be non dimensionalized as T- Tw / Tb – Tw okay and that non dimensional temperature quantity we have taken as ϕ okay and we have also shown that this temperature as it is thermally fully developed this temperature or dimensional temperature will be only a function of R okay.

So this we have already provide in our last lecture so from that conclusion only we will starting over here now as we are having uniform wall temperature in the present lecture so obviously this Tw okay this Tw will be constant value okay this will not be varying with respect to the axial location as we have seen in our last lecture here Tw will be a constant value, so little bit of site changing if we do then we can get T = Tw + Tb – Tw x ϕ which is a function of R only.

Okay it is not dependent on the z because flow is thermally fully developed okay with respect to z with respect to the axial location profile will not change okay, then if we do the derivative of this temperature with respect to z then we can find out ∂ ∂z of T as Tw is constant for uniform wall temperature so this team will be going to 0 okay + here as ϕ is a function of R we need not to do the derivative of ϕ with respect to z okay.

But here you see Tb – Tw is there which can depend on the value of z so here we need to do the derivative now once again this Tw part is once again constant so that derivative does not make sense, so if we do the derivative of T with respect to z this is only coming over this Tb or wall temperature, so it will become ddz (ddTb) dTb (dz) that means derivative of this wall temperature with respect to z multiplied by this ϕR okay so we have seen over here ∂T ∂z = ∂ dTdbdz x ϕR okay.

Next let us see our energy equation we know that for AA ϕ in case of adopter pipe and the energy equations simplified form of energy equation is like this and this convection equation will be axially dropping two terms over here whenever we fully developed flow okay so here you see we are having only w component over here w ∂T, ∂z w is nothing but axial component okay in the convection side and in the ready conduction side we are having our radial conduction as well as our axial conduction okay.

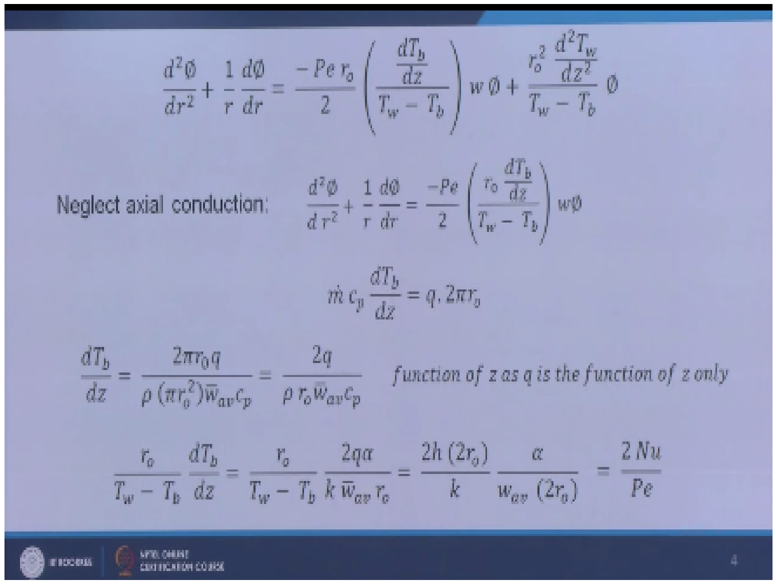
So here you see we are having we are axially neglecting the azimuthally component okay, so and this α is nothing but thermal diffusivity this we have seen earlier okay next let us try to see that how this equation can be non dimensionalized so for non dimensionalization in the left hand side we are having w so that w we are non dimensionalizing with some average value of the w, w bar average into w okay.

And ∂T ∂ ∂z (T) over here we have already seen that is nothing but ddz(Tb) x ϕR so that value we have substituted over here okay and in the right hand side for the radial conduction side we are taking out this Tb – T ∝ for making T to ϕ okay and the axial conduction part we are using once again derivative of this 1 , so second derivative of T with will becoming second derivative of Tb into ϕ okay so that we are keeping over here, so from this equation we have actually found out this type of equation after changing δ δz (T) to δ δz(Tb) okay.

Next let us see little bit of sight change if we do then this radial conduction can be written as w average R^2/ α x ddz (Tb/ Tb – Tw) x w x ϕ okay so w and ϕ you can get from here w and ϕ was there okay and this ddz (Tb) wil be remaining over here okay for the axial conduction part which will be going in the other side with a – sign so this some over here and this axial conduction will be remaining as second derivative of the bulk temperature with respect to z axial direction okay.

Along with that we will be also having a ϕ over here okay so this is the radial conduction part And here we are having this axial conductions abstraction of the axial conduction from the convection part okay. Next let us do little bit of further simplification here you see this term which is there in the first bracket this can be written as in terms of your pick lay number.

(Refer Slide Time: 08:08)



So here what we have done this – pick lay number we have introduced okay and $R0^2$ was there so one R^0 will be remaining so that R^0 we have kept over here okay and the axial conduction remain in the dame form only the sign change we have done as result this – sign became positive okay so this is more or less then equation now already we have told you earlier this axial conduction can be neglected in case of thermally fully developed flow so in that case if this value is smaller compare to the other terms that means radial conduction and convection term.

So this last term for the axial conduction can be neglected so let us take that assumption that neglected axial conduction that means negligible axial conduction we are having so under that this term can be neglected so our equation actually simplified into this form where in the left hand side we are having the radial conduction and in the right hand side we are having actually you convection okay.

Next let us try to understand what is the magnitude of this dTbdz okay so in order to do that let su take a small control volume in our fluid element okay cylindrical control volume in our fluid element and there yiu try to understand the energy balance so if we try to do that let us say that fluid element is surrounded by a fluid coil electrical coil or heating coil which is supplying heat flux q okay.

So this heat flux q will be received by 2 Π R_0 is the perimeter into dz quantity okay so 2 Π R0 x dz is the actual area through which heat is being supplied to that through element and what amount of temperature change the few element will be causing that is nothing but m Cp into dTb okay now that energy balance if you divide throughout by dz so it comes out to be m.Cp dTb dZ equals to $2\pi_{r0}$ dZ, dZ will be cancelling okay so from here you can further see that this dTb dZ can be written as $2\pi_{r0}q$ divided by m Cp and m once second can be written as the mass of that fluid element whatever we have taken. So that is nothing but the density of the fluid multiplied by the volume.

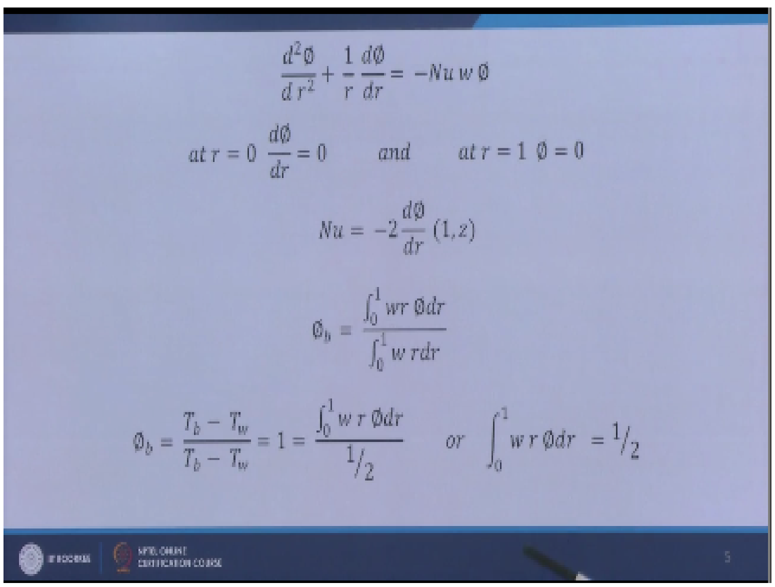
So the volume will be nothing but π^{r2} πr_0^2 which is the cross section area of the duct multiplied by the velocity average, velocity w bar average okay, littler bit of cancellation of terms we can get to q / ρr_0 w_{avg} Cp so here you see only q is changing which is a function of z so we can say that this dTb dZ is only a function of z okay because q is a function of z only and rest parameters are actually from stand okay, so we got that this dTb dZ is actually a function of Z okay and as result what you can do the axial conduction second derivatives of this one will becoming actually one second.

A weight function okay, which can be neglected by here we have taken the assumption okay, next let us try to see that what this star in our equation is becoming so let us try to simplify this one so r_0 / Tw - Tb so r_0 / Tw – Tb multiplied by this dTb dZ so dTb dZ is over here okay let us try to put this value of dTb dZ whatever we have got in terms of q just before this okay so, we have to do the same thing you see 2q now this Cp we have convert in terms of α and K okay. P and Cp is nothing but equal to α / k that comes from the definition of thermal diffusivity thermal diffusivity α = k / ρCp.

So from there we get this equation so 2q ρCp we have convert it to α / k and w_{avg} r0 remains over there okay, so little bit of cancellation if you do and if you consider q / Tw – Tb is it actually equals to h it transfers coefficient h it transfers coefficient h is nothing but heat flux divide by the temperature change okay average chill ball temperature change, so you can write down this one as h. So ultimately this term whatever is in the first bracket over here you can write down 2h α / K w_{avg} if you multiply $2r_0$ at denominator and numerator the you can find out this is actually.

One non-dimensional number and here also we are getting another non-dimensional number this is nothing but your Nusselt number $h2r_0$ / K based on that diameter and here also we get the other non-dimensional number which is nothing but α w_{avg} into 2r or 0 by α which is nothing but Pr number okay, so we can write down this whole term as 2 Nu / Pe this two will be coming from here okay. So ultimately this term over here in the first bracket can be written as 2Nu number by Pe okay.

(Refer Slide Time: 13:38)



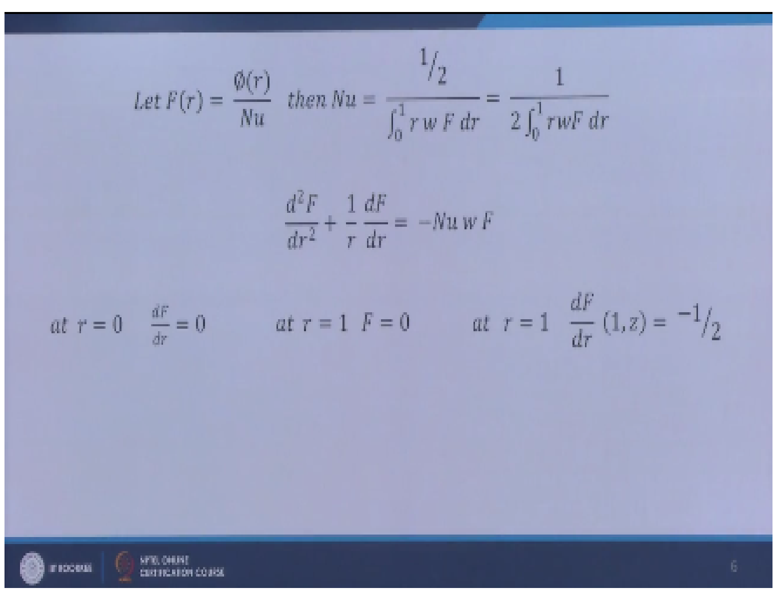
So if we proceed further so our equation will be simplified like this because earlier there was Pe / 2 and we were getting 2Nu / Pe so Pe numbers gets cancelled out so ultimately you get a ∂^2 $d^2\phi$ is r2 + 1 / rdϕ which is nothing but radial conduction part is equals to – Nu wϕ okay, and subsequent boundary conditions will be coming like that at r = 0 in dϕ dr = 0 which is at the axis of that u okay, so there we know gradient of ϕ across the axis in the radial direction and this is at the wall at r = 1ϕ = 0 because ϕ we have defined as T – Tw/ Tb – Tw. So that will be coming and 0 because Tw – Tw now moves to 0 okay.

This is another boundary condition which we have obtained or we have found out at the wall okay at the wall, r = 1 for our full thermal fluid this has been proved in our last lecture so we are not going into detail of this proof of this one we are straight away taking this equation from the last lecture so if you want to see that details of this one please visit our last lecture okay, so this equation actually will be helping us to find out the value of Nu okay after solution of this ets of equations right.

So let us proceed further before going there let me introduce the bulk temperature this also we have introduced in our 11^{th} lecture where we have shown that bulk non-dimensional has bulk temperature can be written as 0 to 1 w or ɸdr and divided by 0 to 1 w rd r okay and bulk temperature they finish and here is nothing but Tb – Tw / Tb – w because ɸ we have considered as T – Tw / Tb – Tw okay so we get from here that ɸb is nothing but equals to 1 so we can get the value of this integration is actually equals to 1 okay, and has we know that in thermally developed zone w will be getting some of parabolic profile.

Hagen profile so if you put the value of w over here and integrate from 0 to 1 will be getting the value of this integration will ne ½ okay, so from here we get from here we get that 0 to 1 w or ɸd is nothing but it told to ½ okay so this will be required in the later portion once we get the profile of ɸ okay next.

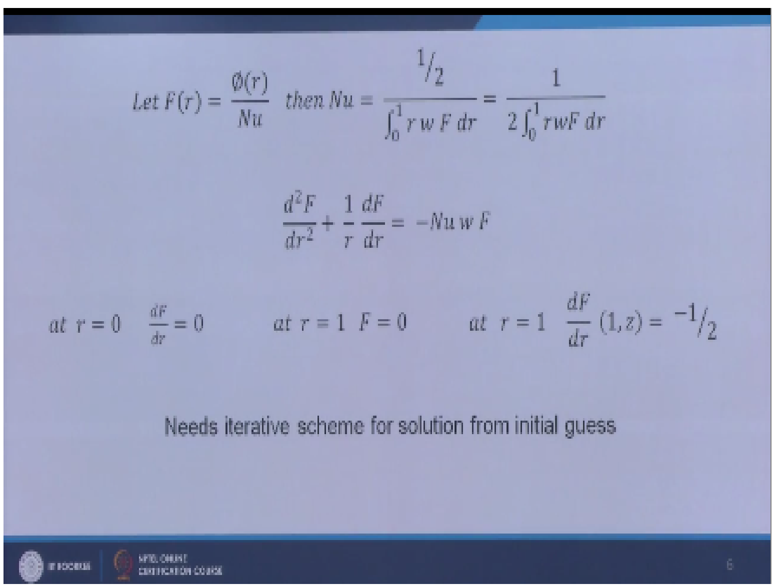
(Refer Slide Time: 16:26)



To solve this sets of equation and simplify this further especially this equation simplified further especially this equation simplified further what we do and let us consider that ϕ is actually a Nu number into F okay here f and ϕ both are function of r okay so if you do so then you see from previous set of equation here you see Nu term number was – 2 dr of ϕ at 1, z from there we can write down Nu is nothing but equals to ½ divided by 0 to 1 r w Fdr okay, and here little bit of multiplication with to in the denominator and numerator we can get Nu term is nothing but ½ into 0 to 1 rw F dr okay.

Now this r into F in r into w into F that whenever we integrate from 0 to 1 that we have already written over here this kind of equation will be coming over here okay, so let us see that how we can explicit or replace this ϕ with respect to F and reduce the sets of equations so if you do so in the previous equation in this equations and this boundary condition if you put this.

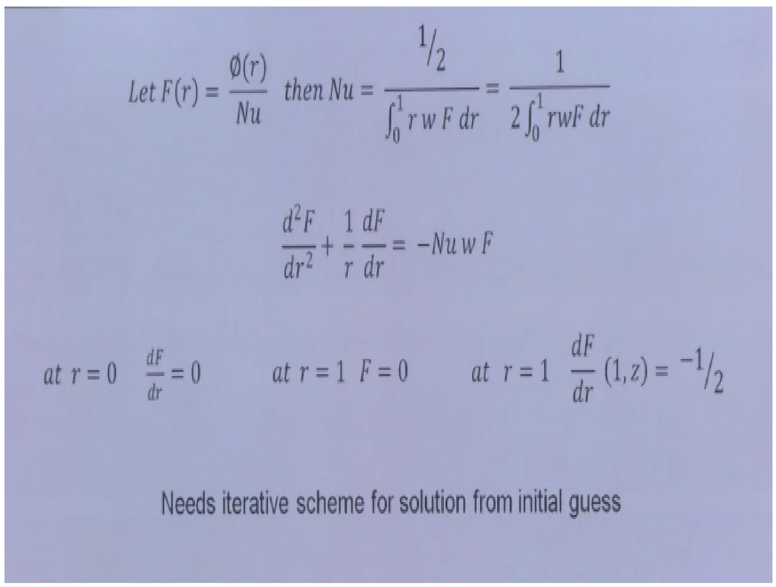
(Refer Slide Time: 17:44)



ϕ Nu into F then ultimately you will be getting this type of equation d2F dr2 + 1 / r df dr = - Nu w F thus equation will not be changing its form because in both side left hand sides and right hand sides we are having F, okay now let us talk about boundary conditions here you see these two boundary conditions more or less remaining same whatever we have seen in the form of ϕ only ϕ is being changed to F but measure change is coming over here r we have seen that in the boundary condition we had Nu, now by replacement of this Nu into ϕ into F by this ϕ we are changing.

This boundary conditions like this that dr of F at 1, z is becoming – ½ there is one Nu in this equation okay, so now this equation along with this boundary conditions is little bit typical to solve analytically because in both the sides we are having a over here no while we are having constant so this is little bit difficult analytically it is solve, so what we do take the help of numerical scheme. So it needs actually iterative numerical scheme for solution and that to we have to start from 1 initial guess okay.

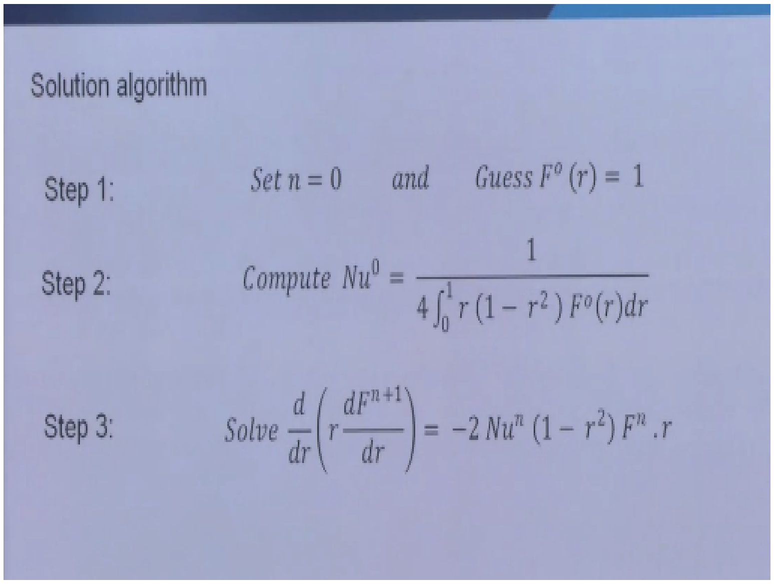
(Refer Slide Time: 19:09)



So let us see that what will be the numerical scheme to solve this sets of equation so here I will be presenting in some solution algorithm at the beginning at that we have mentioned we have to start from some initial guess, so we set some parameter n let say the n determines that what is the state of your guess, okay so set n = 0 and let us guess the value of F. It is very authentic game F is nothing but all F at different r location is 1 okay you can start from any other guess no problem okay, depending on would guess number of iterations in between be value okay so with this guess of F of r.

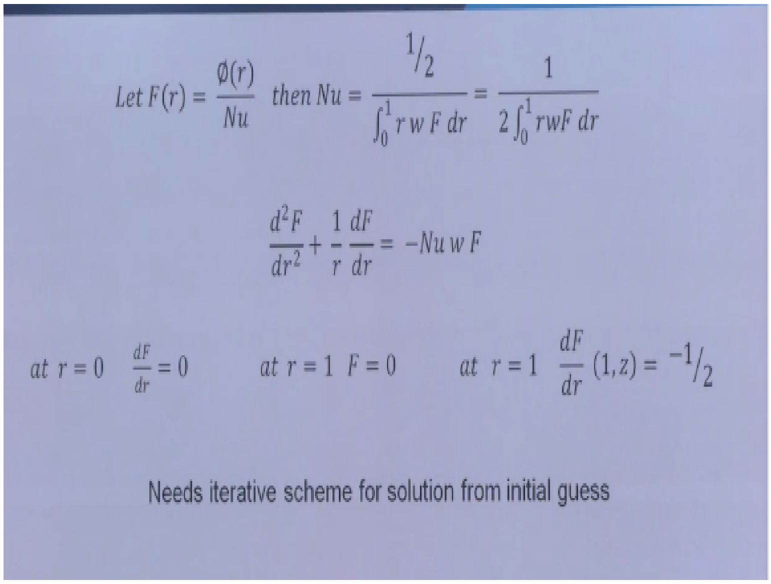
What we can do immediately find out the Nusselt number, okay. Already I have shown you that Nusselt number is nothing but ½ x 0 to 1 r w F dr once you have the value of f so what we can do w is also a known profile so we can find out the value of this integration and ultimately we can get the value of Nusselt number so with the initial guess of F.

(Refer Slide Time: 20:25)



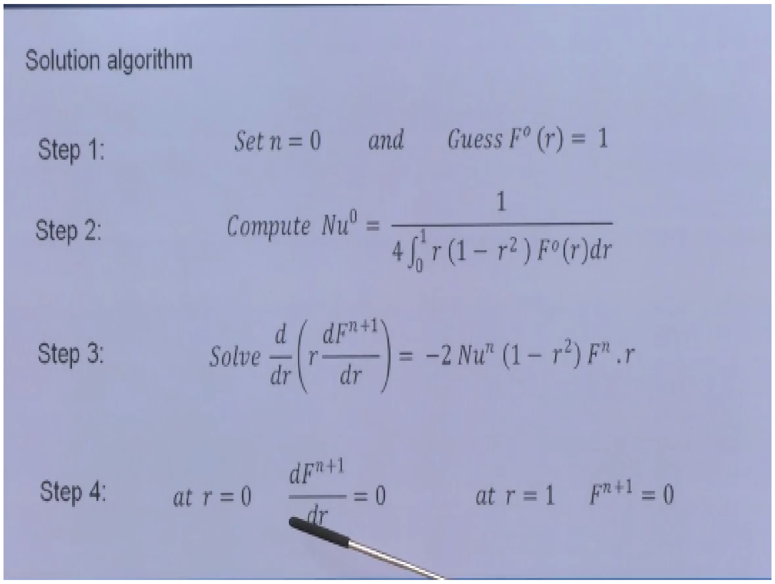
We are doing the same we first find out the Nusselt number, okay Nusselt number and we are mentioning this Nusselt number as at level N = 0, so Nusselt number 0 is nothing but equals to 1/ 4 x 0 to 1 $r(1-r)^2$ $f^0(r)$ dr okay, so earlier it was two now the w profile is nothing but 2 x $(1-r)^2$ Eigen possible profiles so that is why it became 4, okay. So once you compute the Nusselt number at 0th iteration level. Then what you can do, you can try to solve the sets of equation whatever we are having.

(Refer Slide Time: 21:04)



Let us show you once again the set of equation this one, so what will be considering in this set of equation that in the right hand side we are the value of F at the next iterative level and in the sorry in the left hand side we are having the value of the F at next iterative level and in the right hand side we are having the previous iterative level, okay. So here this F will be considered at previous iterative level. So in that same way and here you see if you multiply the whole equation with R then you will be finding out this term can be written as.

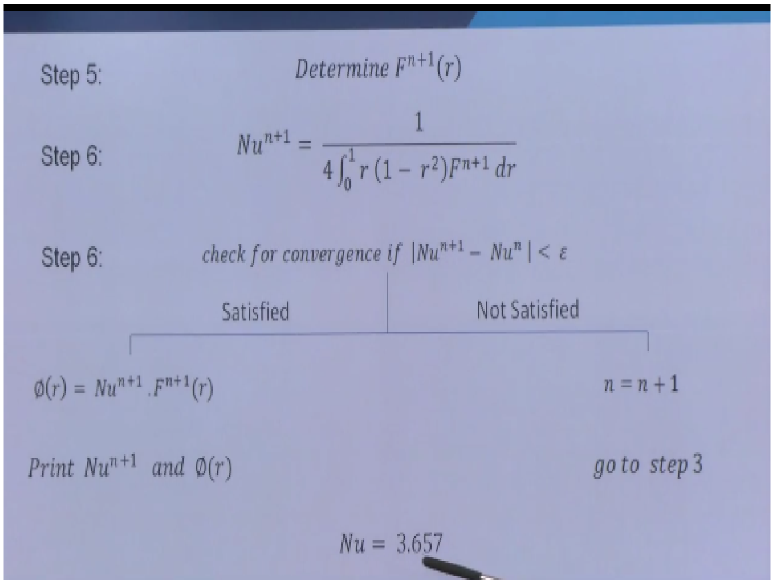
(Refer Slide Time: 21:41)



d/dr(r dF/dr) okay so you see if you do the derivative of this equation then we will getting actually $dF/dr + r\,\delta^2F\ r\ d^2F/dr^{2.}$ So that is nothing but if you multiply this equation with r then you will be getting the same thing okay in the left hand side atleast, in the right hand side you see we are taking the same thing r multiplication is over here but in this side we are considering F based on N iteration level.

In the left hand side we are having n+1 iteration level so for N = 0 this will be F_0 and this will be F_1 okay, so once we get this equation this side is becoming a constant making the integration of this equation and find out the value of F_1 or F_{n+1} will not be difficult. So once you get the value of F then next task is to find out or if you have to actually get the values of F using this boundary conditions also. This boundary conditions whenever you will be integrating this one will be required, okay. So at r = 0, $dF^{n+1}/dr = 0$ and r = 1 F(n+1) = 0 okay.

(Refer Slide Time: 23:06)



Next once you determine the value of n+1, F(n+1) next task is to get the value of updated value of Nusselt number earlier you had Nusselt number at N, now you will be finding out a Nusselt number of n+1 using the value of f^{n+1} okay, so that we are doing over here using the same formula ¼ x 0 to 1 r(1-r^2) F^{n+1} dr here we are putting the value of F at n+1 at iteration level, okay. So once you get the new Nusselt number so you have now two sets of Nusselt number one is at the old time step another one is the new time step.

I should not say time step iteration steps. So you have to compare those, okay. The two values of Nusselt number and check for convergent whether you have obtain some converge solution or not okay. because you have started from initial guess with respect to progress in the iteration number you will be reaching to the convergence, so let us say we have said some sort of limit epsilon which is very small.

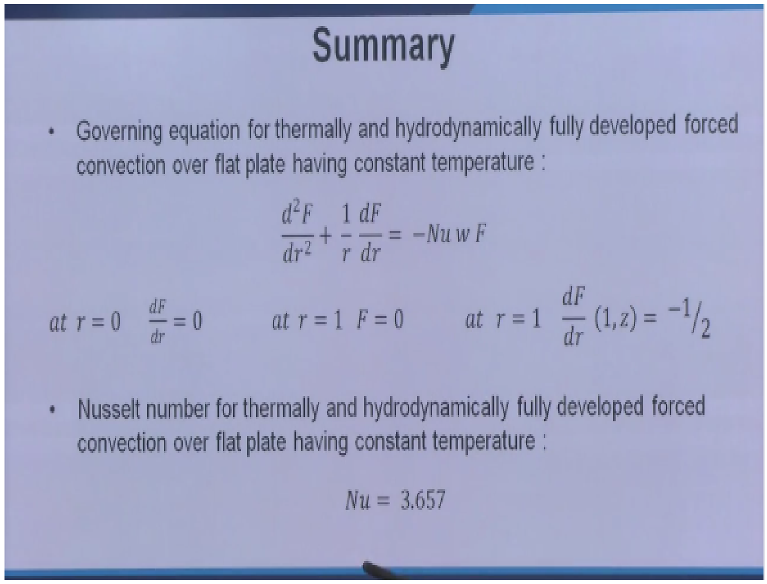
Now if you find out that between two Nusselt numbers we are having difference very small okay smaller than the limit whatever you have considered then we can say that our scheme has converge, other from here what you can do, you can check this criteria and you can actually find out whether it has satisfied or not with this condition, if it is not satisfied then what you need to do. You just make n = n+1 and go top step 3, step is nothing but once again solving this equation, okay. So for example you have started with 0 and found out F1 and you have got that Nusselt number at 1th iteration level – Nusselt number of 0 iteration level is actually bigger than some small value epsilon value, then what you need to do, you need to make that 0^{th} level to 1th level.

And then once again solve this equation for F2 okay, so in this process F2 will be giving you F3 and so on, if you proceed it further at some point of time you will finding out that your condition for checking of convergence is satisfied that means you have reached to some steady Nusselt number value, so in that case what you do, you find out what is the value of Ø by multiplying that Nusselt number with F.

Because whenever you come out you are having a new value of Nusselt number and new value of F you can multiply this to find out the value of actual Ø okay and subsequently you can print the value of Nusselt number and Ø, okay. So if you follow this procedure write a small numerical code then you will be finding out that for this type of situation where you are having thermally and hydroponically fully developed duct with constant wall temperature.

In that case if the program is actually being converge and finally the Nusselt number will come out to be 3.657 okay. So this Nusselt number whenever you are printing this will come out to be 3.657 okay. So this is very important Nusselt number for constant wall temperature duct at thermally and hydraulically fully developed region is 3.657, okay. So let me summarize what we have understood in this lecture.

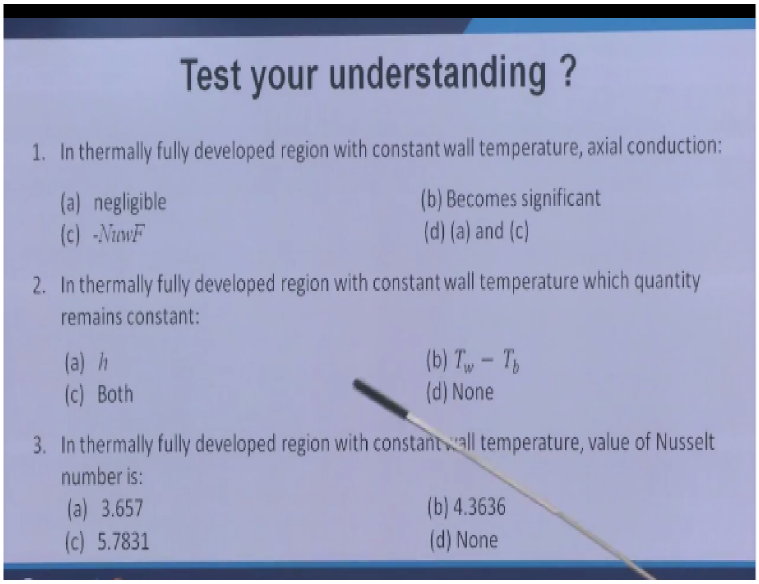
(Refer Slide Time: 26:33)



So first we have understood what are the governing equations for thermally and hydro dynamically fully developed forced convection inside a tube having constant temperature okay, so the equations are like this we have seen in terms of F where F is nothing your Ø a new okay Nusselt number and we have got this boundary condition the last boundary condition has been modified by introduction of Nusselt number.

And by using numerical scheme iterative numerical scheme I have showed you that how to solve this sets of equation and final we have mentioned that if you solve this equation by durative scheme the Nusselt number comes out to be 3.657 okay. Just like other lectures let us also text your understanding at the end of this lecture, so we are having three questions once again.

(Refer Slide Time: 27:20)



First question goes like this, in thermally fully developed region with constant wall temperature axial is negligible becomes significant you cannot neglect becomes –NUWF or d) (a) and (c) both okay, so already we have done elaborate discussion on this so probably you can understand that axial conduction can be neglected okay and we have neglected that one to obtain the equation of AF, okay.

Finally which I have shown you in the last slide in the summary, in the second question goes like this, in thermally fully developed region with constant wall temperature which quantity remains constant, okay. We are having 4 options h, $T_w – T_b$ both are none okay, so obviously the correct answers will be only h remains constant, okay. Then last question is like this in thermally fully developed region with constant wall temperature.

Value of Nusselt number is 3.657, 4.3636, 5.7831 and d) is none in the last two last slide we have actually derived that one, so the value is 3.657 okay. With this I will be ending this lecture so please visit our next lecture which is about thermal n terms region, okay. And that too will be considering at uniform wall temperature, about this lecture or about any other confusion if you are having any doubt please keep posting in all our discussion forum, thank you.

For Further Details Contact

Coordinator, Educational Technology Cell

Indian Institute of Technology Roorkee

Roorkee – 247667

E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in

Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team

Sarath K. V

Jithin. K

Pankaj Saini

Arun. S

Mohan Raj. S

Camera, Graphics, Online Editing & Post production

Binoy. V. P

NPTEL Coordinator

Prof. B. K. Gandhi

An Educational Technology cell

IIT Roorkee Production