INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NPTEL

NPTEL ONLINE CERTIFICATION COURSE

Convective Heat Transfer

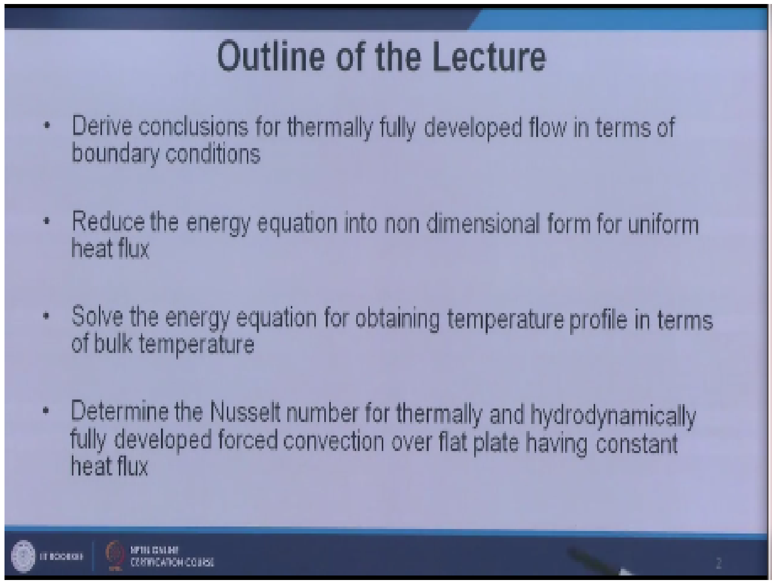
Lec-13
Thermally and Hydrodynamically Developed Flow: Uniform Heat Flux

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee

Hello, welcome in the 13th lecture of my course on electric heat transfer. In our last lecture we have discussed about flux flow okay where velocity was almost constant okay, hydrodynamically the boundary layer is not developed. Here we will be considering about thermal layer hydrodynamically developed flow that means hydrodynamically boundary layer is already developed okay.

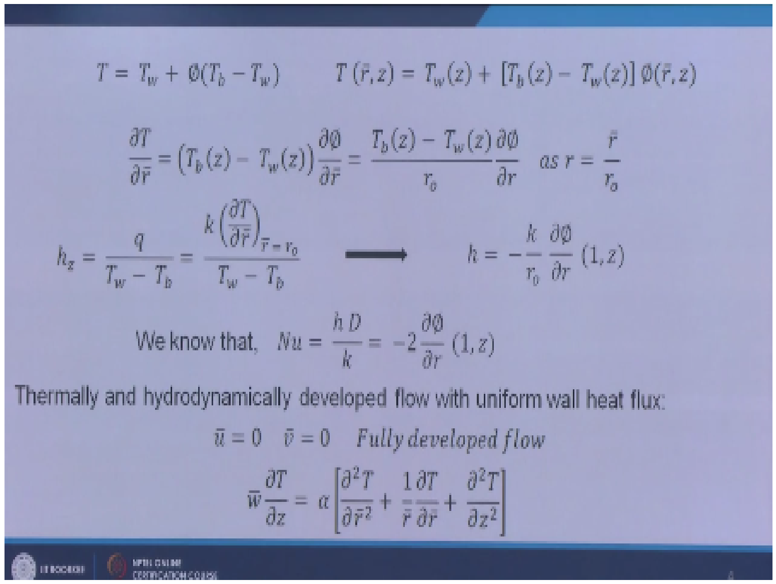
That to in this lecture we will be considering uniform heat flux, so that means the pie is actually rapped up with some coil electric coil let us say heater coil, let us say which is supplying uniform heat flux okay. So let me first tell you that what we will be covering in this lecture.

(Refer Slide Time: 01:07)



In this lecture we will be first deriving conclusions for thermally fully developed flow in terms of the boundary conditions, then we will be discussing or we will be reducing the energy equation into non dimensional form for uniform heat flux case. So we will be considering only by uniform heat flux case over here in this lecture. In the next lecture we will be discussing about uniform temperature case. Solve the energy equation for obtaining temperature profile in terms of bulk temperature and lastly determine the nusselt number for thermally and hydrodynamically fully developed forced conviction over flat plate having constant heat flux okay.

(Refer Slide Time: 01:42)



So let me start with the topic as we are only interested in thermal fully developed region, so as it is thermally fluid developed so the heat transfer coefficient will not change with respect to time, with respect to radius at least for hz will be keeping as constant okay, hz is constant means as it is fuly developed it will not change with respect to z, so hz is constant okay. As hz is constant let me see that what are the other quantities, for example, bulk temperature okay.

Bulk temperature is obviously the integration over the radial plane, so here you can see this will definitely not depend on the radius and so it is the function of z only okay. On the other hand wall temperature, wall temperature obviously it will not depend on the radial plane okay, it will be only and it is only varied are not which is the tube diameter, tube radius okay. So T is actually function of z, so here we are considering Tw is actually a function of z okay. So both we have got as a function of z and hz is actually constant for thermally fully developed region okay. Next let us see by definition this heat transfer coefficient h is nothing but Q/Tw-Tb okay, wall temperature minus of wall temperature okay.

So here you see Q can be written as k ∂T/∂r at r=r0 that means at the wall okay. So here we get k∂T/∂r at r=r0/w-Tb, now here you see Tw and Tb both are function of z, so what we can do this term we can take inside the ∂/∂r term, so there we have done ∂/∂r(T/Tw-Tb) because this Tw-Tb is actually not a function of r okay. So as constant that can be taken inside. So actually this T/Tw-Tb now it is becoming a function of r as T is function of r so this whole term is becoming function of r okay.

And we have to make hz equals to constant so obviously this has to be a function of r only, now we will be putting the derivative and then putting the limit r=rbar r=r0 and ultimately this hz will become a constant okay. So let us define non dimensional number like this ϕ = T-Tw/Tb-Tw okay. So if you remember you know last lecture we have considered Ti=Tw here let us define with Tb-Tw okay. Why because we want to make this ϕ only as a function of r okay.

So this T/Tb-Tw is actually a function of r in the thermally fully developed region okay. Now if you define this ϕ in this fashion then definitely it is ϕW at wall the value of ϕ will become 0, because T becomes Tw okay. Then let us see how the derivative looks like so T as for the definition of ϕ becomes Tw+ϕ(Tb-Tw) here I have showed the dependences based on the r and z coordinate for different parts of this equation.

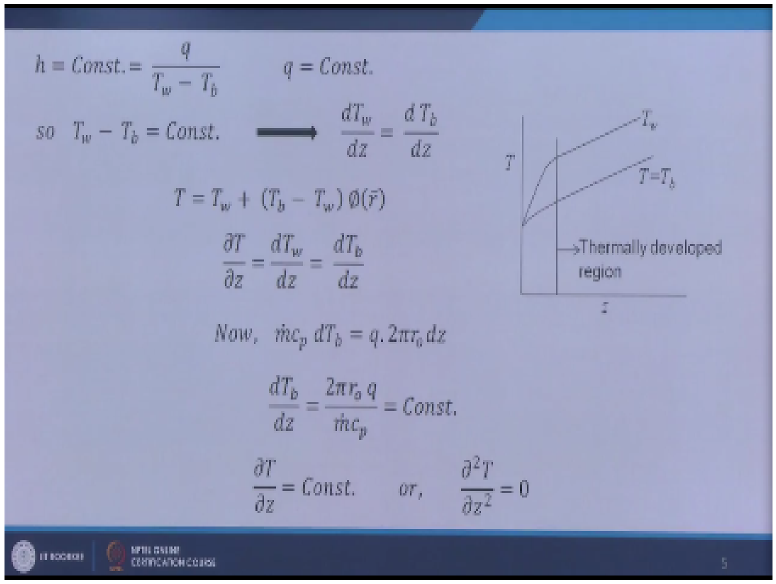
Now if we go for the derivative with respect to r first okay, if we go for the derivative with respect to r first this is the function of z we need to do, this will also be constant, this is not a function of r, here it will be only becoming the derivative. So only you see this is remaining as constant and this is becoming ∂/∂r okay. And now if we put r=rbar/r0 then this rbar needs to be transferred to r with 1/r0 multiplier with this term okay.

So ∂T/∂rbar becomes this one okay. if we proceed further for the heat transfer coefficient so for the heat transfer, from the heat transfer coefficient we can see now this becomes as for the non dimensionalization of the temperature it becomes actually –k/r0∂ϕ/∂r okay, so ∂t/∂r okay you can get from here ∂T/∂r is actually ∂ϕ/∂r multiplied by this term okay. So here we are having already Tw-Tb Tw-Tb and here Tw-Tb will be cancelling out with a minus sign okay.

So this minus sign then K/r0 ∂ϕ/∂r at 1, z this 1 is nothing but r=r0 okay, r=r0 means rbar equals to r0 means r=1 okay. Then if you go farther for the evaluation of nusselt number which is nothing but hD/k h(2r0/k) then from this equation we get nusselt number is nothing but -2(∂ϕ/∂r) at 1, z okay. Now let us see for thermally and dynamically developed so with uniform heat flux which we are suppose to discussing this lecture, for that case we definitely has a fully developed U and V will be 0 okay.

So by equation we will be turning into this one in the left hand side we are having the conviction, but this time W is not a constant and in the right hand side we are having the conduction terms okay, a numerical conduction has been neglected.

(Refer Slide Time: 07:13)



Now let us proceed further for constant heat flux case, so if the heat flux is constant and as it is fully developed h is already constant, then we can get obviously Tw-Tb needs to be constant okay. So here q is constant, h is constant so obviously it will be constant. Now if this is the case I can show the temperature profile in this fashion let us say this is the z axial detection of the tube and here I try to plot the T, so after this entranslate that is the thermal developed region you can find out though due to supply of the heat flux temperature, wall temperature is changing as well as the wall temperature will be changing keeping the gap always constant. Tw-Tb is always constant okay.

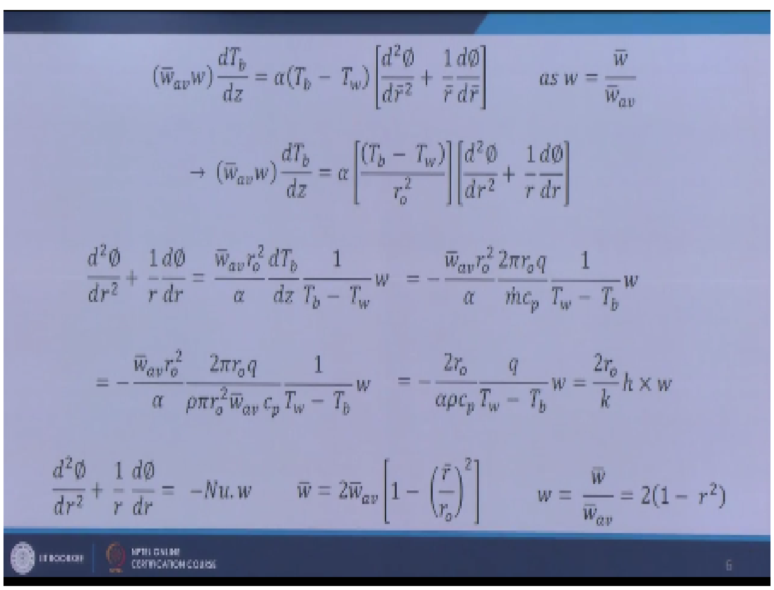
So we will find out from here as it is constant so d/dz of Tw will be obviously d/dz of Tb okay d/dz of T, d/dz of Tw is equals to d/dz of Tb. Let us proceed further here let us do the derivative of this temperature in form of ϕ with respect to z so ∂T/∂z will remain here you see ∂T d/dz(Tw) because this is the constant of now which we have proved over here and ϕr is not dependent of z. So we get ∂/∂z(T) is nothing but d/dz(Tw).

And here we have shown that d/dz(Tw) is nothing but d/dz(Tb) so ultimately d/dz(T) becomes d/dz(Tb) okay. Now let us try to do little bit of balance of energy from the conviction and whatever heat we have supplied from there. So let us say m dot is the mass attached with the pipe wall, so that is taking the heat from the pipe, so m dot into CpdTb=q.2Лrdz is the area so with the heat is being supplied 2Лr is the perimeter multiplied by dz in which the heating coil is wrapped up.

So if you do this kind of balance then we can find out dTb/dz is become 2Лr0q/mdotCp we are using the right hand side all terms are constant, so I can write down dTb/dz equals to constant. As dTb/dz is equals to constant then definitely ∂T/∂z becomes constant, so ∂T/∂z is constant over here okay, because we have already proved ∂T/∂z is nothing but dTb/dz okay so as this is constant definitely this term will remain constant.

And if this is constant second derivative of T/z will be obviously equals to 0, so that means we can obviously neglect an axial conduction in case of thermally fully developed region.

(Refer Slide Time: 10:12)

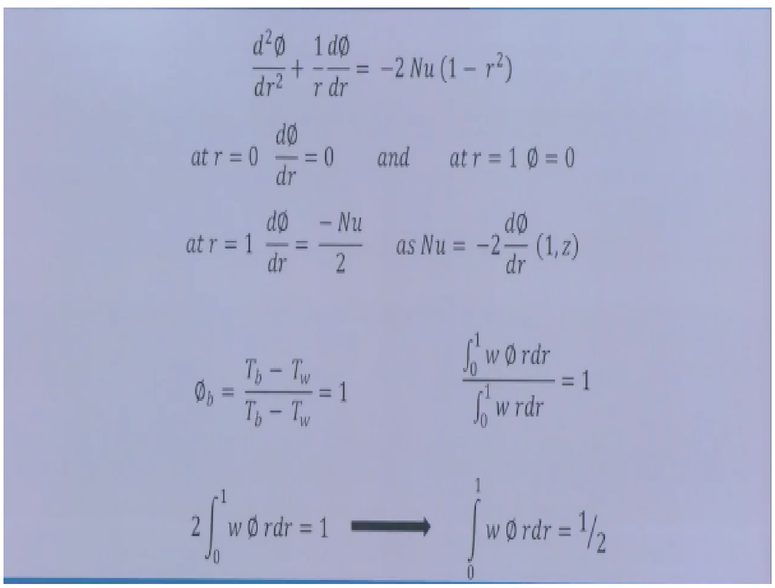


So in the equation last term we have brought this axial conduction term we have brought and tried to non dimensionalize that one with respect to w=wbar/wbar average okay. So as a result here wbar average came okay. and we tried to convert the ϕ to θ sorry, we convert the T to ϕ and we have written actually $\partial/\partial z(T)$ to $\partial/\partial z(Tb)$ okay which we have already proved. So after little bit of site changing we can find out that whenever we convert this rbar to r $r0^2$ also comes out, because we have considered r is nothing but rbar/r0 okay.

Now little bit simplification if you do then $\partial^2\phi/\partial r^2+1/r\partial\phi/\partial r$ is becoming W average r^2/α dTb/dz(1/Tb-Tw)w okay. So if we further do the simplifications then we will be finding out that we are having over here 2r0/k(hxw) okay where this 2r0h/k is nothing but your nusselt number okay. So this becomes nothing but your nusselt number, here one minus sign is missing so this is becoming minus nusselt number into w.

So my equation turns out to be this one $\partial^2\phi/\partial r^2+1/r\partial\phi/\partial r$=- nusselt number into w okay. Now where this w is nothing but your parabolic profile like flow we are considering so this is nothing but wbar is like this so w is becoming $21-r^2$ okay.

(Refer Slide Time: 12:08)



So my final equation once I put value of w as 2 Nu (1-r 2) this is my equation, let us also see boundary conditions so the boundary condition are like this at r=θ definitely needs to be boundary D θ =2 to l tat earns at the wall okay here r =to 1 θ5 in the number by 2 it is proved in the likes back, here we are proved, that r= 1Qϕ Q R is nothing Q R okay. It is coming as one boundary in the condition of my equation okay, so his is one equation ant the wall, because we have all proved this okay.

Next let us try to see the value of the 5V before we find 5V in Tb_Tw/Tb-Tw+1 and the ф in the obviously in the Tb_T Tw/Tb_Tw=1 in the nothing but it is =to 1 from the concept of the 5 V, which we are discussed in the 11 lecture we can right down that ф V is nothing but the G and the R D /2 in the 0 to W r d r its okay. Now the is equals to 1then what we can write down as we know the file of W its is nothing but 1_R^2 this integration very simple but its look me nothing but it is equal to t5 the it is okay so it is nothing but sorry, but it is nothing but then it is half it will becomes 0 to 1 W ф and the R D R is equals to the half okay.

Next let us try to solve this equation let us consider that ф in that is nothing but the assent number in to F okay. Let us try to the solve equation do you simplify for the let us consider the ф it is nothing but the assent number in to the F okay, if you consider that the equation is the trans out to the line this it is the assent number nothing but the sides which are they are consider if you equal to the assent number F=to ф it is the assent number, so if we actually % in the assent number in to the ф it is the assent number that can be cancelled in the both sides.

So it will become in the simplistic in nature ,for the corresponding boundaries they are like this is at the centre of the tube, and the axis s of the tube it is at the wall this is also at the wall coming from the nozzle of the condition okay, this condition is put in the ф=to assent number into the F then the assent number we are cancelling from the both side you look at the boundary condition, let us now try to see on the solution the solution is coming very simple.

So if you do little bit of change of the order this will become in the d/dr and the r =to the _2 in to the R –R^3 okay, after once the equation no after the equation the multiplication ,R if you do the multiplication by R then it will come out to be like this okay. Then let us go for the those the after integration once tape it will involving once constant c1 okay, this is from the boundary condition okay ,at r = to 0 the nit will become you the 0 so from here to find out c that also equals to 0 ,if you do once step further the integration.

(Refer Slide Time: 15:36)

$$F = \frac{1}{2}\frac{r^4}{4} - \frac{r^2}{2} + C_2$$

$$at\ r = 1\ \ F = 0\ \ C_2 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

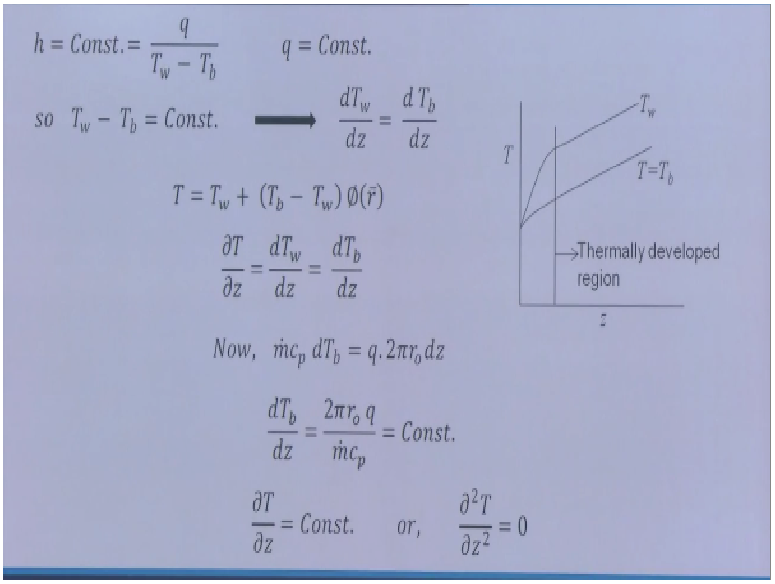
$$F = \frac{r^4}{8} - \frac{r^2}{2} + \frac{3}{8} = \frac{1}{8}\left[3 - 4r^2 + r^4\right]$$

$$Nu = \frac{\emptyset(r)}{F(r)} = \frac{\int_0^1 r\,w\,\emptyset\,dr}{\int_0^1 r\,w\,F\,dr} = \frac{1}{2\int_0^1 r\,w\,F dr}\ as \int_0^1 r\,w\,\emptyset dr = \frac{1}{2}$$

Then you will find if it becoming one by two R4, then we will finding out it will becoming then it will come by ½ and R^4 it6 is r^2 / 2 the n=c2 ϕ okay, this can be also by putting the this is the boundary condition then the r is equal to thee =to 0 .then 3/8. So if finally becomes in the three r^4 it is finding out the it the number as per the consideration and the assent number ϕ /3, we have consider in the assent number so the assent number ϕ / 3 now it can be also to 0 then it is the rf then it will be at the W F D I okay.

Now o to R if the D R now the 0 to 1 and the W D ϕ it is the found of the way the value of the half over here it is the value of the value and the half over here.

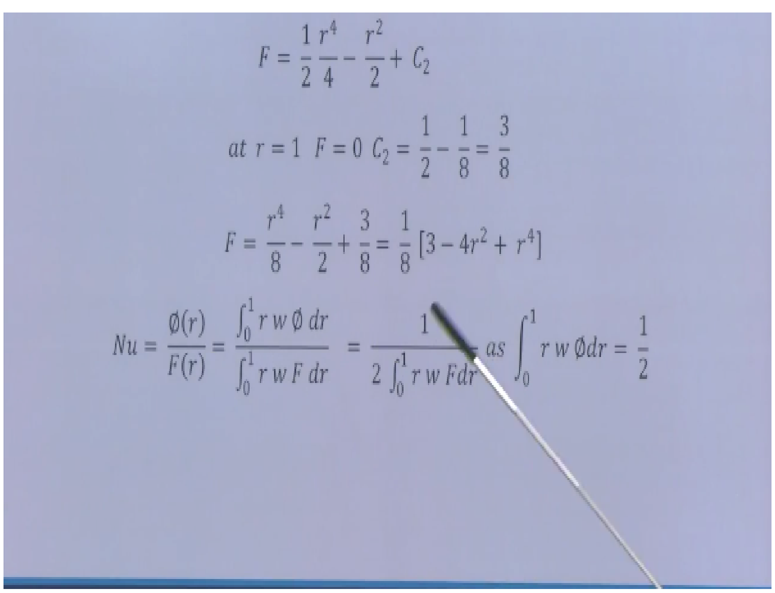
(Refer Slide Time: 16:39)



We are found out the over as half in over here okay 0 to the in the w 5 r D half ,that we can put obviously in the equation to get the 0 to l and r W F D R =to what is the number in the assent number on the assent number ,now let us try to find over the what is the value of the integration it is also know in the function it is the f that we cannot put in the R F it is the 2 * R in the 1 – r ^{2} and it is multiplied in the F they have put in the f little bit of algebra here also.

If u integrate from the o to 1 then it will be giving you in the 11q and /96 so further in the v find out the assent number because we know that the assent number.

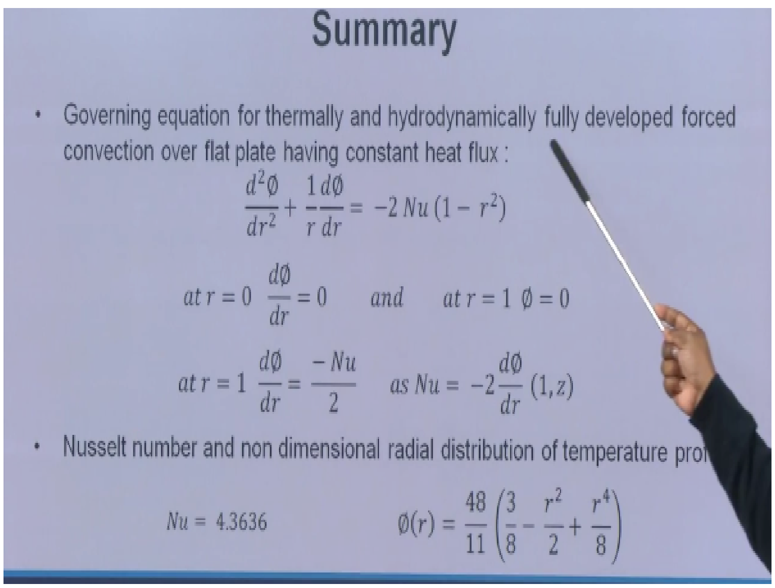
(Refer Slide Time: 17:39)



It is nothing but the half 0 to 1 W F D R, so the 0 to 1 R F D R then we will found out intersect in the 9/1 and it comes in the 4/96 it is the half 0 to 21 and the f d r so that the 0 to 1 then it will be F D R it will become in this 19 11/96 and it come to 4 .6 then okay. Then we find out the assent number is not in this fashion so as we know the assent number so the obviously find out the 5, which is nothing but the 5 in the assent number F.

If the assent number it is found out the 48 / 11 so that we have gone over here in front of the F .this was the actually found in the so in this way actually collected the % it is the profile okay, which is dependent on the radius as well as the found out the assent number, so let us see the what we have actually under stood in this lecture in this lecture actually discuss the governing equation and the thermal and hydro and it is fully develops in this lecture.

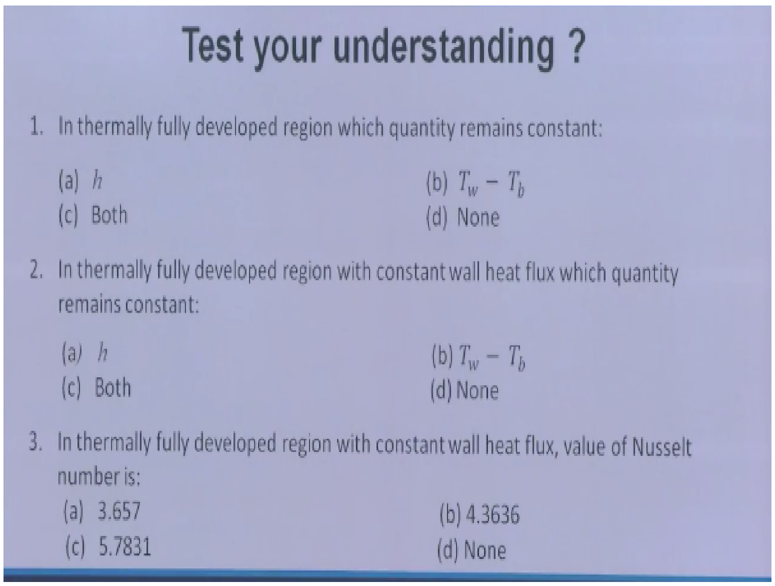
(Refer Slide Time: 18:35)



And it is fully developed in this is very important fully developed in the constant in the lecture over here a back plate in this constant heat flux case in the scene that the equation come in the in the =1/5 in =-2 assent number in to 1- r ^{2} okay .the boundary the condition we are established as the r= to 0 θ 5 r = to 0 and there = to 1 and the 5 = to o0 so okay. Also we have seen that at the wall it can be found out θ 5 θ r it will be obviously – and in the assent number by 2 .this we have proved from the equations in that beginning that will become in the understood that the assent number is nothing but okay.

And whenever we have tried it to find out the assent number and the number of the distribution of the temperature profile and we have actually obtain in the assent number and the 4 36 and the temperature becomes 48/11 and the three in to the r 4/ 8 all we have found out the thermal and the hydroponically developed in the forced convection okay.

(Refer Slide Time: 20:06)



And here we can take the mistake in the black plate in the constantan we have three question and so thermal fully developed region .so we have four answer over here first one so H and second on e T W T B and you can say both and the and the none I have to choose and the weather the h and the Tw/Tb or the both none of this an T W – T q so obviously we have understood in the elaborately discussed in this file in this lecture the correct answer is h. So this is constant in the alert region and when ever heat flux remains in the constant then it will other futures, so also comes in the picture but only for the picture the thermally fully developed region on the h we are remaining constant okay.

At it is related to this and it is only fully developed region with the constant heat flux now we are applying now the condition with the constant which quantity can be remained in the withstand the obviously in the developed in the constant and due to the constant the heat flux it can some more less H T W /T b and both the none and we can consider the find out that we have the proved ni the constant heat flux and thermal heat fluxed region T W_TB is also the correct answer the H as well as the T B is the remains the constant.

The last question is in the thermally fully developed region with the constants heat flex the value of the assent number 3.467 and there is 4.366 5.783 and or none so this we are in the assent number the obviously is 4.3636 so with I am ending this lecture in our next lecture we will be discussing about thermally hydroponically developed floor at this time we are considering the uniform wall temperature in this inform the heat flux but here we will consider uniform now wall temperature please do not forget to oppose it in discussion for us thank you.

For Further Details Contact

Coordinator, Educational Technology Cell

Indian Institute of Technology Roorkee

Roorkee – 247667

E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in

Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Production Team

Sarath K. V

Jithin. K

Pankaj Saini

Arun. S

Mohan Raj. S

Camera, Graphics, Online Editing & Post production

Binoy. V. P

NPTEL Coordinator

Prof. B. K. Gandhi

An Educational Technology cell

IIT Roorkee Production