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NPTEL ONLINE CERTIFICATION COURSE

Convective Heat Transfer

Lec-11
Forced Convection in Ducts

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Hello welcome in the 11th lecture of convective heat transfer course. In this lecture we will be discussing about convection inside a duct okay. Till now we have learnt what happens in case of conviction over a flat plate and we have also practiced sums on that one okay. But here if flow is occurring inside a duct how a conviction mode comes inside that we will be discussing over here. Today we will be discussing about the basics of the conviction inside a duct okay. So in this lecture we will be actually understanding forced conviction inside ducts.

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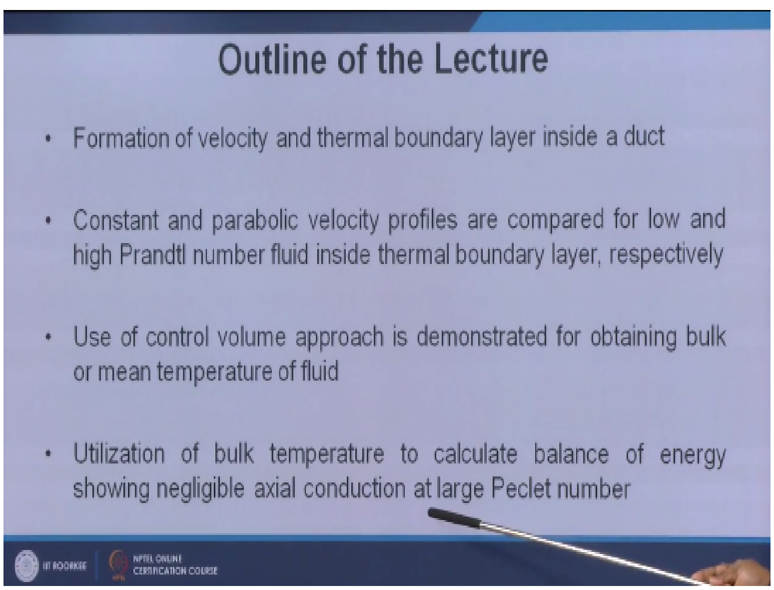

As outline of this lecture I would like to mention that at the end of this lecture we will be understanding what is the velocity and thermal boundary layer inside a duct okay, how those are being found. We will be also understanding that how constant and parabolic velocity profiles can be considered for low and high prandtl number fluids inside the duct okay, in case of thermal boundary layer okay.

We will be also understanding from control volume approach what is the bulk temperature of the fluid and what is the mean temperature of the fluid okay. So bulk mean temperature of the fluid we will be understanding from the control volume approach inside the duct. Also we will be utilizing the concept of bulk temperature and we will be calculating balance of energy to show that actual conduction at large peclet number is having negligible effect okay.

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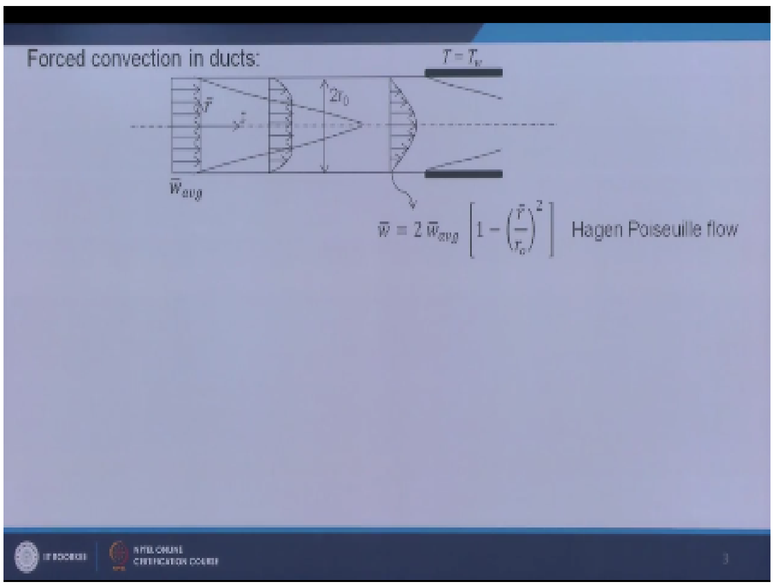

So first let me show you that what happens whenever fluid flows inside a duct. Now from your basic knowledge of fluid mechanics we already know that if we are having a duct, so let us say this is a duct having centre line this one okay, so this is a duct. So inside this duct if some fluid actually enters having let us say uniform velocity U bar average and then we can find out just at the wall due to mostly boundary condition the fluid comes to stand still or rest so due to most of the boundary condition immediate next to wall we are having 0 velocity.

But to hold the continuity obviously we will be finding out that inside the duct we are having finite amount of velocity. So velocity increases and then reaches to the value of Ubar average okay. And this pattern continues until you get a fully developed velocity profile, parabolic velocity profile like this okay. And we know that till that point from both the sides of the wall the boundary layer propagates and it fuses at a point okay at the center line up to that we call that as entrance length okay boundary layer entrance length for the velocity.

So up to this portion we are having the boundary layer, but at this point, at the center of the tube both the boundary layers have actually merged okay. And from here onwards velocity profile is not changing and you can have a parabolic velocity profile like this which is called actually Hagen Poiseuille flow for a tube okay, having radius R0 okay. So let us say in case of forced conviction inside duct which has already obtained the parabolic velocity profile like this, we start hitting that you do all okay.

So we have given let us say a hitting pad around the tube okay which is having constant temperature Tw okay. So immediately next you will find out that from here thermal boundary will start, whether what is thermal boundary layer already we have got the idea that whenever we are having heated plate so there the temperature is quite high compared to the fluid whatever it was flowing inside this okay. So let us say the fluid temperature is some Ti okay, so this Tw will be higher than Ti.

So ultimately we will be finding out that there is temperature gradient okay and that temperature is actually becoming fisting temperature soon. Now the locus of the points which has actually reached up to 99% of the fisting temperature is actually called a thermal boundary layer okay. So here also we can see from both the sides of the duct we are having propagation of thermal boundary layer okay, and these two thermal boundary layers should be margin from two at the velocity boundary layer. And this region is actually called thermal entrance length okay.

So let me show you what happens whenever these two things velocity boundary layer and thermal boundary layer actually comes in contact with each other okay. So here we have shown both the effects actually divide from each other in the first part of that we have the development of the velocity boundary layer and next we have started the development of thermal boundary layer. But if this padding of heaters we start from the beginning of the tube then we can say simultaneously the velocity and thermal boundary layer will be growing okay.

So in that case which one will be dominating whether the velocity boundary layer will be dominating or thermal boundary layer will be dominating that is very important to see. Already we have seen in case of flat plate that prandtl number of the fluid which is flowing over the flat plate determines that which boundary layer will be dominating here also the prandtl number of the fluid flowing inside the duct will be determining that what is the dominance which one is having thee dominance in these two okay velocity and thermal boundary layer.

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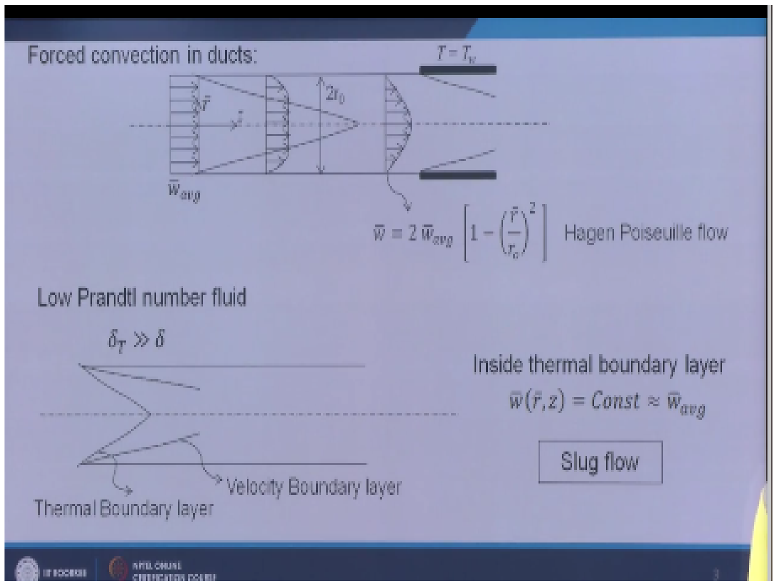

So first let me show you what happens for low prandtl number fluid, in case of low prandtl number fluid as we have already seen in case of low prandtl number fluid for a flat plate thermal boundary layer actually will be thicker compared to your velocity boundary layer. So here also you can find out that both the boundary layers are starting from the initiation of the tube, so thermal boundary layer thickness is having higher compared to the velocity boundary layer.

So the entrance length for the thermal boundary layer will be smaller compared to the velocity boundary layer. So that means thermal boundary layer will be fully developing okay, before the velocity boundary layer develops okay. So you can find out in this case that in immediate of the portion of the thermal boundary layer the velocity will be more or less fisting velocity, only in the small portion here inside this velocity boundary layer there will be gradient, but elsewhere you will be finding out the velocity is actually fisting velocity.

So this type of flow where low prandtl number fluid is involved we call flux flow. And in this case the velocity mode enters the times hence thermal boundary layer will be constant and we can call that velocity as U average or incoming velocity U average okay. So this is special case where in case of low prandtl number fluid thermal boundary layer actually dominates in compression to your velocity boundary layer okay.

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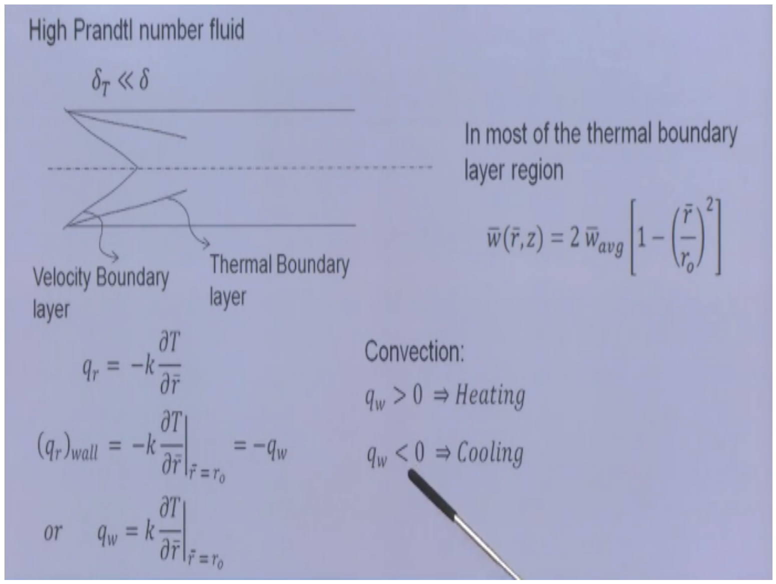

Let us see the other extent for high prandtl number fluid, in case of high prandtl number fluid just the opposite happens, that means the velocity boundary layer dominates over the thermal boundary layer. High Dominic entrance length is small are compared to your thermal entrance length okay. So here you can find out that both the velocity boundary layers are propagating towards center and the marge is quickly okay.

Till that time the thermal boundary layer has not developed, it is taking longer length scale to develop and somewhere over here it will be actually merging at the center and this much big length is required for the entrance, thermal entrance length okay. So in this case we can find out ΔT which is thermal boundary layer thickness is smaller compared to velocity boundary layer thickness Δ okay.

One interesting thing here is to observe that in fact, thermal boundary layer which is smaller compared to the velocity boundary layer thickness you will be finding out the velocity profile will be always following a parabolic law, because this is coming below the velocity boundary layer. So obviously in fact velocity boundary layer will be having a parabolic profile which we have already shown over here okay.

So in fact thermal boundary layer always it will be having a parabolic velocity profile. So parabolic velocity profile I have already shown you in case of a tube having R0 radius will be coming in this fashion from high parabolic ratio. So in most of the thermal boundary layer region you will be finding out velocity profile is like this okay. A few derivations are there for example, in the leading age the velocity profile becomes little bit modified, but those things we are not considering over here, that is why I have written most thermal boundary layer region okay.

Next let us try to understand that what happens to the wall heat flux okay, as we have considered the wall temperature as Tw what happens to the wall heat flux. For understanding that we will start from the radial conduction, so radial conduction heat flux Qr can be written as $-k\partial T/\partial r$ okay.

So R is the radial coordinate that inform that the axis of the Q. and it will be maximum at the QR okay. so if the we tried to find out what happens in the radial conduction to the all .so the Q R at the wall will to be find out .so in this case the wall means the R will become R 0, I have shown over here or0, is the Q radius .so here also you have to take this derivative $\partial T/\partial R_{r'=r0}$ okay.

So let us write down the Q n Rat the wall as _Q WE okay, we get the Q W +And r r=r0 okay, if we Q W greater than the 0 ,if the Q W is greater than the 0 then you can find out del t at also greater than 0 okay. So as well as this wills 0 this situation was the heating offers okay. The temperature of the how to the ward side high and the concrete and the temperature of the inward side just the opposite what happens the laser, that means we are having the cooling a cooling cloth rapped along the heater in that case we are find out the that is also negative, so that means that radically, how do downward the which having the lesser temperature in to the to the inner fluid okay.

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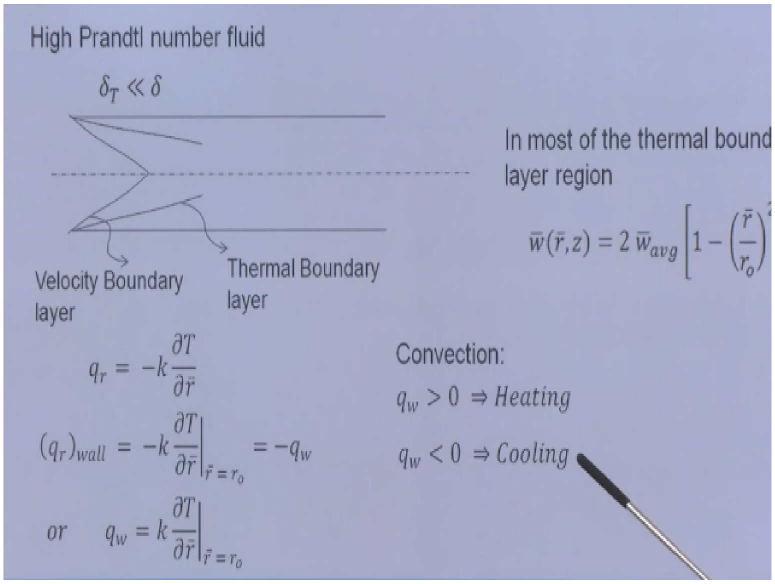

So the is the case of cooling right, now let us consider how are the first case that means that load the angle number with the case of the slug flow when the velocity almost is constant, okay.

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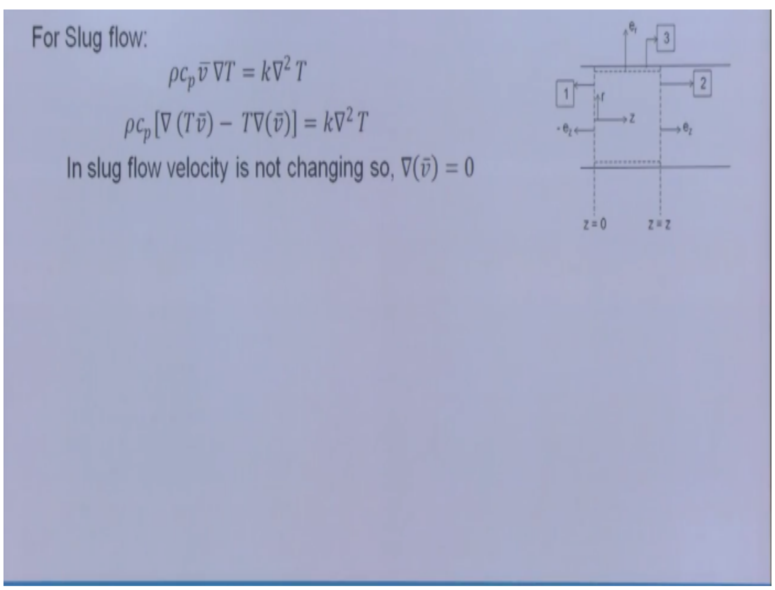

In the out from the contour volume in the approach the vertical s to simplified energy ratio, so remember this is for the slug flow that means in the timer in the constant okay. So we can equate the heat transfer due to the convection to the heat transfer and due to the conduction this CP and in to the ΔT =to K^2 ΔT, this is the actually the especially derivatives the delta can be written as the X + for the coordinate systems. So let use little bit modify this one this the row CP and we can write down as the del of the T _T of ΔV okay, so the ΔV is the T V = to the T of del v +and this from we have written in this passion okay, here you now this you see whatever we can do this V is 0 from the continuity, so this vector we can drop.

So in case of slug flow velocity is not in the changing we know ,so let us more less constant , so this is can actually okay 0 ,this term is convey to the 0 okay, so the ultimately we find out the or w cp in the in the k and now we will take a control volume and then in the pie plan ,here we have taken a small element and we actually we found out the contour volume of ,in the form this is a small cylinder which is having the radial in the E R and the axially EZ okay, and _E Z respectively, okay these are the r and the Z code nets okay, and if you see minutely we are having the three phases of this cylinder the first face is.

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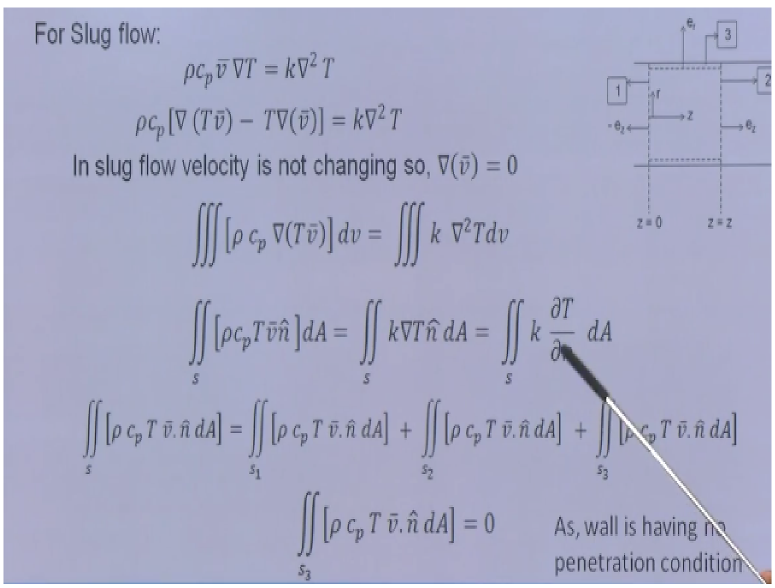

In the one face which is the radically in the left hand side direction ,the the second face is +racially in the right hand side detection and the third face is the cylindrical surface which is having E R F in the detector okay, let us consider that this is the small cylinder having thickness and high we are having say the Z=to x it is actually Z ,now it is the integrate the energy equation what is that e are finally find out the over this hole volume ,this if we integrate that so we are having in the over e S DV ,lecture term here we n the volume of to okay,

Now let us one by one first we show Δ you the convection here we use we are having in the Δ V okay, so if we user the volumetric the surface of the transformation and the arithmetic's so you can write down the ΔT V in the actually in the perpendicular direction okay, from the intendments in the day, so to in the N cap is A over here ,in the case of the DV so they do as converted over here so the volumetric is also to the right ,so here you see as this is the row cp in the in the cap ad is you can in the k and the n cap dA in that ,here you see that ,we can right in this fashion, okay

If you see from the contraction side from the contact onside K*to X^2 in the, so here the double derivative his was the derivative and the single of the derivative over here Okay, And this ΔT in the of t in the is actually product in the two vectors in the T okay, so this is the convection side of the conducting side okay. So now let us see the first convection side and the convection side is using in the having in the surface interior in that.

There are three surface over here and one two and three and, so I am witting on the s on S 2v and S3 okay, now the specfically6 if you see the three side this is actually A cylindrically surface it is the actually the cylindrically surface in the so if you see the cylindrical surface they are as it is they are having the wall .so this is the actually in the wall in the tube wall ,having no actually no penetrate equation. So the velocity in the radiation racially in the out ward direction around the pipe line it becomes in P.A in the cap becomes 0 okay as well as in the holing interview it will become in the 0 right.

In this term you can be dropped out okay ,so we have obtain that this ace okay ,after than let us see this contraction term so the surface integral of the ΔT at the D A surface integral of the DA.

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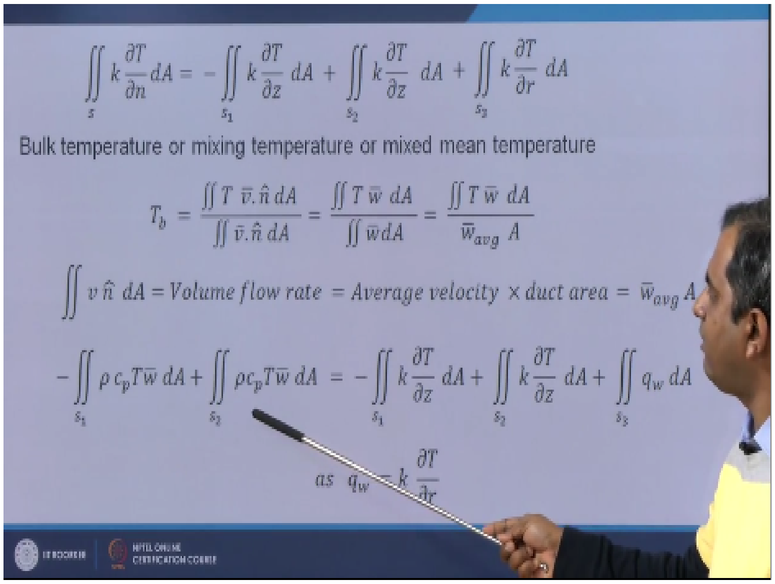

This will become s2 sand s3 okay now what I have done as I know the detecting of the lane in the t Z and 2 axial we have taken along the Z direction ,here I have put the +sign over here the radial direction I have actually retain the ΔT and the R will for the three surface Δ and ΔR which is the cylindrical surface ,now let us define a new concept which is call in the wall temperature in the temperature mixed mean temperature what is wall temperature, the wall temperature ca n be T B let us define that one as T B so the T B actually surface integral of T V and in the D A then divide n this the volume.

Then it is the surface okay ,so this we will be using for the wall temperature so the fluid is coming out what is the temperature of the fluid that will be finding out so for in finite in the small AD are we are finding out the temperature the temperature *to D in the D V 1 b the V1 .so in that we all, the wall temperatures okay, so if you simplify for this one the our case it was the we are having only in axial velocity radial velocity is not in the D A ,so in that case we V will become surface integral of the T W dA surface integral of WdA okay.

And the surface integral of WdA can be written as W average into A where A is the pive diameter okay, pie cross sectional area okay. So here you see Tb or bulk temperature becomes surface integral of TwdA/W average into A. Now here we will be also trying to find out that what is this surface integral of v ncap dA so v ncap dA as already we have showed this is nothing but W average into A which we have used over here okay.

Then let us try to go back once again the equation what we have derived so here we had you see the conviction side S1 and S2, S3 actually we have cancelled.

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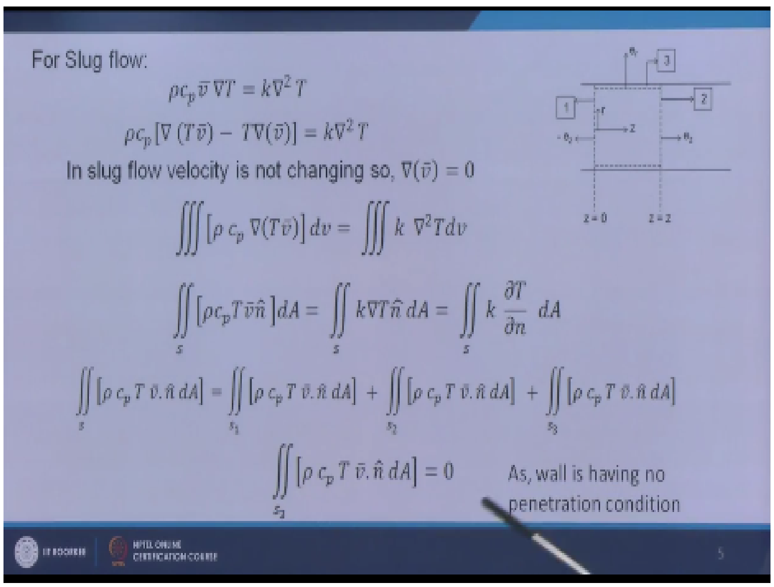

Mid 0 over here so this actually we have made 0 okay, by virtue of the wall open it penetration.

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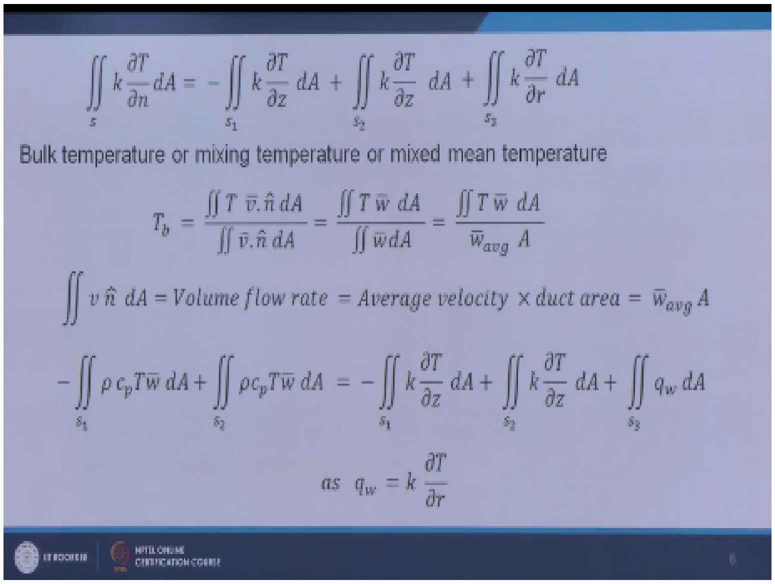

So that S3 term is not there in the left hand side the right hand side we are having all three terms over here, and in place of this K ∂T dealer we are actually witting Kw okay, so this is actually qw into dA right, now using concept of mixed mean temperature as we have derived in the previous slide we can write down the left hand side we see left hand side we are having integration of Tw dA okay so here it is Tw dA so using this concept we can write down this one has Tb into wave into A okay.

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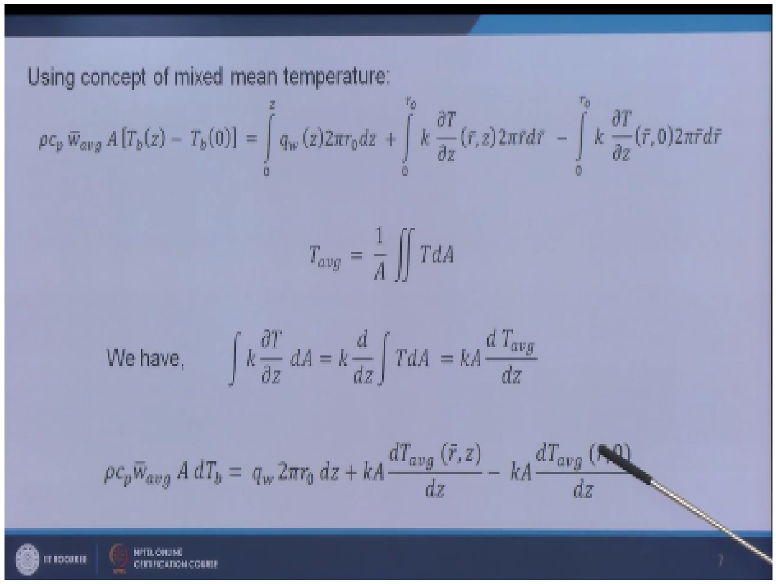

So here we are doing that 1Tb into w_{avg} into A okay, so first one is for S1 which is at – of S1 which is at x = 0. So that one will be coming as – Tb0 and second one is actually at S2 okay so which will be actually coming to Tb of z okay, so this is becoming ρCp wavg A Tb z – Tb0 okay. On the other hand right hand side becomes as usual the same okay whatever we have seen over here only thing we have done is that this dA we have written has 2πr into dr okay, so 2πr into dr for this two cases and this one will be $2\pi r_0$ which is the perimeter of the tube we multiplied by the z okay.

So this z will be 0 to z right, now let us also try to derived what is the average temperature okay so average temperature can be written has 1/A surface into L of T into dA okay, now we will be trying to use this concept of average temperature for finding out what happens over here okay, in the conduction side so let us try to see if we write down integration of K ∂T ∂z of T into dA which was here and here, so that will come out K can be taken out of this integration and the order of integration in differentiation can be so up.

So K ∂z of integration of TA okay TdA, so integration of TdA can be linked up with average temperature so this becomes K into ∂ ∂z of T_{avg} into A okay and if, it is a pipe line is not changing so A can be taken out so this becomes KA dbz of Tavg okay, so this conduction term actual conduction term becomes KA d dz of Tavg so let us use this one over here in these two terms and simplify the equations so here you will be finding out this term has we have written so

$T_b z$ – Tb0 we have written has dT_b change in bulk temperature this qw z if we consider that throughout the pipe the heating or cooling.

Is not changing its rate so that can be taken out 2πr0 is can be also taken out 0 to z Δz will give us dz okay for infinite small length dz okay and here in this case both the equations will be giving me K into A just of this form dTz of Tavg first one is actually z and second one is z0 okay subsequently these two plates 1 and 2 okay.

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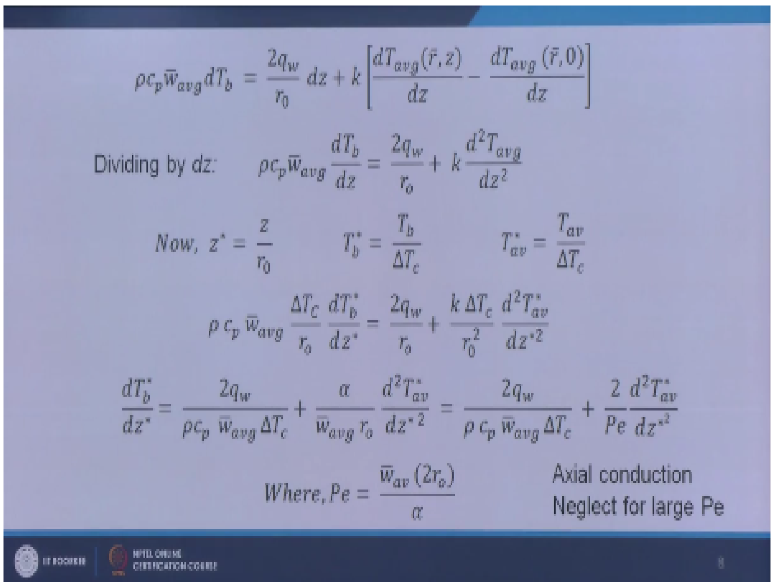

So we have got this equation finally this can be also reduced or throughout this equation we can divide by dz and we can write down ρCp w_{avg} dT with dT_b q 2/ 2qw / r_0 into dz plus K into dT average dTz of T average at z and Tbz of Tavg at 0 okay, so if you divided it by once second dz then you will be finding out ρCp w_{avg} dT_b / dz here dz cancels out 2qw / r_0 and finally if you divide it by dz it will become the second order derivative of T_{avg}, so ∂2 ∂z^2 of Tavg okay so ∂^2 Tavg / ∂z^2 so this term is actually nothing but your axial conduction.

Now let us try to find out what happens whenever we do non dimensional lines so let us non-dimensionalize the axial direction z / z* so z* = z / r_0 okay, the temperature let us non-dimensionalize Tb* = Tb / ΔTc and average temperature so this is bar temperature non-dimensionalization average temperature is a non- dimensionalization average temperature is a non-dimensionalize it one second by ΔTc so T*avg = Tavg / ΔTc so here we are having the bulk temperature but this side we are having the average temperature okay so if we do that then we are finding out.

ρCp wavg and here this becomes ΔTc and z will be releasing $1r_0$ so ΔTc/ r_0 into dTb*/ dZ and the right hand side the first term will be remaining at equal same because there is no derivative over here for the second term you can see ΔTc / r^2 will be released because Tavg will be releasing ΔTc and Z will be releasing r and we are having Z^2 so this is ^{r2} okay, so if you reduce it further the you will be finding out ∂T ∂Z* of Tb, Tb* so this whole term if you take in the right hand side by dividing when we will be finding out this becomes 2qw ρCp w_{avg} / ∂Tc okay and this last term will become α by wavg into r0 okay.

Little bit of reduction will be giving you this one is nothing but your pick line number okay based on the lane scale of the tube radius okay, if we do just on the tube diameters so this will become 2/ decline number okay, and this qw term will be involving over here like, this okay, so as usual I have told you this becomes the this becomes the axial conduction now if you have large pick line number in this one so what we can do this term you can neglect so I have told you that this axial conduction in case of large pick line number.

Can be neglected because it is having the having the coefficient of 1/ p sorry 2/ Pe over here, okay so the axial conduction can be neglected for large p in cases decline number cases okay.

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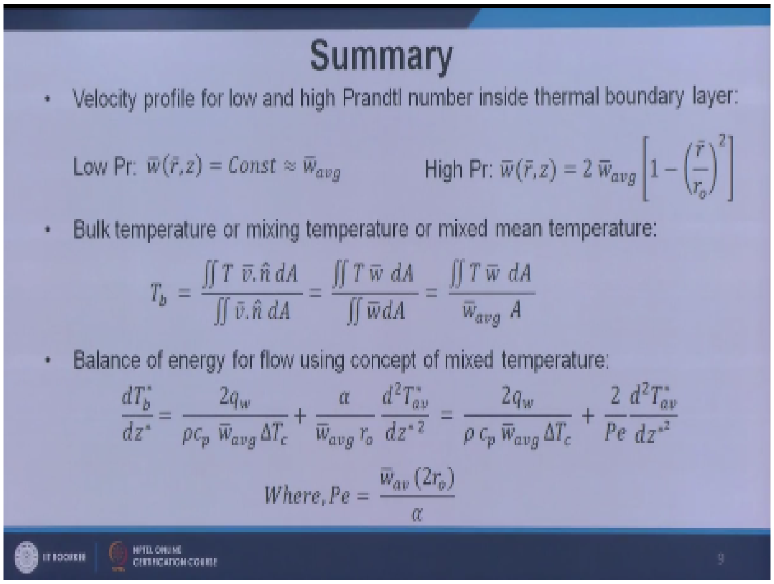

So to summarize this lecture what we have done in this lecture the velocity profile for low and high prandtl number inside thermal boundary layer of that we have found out for low prandtl number we have seen w is actually remaining contending most of the cases most of the positions, and for high prandtl number w is becoming a parabolic profile, okay in case of tube it becomes the hagnes flow we have define the bulk temperature on mixing temperature or mix in temperature in this fashion. So Tb is actually in surface integration of T_w / w_{avg} into A okay.

And whenever we have balance the energy okay using the concept of mixed temperature and average temperature, we found out this equation okay dz* of Tb is * = 2qw / ρCp w_{avg} into ΔTc + 2/ Pe number Δ2 ∂Z^2 of T_{avg}* okay so here we have shown for large Peclet number this term can be neglected okay so all these things we have discussed in this lecture.

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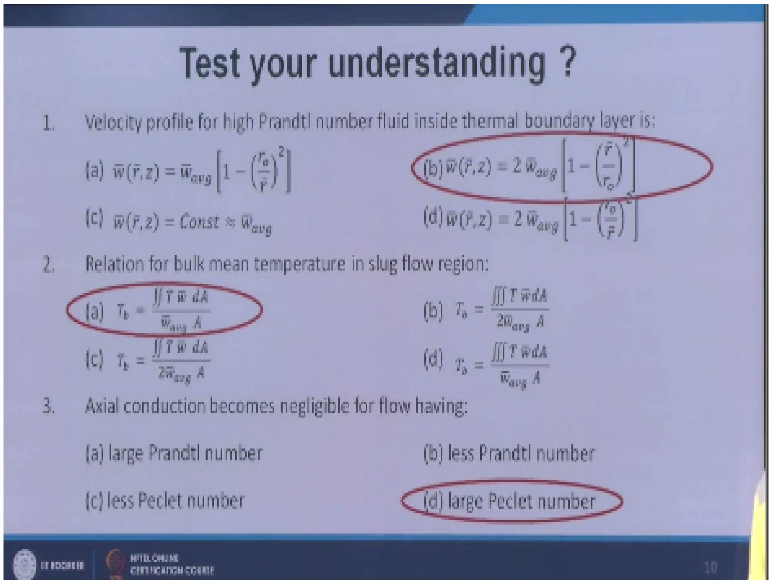

Let us test your understanding at the end of this lecture so we are having three question over here first question velocity profile for high prandtl number fluid inside thermal boundary layer is so it is high prandtl number fluid, so we are having four options w as a parabolic profile over here based on r_0 / r bar remember this is r_0 / r bar I have written second one is parabolic profile one second but this term r bar / r_0 okay, third one is constant okay and fourth one is same but one second over here r_0 / r bar but I have put it 2 over here okay.

So what is the correct one obviously the correct one is this one okay this we have already shown this is Hagan flow okay next relation of the bulk mean temperature in slug flow region what is the relationship of bulk mean temperature, so we have your having four options Tb = integration of T w dA / w_{avg} into A Tb = integration of Tw dA / $2w_{avg}$ by A here Tb = integration of Tw dA / $2w_{avg}$ into A okay, and here by this is these two are actually surface integral and this is actually a volume integral okay and here we are having the bulk temperature is equals to Tw dA / w_{avg} into A okay, so here obviously the correct answer is the first one we have showed.

So when a tan so this is the correct answer here the number of integrations are not matching so these two will not be the correct answer okay, then the last question is axial conduction becomes negligible for flow having large prandtl number less prandtl number less peclet line number or large peclet number so four options we are having so by now probably you have understood what is the correct answer obviously the large peclet line number is the answer so in case of large peclet number because the coefficient was 2/ P the axial conduction can be neglected okay so with this I am ending this lecture than k you very much in the next lecture we will be discussing about thermally developed slab flow in a duct okay so keep on posting your queries in our discussion for thank you.

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