Engineering Economic Analysis
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Lecture 09
Compounding Frequency of Interest: Nominal and Effective Interest Rates

Welcome to the lecture on compounding frequency considerations. So what so far we have seen that normally the interest is compounded once in the year that is known as compounding once in a year. Now many a times in case of loan agreements or many a times when we repay our loan, the compounding is done more than once.

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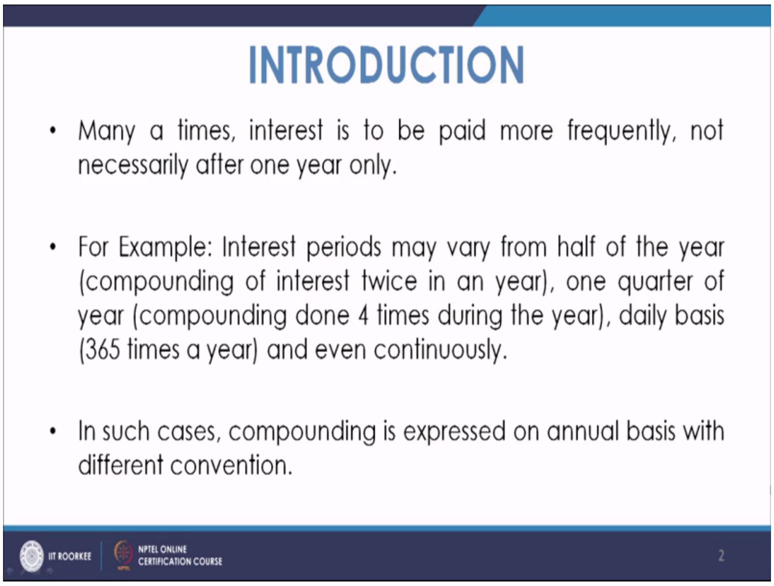

So that is, interest is to be paid more frequently. So what normally is the practice that you have to pay the interest once, normally it is more than once, it may be four times, it may be six times or it may be even daily basis. For example interest period may vary from half of the year. So suppose the interest rate if it is calculated every six month this is known as i mean that this compounding of interest twice in a year.

So and if you do 4 times so that is known as compounding done 4 times during the year. So basically after every quarter the interest is charged. Here it can be on daily basis and sometimes even on continuously. So in this lecture we will try to see how this affects basically the interest rate if you calculate on a particular period.

So if the interest compounding is done suppose 2 times during the year, basically during the year if you take as a whole the interest rate otherwise it will be more. So what is seen is in such cases, compounding is expressed as annual basis with some different convention. So basically the way we call them, it is somewhat differently how we call it them.

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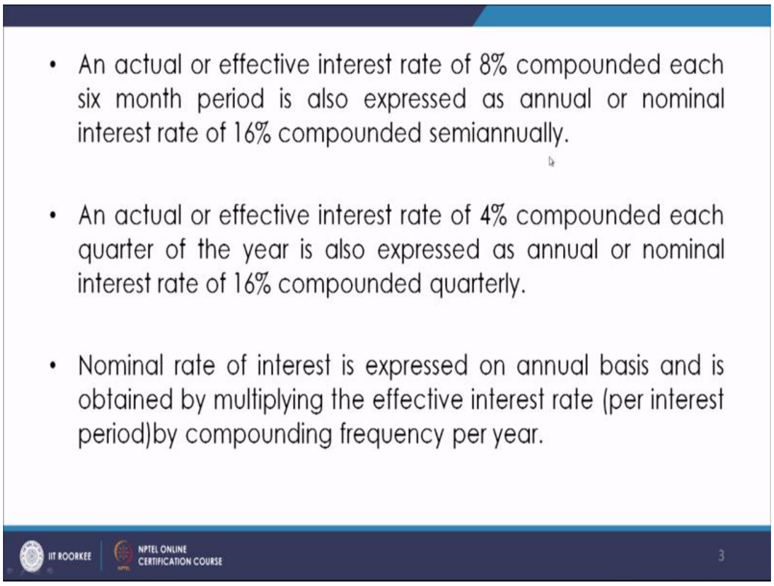

So basically the actual or effective interest rate of 8% compounded each six month period. So this is also expressed as annual or nominal interest rate of 16% compounded semiannually. So if we talk about an interest rate of 16% compounded semiannually means the annual interest rate is 16% that is known as nominal interest rate but this semiannually means the interest rate is calculated every six month.

So basically effective interest rate will be since it is done two times, effective interest rate, it will be since it is done two times, we will divide by 2 so 8% per six month period. So this is known as effective interest rate. So likewise suppose effective interest rate of 4% compounded each quarter of the year, you can also term it as a nominal interest rate of 16% compounded quarterly.

So these are the different ways by which you express the effective interest rate and the nominal interest rate. Nominal interest rate is expressed on annual basis and is obtained by multiplying the effective interest rate per interest period by compounding frequency per year. So this is the definition of nominal interest rate and this is how you basically express effective interest rate in terms of nominal interest rate.

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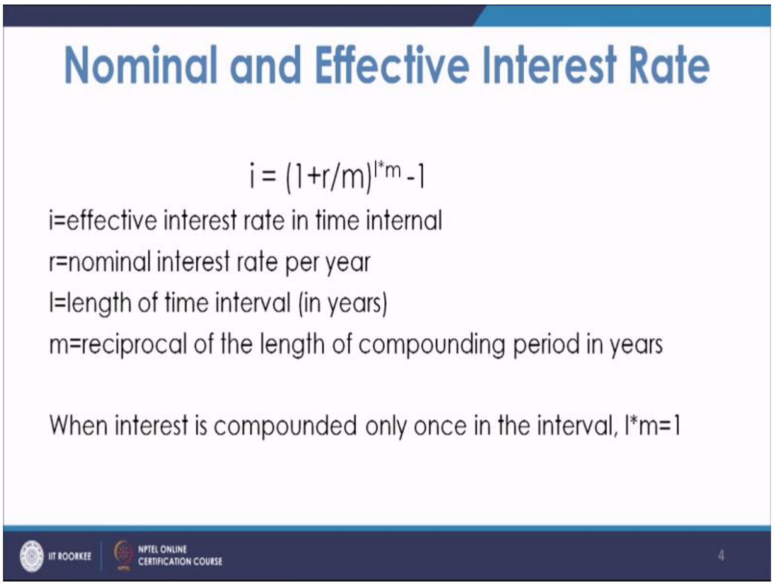

Let us see the relationship between nominal and effective interest rate. So this is the formula for finding the effective interest rate i in a particular time interval, r is the nominal interest rate per year. So nominal interest rate is basically expressed on annual basis so that is why nominal interest rate is per year. l is length of time interval in years and m is the reciprocal of the length of compounding period in years.

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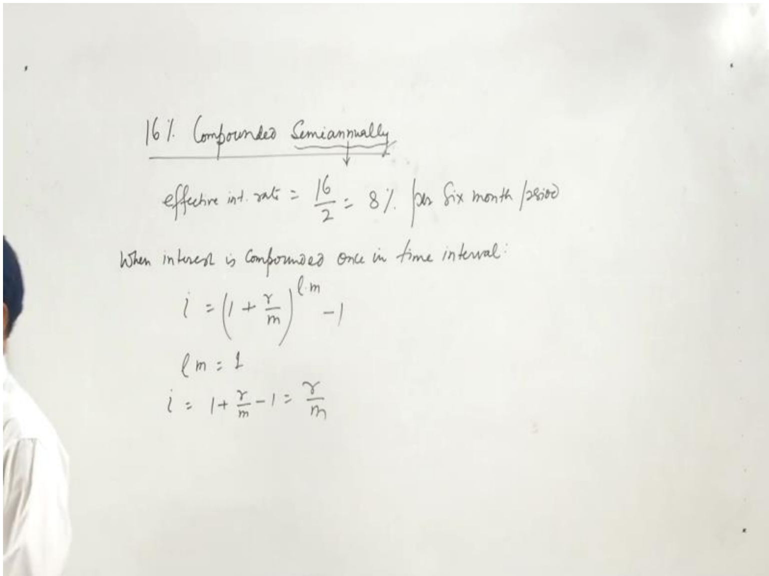

So this relationship is used to calculate the value of effective interest rate for a particulartime period or time interval. Now when the interest is compounded once in the time interval, in that case l into m becomes 1. So in that case what you could see is, when interest is compounded once in the year, once in time interval, what we see that as i is expressed as 1 + r by m raised to the power l into m - 1.

So once interest is compounded once in the time interval, in that case LM is 1. So effective interest rate will become 1 + r by m - 1 so that is r by m. So in that case, effective interest rate will be calculated by dividing the nominal interest rate with m. So number of times you compound basically. Now further we will look at different examples and we will see how this effective interest rate is calculated when there is nominal interest rate.

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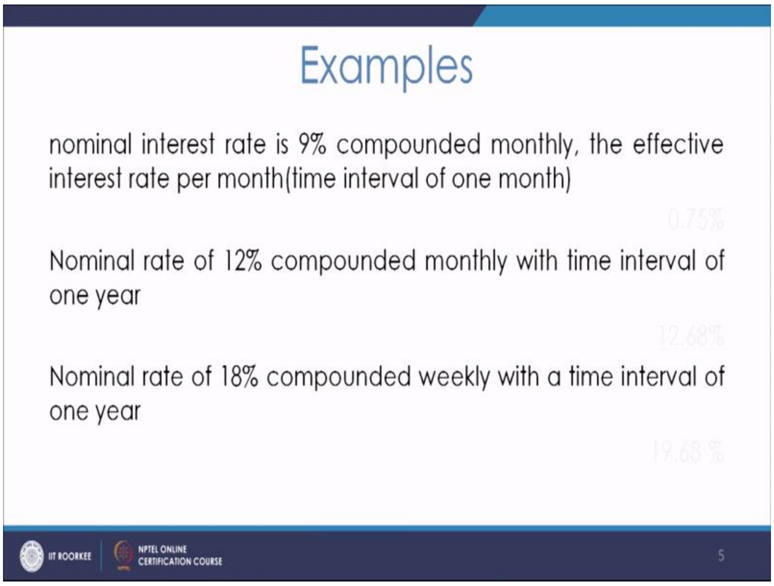

So let us see, the nominal interest rate of 9% compounded monthly, the effective interest rate per month that is time interval is one month. So let us calculate this. So if you take the first case, what we have seen so far is the effective interest rate is 1 + r by m raised to the power lm- 1. l into m is also the number of compoundings during the time interval. It is also expressed sometimes in another term like C.

So you can have the expression like 1 + r by m raised to the power c - 1 where c is nothing but l into m and c is normally greater than equal to 1. So this is the general relationship between the effective interest rate and the nominal interest rate.

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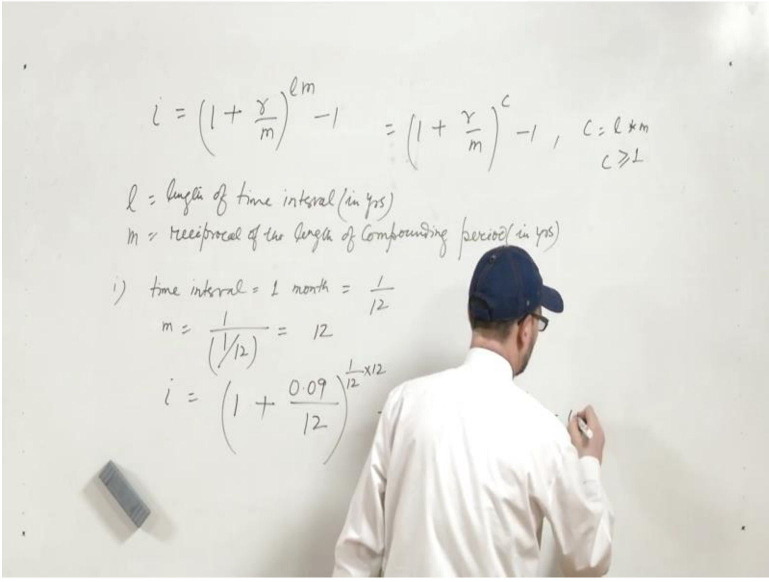

Now let us see the first example, you have to calculate the effective interest rate per month. It means as we see, in the earlier case, your l is length of time interval in years and m is reciprocal of the length of compounding period in years. Now in this case, as we see the first example, in this case your time interval is given as one month. Time interval is given as one month for which you have to find the effective interest rate.

So if you express it in years it will be nothing but 1 by 12. Then the effective interest rate length of interval is one month. Then m, m is reciprocal of the length of compounding period. Length of compounding period is nothing but 1 by 12, so it would be reciprocal of 1 by 12, 1 by 1 by 12. So it comes as 12. In that case, effective interest rate i will be, 1 + r, r is your 9% so it is .09 divided by m.

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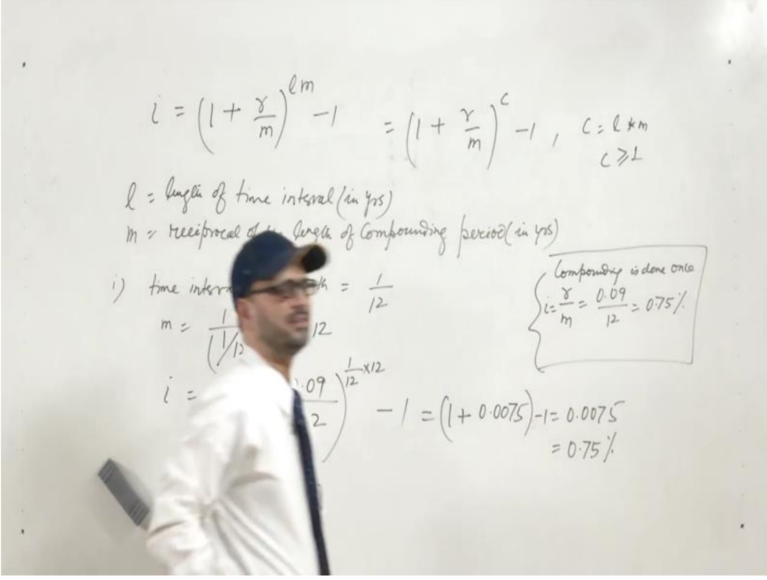

So this m is reciprocal of the length of compounding period that is 1 by 12 that is 12 to raised to the power l, l that is 1 by 12 into m is 12 - 1. This comes out to be 1 + 0.0075 - 1 that is 0.0075 which can be **sss** said to be as 0.75%. So this way you calculate the effective interest rate in the time interval of one month.

This is also an example of that case where basically the compounding is done once in the time interval that is why you can have the value as r by m directly. So it will be .09 by 12 directly and get, you can also get it directly by r by m because compounding is done only once, once in the time interval. So i will be r by m so it will be .09 by 12 so it will be .75%. So this is also an alternate way to calculate the effective interest rate in this case.

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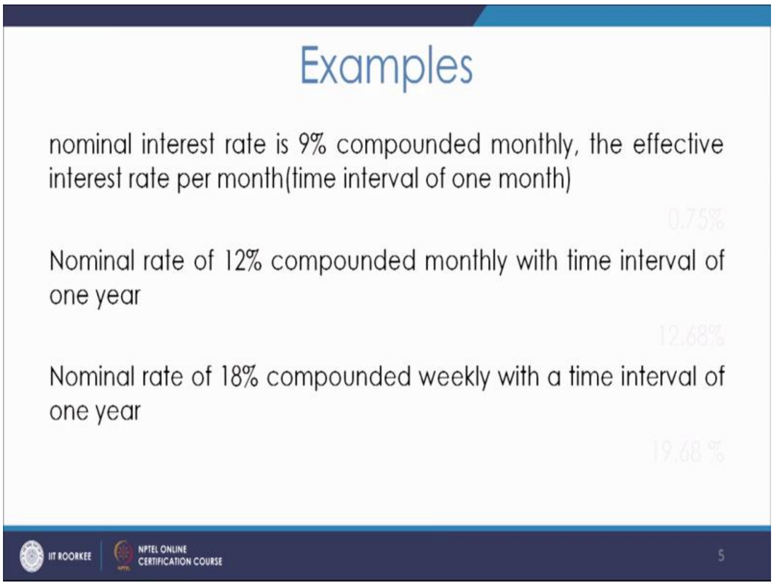

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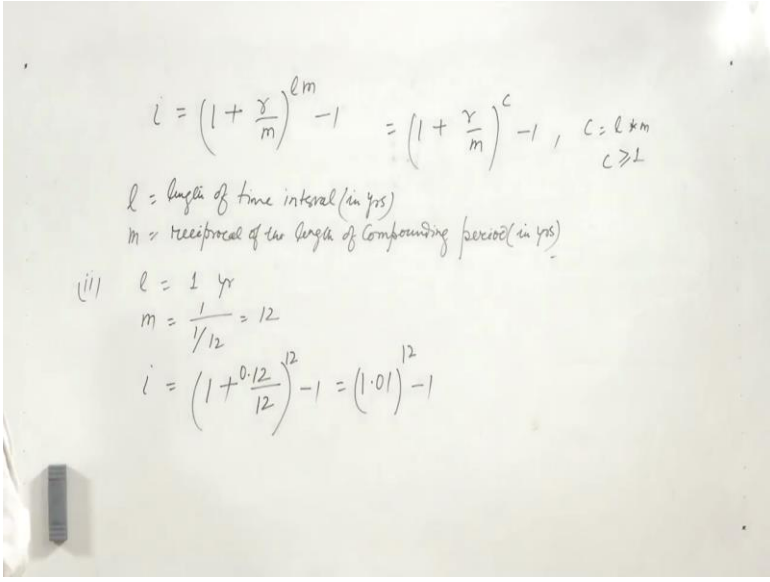

Let us go to another example. Next example is, nominal rate of 12% compounded monthly with time interval of one year. So in this case, your time interval is one year. So l is one. Nominal rate of 12% compounded monthly with time interval of one year. Now the m, m value is nothing but the reciprocal of the length of compounding period in years. Compounding period is monthly. It means 1 by 12 years and reciprocal of that, so 1 by 1 by 12 that is 12.

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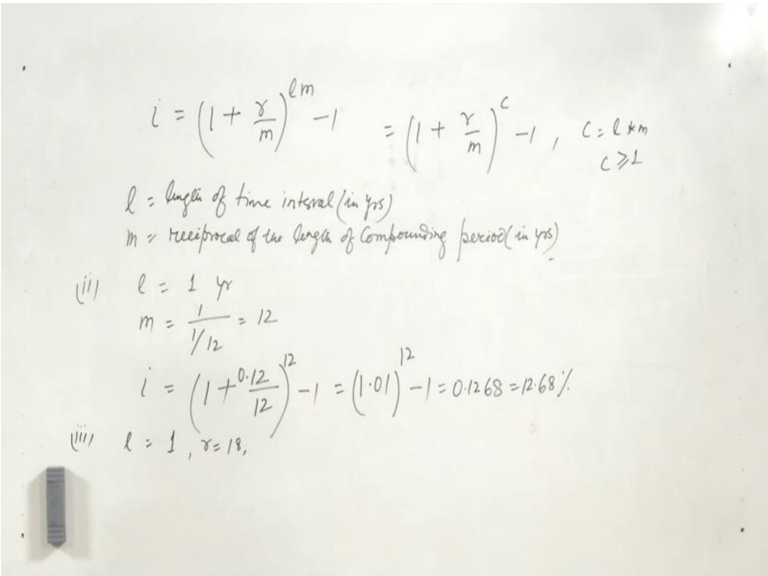

So what will happen, the value again calculated as 1 + r is 12% by 12. 12% so .12 by12 then raised to the power l into m 1 into 12 and - 1. So it will be 1.01 raised to the power 12 - 1.

And this if you calculate, it comes out to be .1268. So it will be equal to 12.68%. So basically what you see is, the nominal rate is 12% compounded monthly.

And if you take the time interval of one year, what you see is effective interest rate is more because the compounding is basically 12 by 12 that is 1%. Effective interest rate is 1% per month, so this 1% per month when it is compounded again and again after 12 compounding times, the effective interest rate over the year becomes 12.68%.

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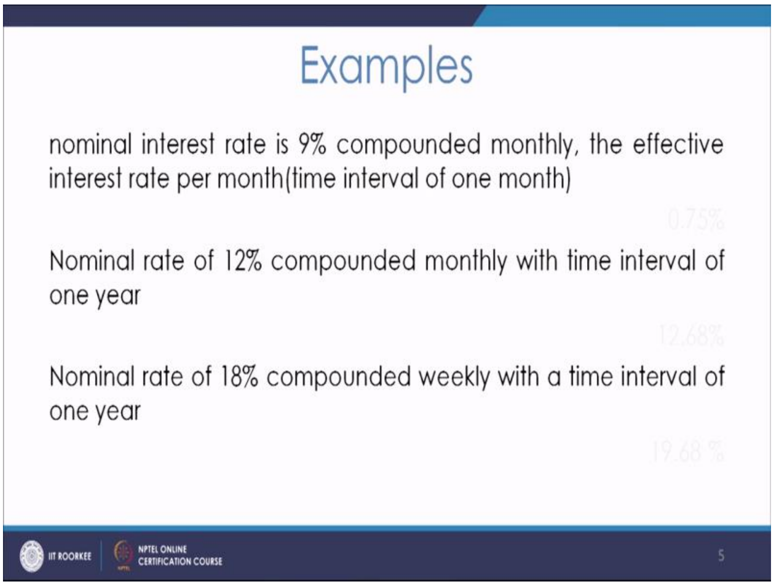

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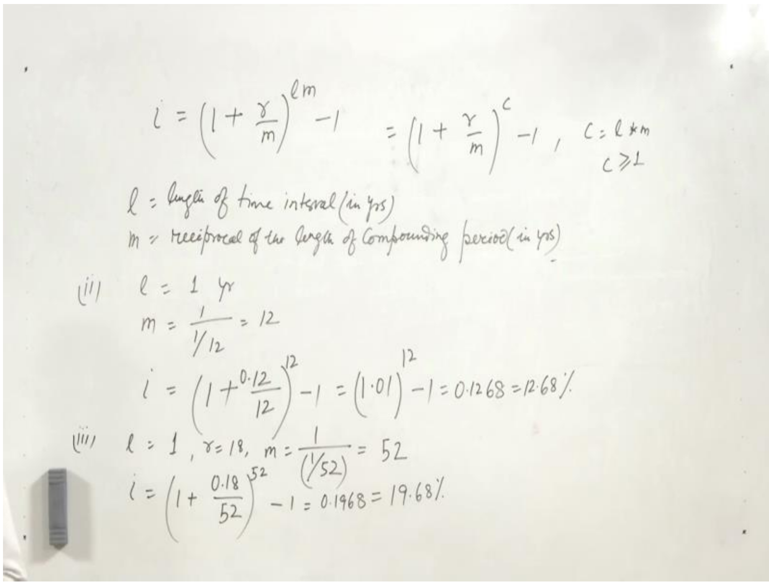

The third case is nominal rate of 18% compounded weekly with a time interval of one year. Now by this time i hope it is clear to you. In this case, l time interval that is known as one year so it is 1 in year terms. r is 18, and compounding is weekly. So basically in year terms m will be reciprocal of the compounding period in years. So in year terms, m will be reciprocal of 1 by 52 years.

A week can be expressed as 1 by 52 of years. So in this case m will be 52. Now if you calculate, again use this formula, effective interest rate with a time interval of one year it becomes 1 + r that is .18 divided by m 52, this will be raised to the power of l into m. So l is 1 and m is 52, so this raised to the power 52 - 1. And this can be found out to be .1968. So this will be equal to 19.68%.

So this is how you calculate the value of effective interest rate when the compounding is done at different frequencies.

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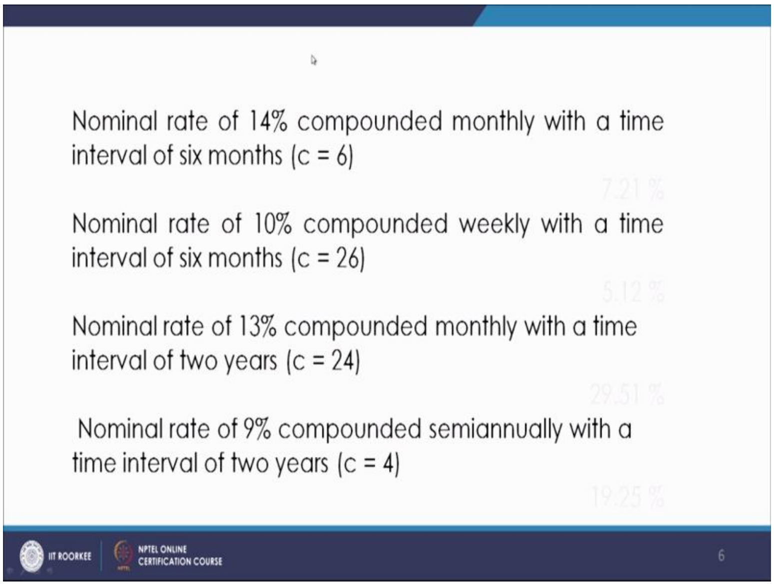

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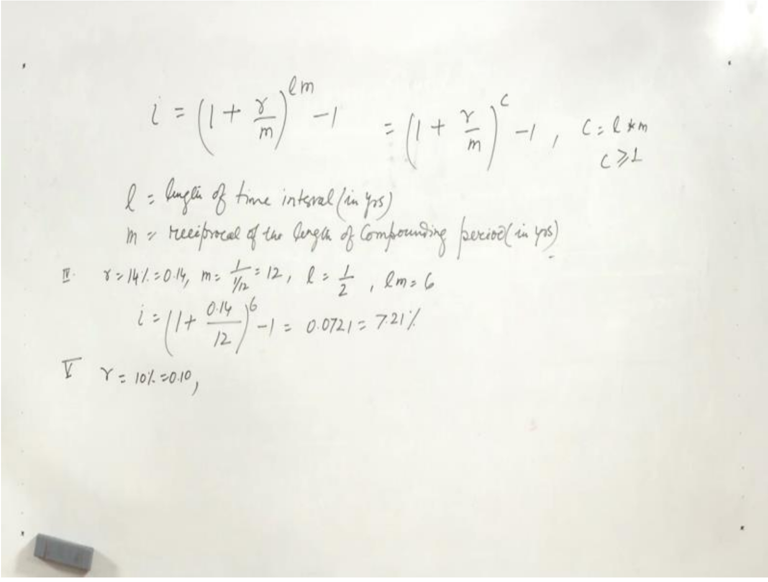

Let us see another example, nominal rate of 14% compounded monthly with a time interval of six months. Now here what you see, you r is 14% that is .14 compounded monthly. So compounding period is 1 by 12 of the years m will be 12. m will be ratio of 1 by 12, reciprocal of 1 by 1 by12 that is 12. Here time interval is six months. So if you look at the l becomes, if you express this six month in years, it will be 1 by 2.

So l into m that is also expressed as c, it is 6. So effective interest rate can be calculated as 1 + .14 divided by m that is 12 raised to the power 6 - 1. And this will be calculated as .0721 that is 7.21%. This is an example which shows time interval is only six months, so basically otherwise it can also be expressed as effective interest rate of so basically you are effectively using 7% interest rate every six month.

Had it been semi annually but here compounding is done monthly so in that case in the six month period the effective interest rate becomes more than 7 so it becomes 7.21.

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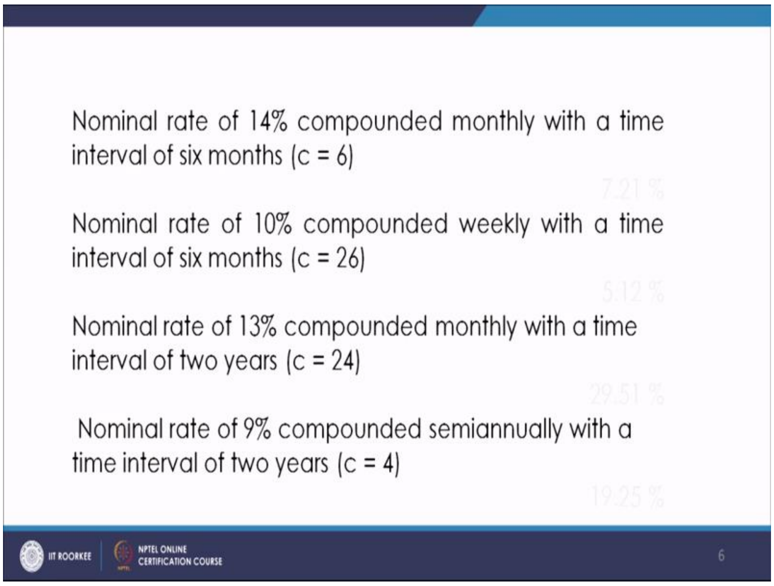

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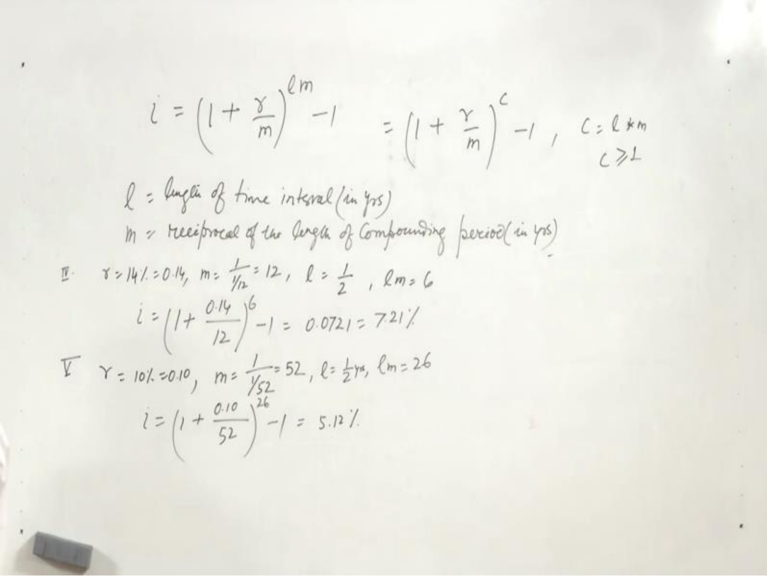

Next example like nominal rate of 10% compounded weekly with a time interval of six months. So this can be further found bylooking at r is 10% or .10 compounded weekly. So m will be reciprocal of 1 by 52 years, so it will be 52. Length of the time period interval is six month. So again l is 1 by 2 years, so l into m becomes here 26 that is also c.

So in this case, effective interest rate can be calculated it as 1 + .10 divided by m that is 52 raised to the power 26 - 1. And this way you can calculate the different values, in fact it comes here as 5.12%. So then you can see another example, nominal rate of 13% compounded monthly with a time interval of two years.

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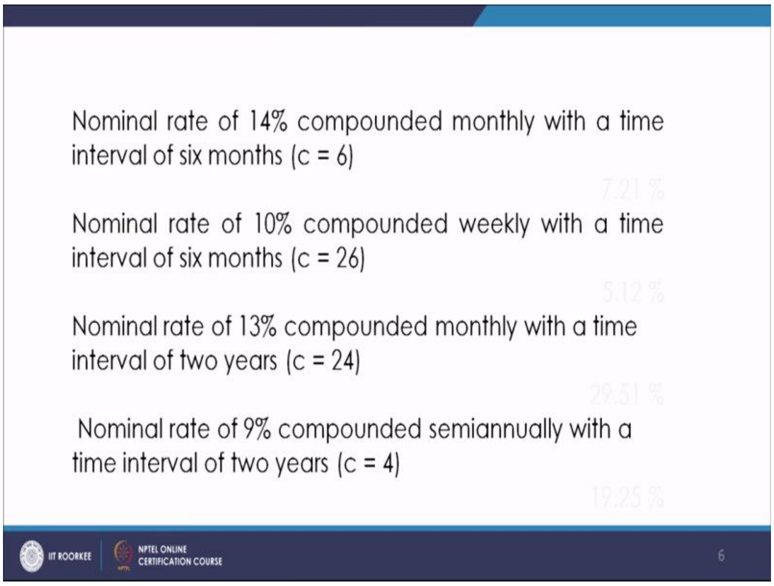

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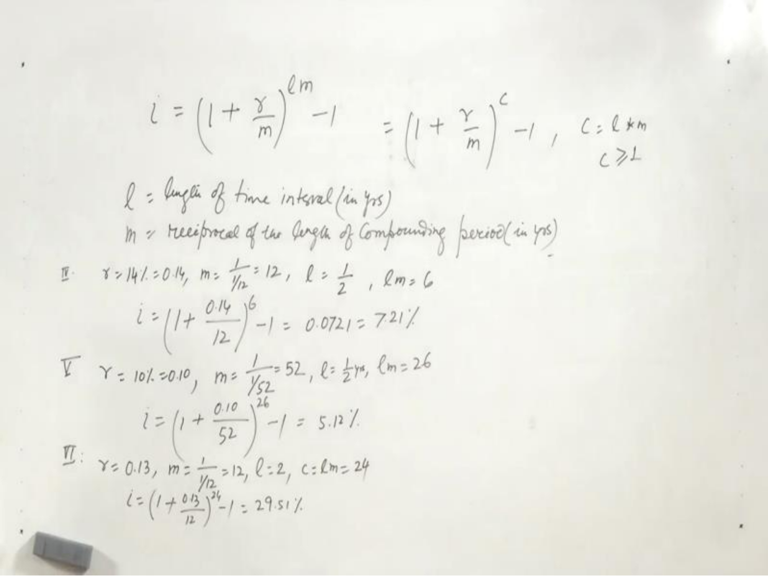

Now in this case what you see is your r is 13%, so r is .13. Compounding is done monthly, so m is reciprocal of 1 by 12 year. So m will be reciprocal of 1 by 12 so it is 12. Time interval is two years, so it will be l will be 2. So in that case your c that is l into m it becomes as 2 into 12 24. So effective interest rate you can calculate as 1 + .13 divided by 12 raised to the power 24 - 1 and that comes out to be 29.51%.

So in this case you can have different type of problems and you can solve on your own. Now we discuss about the case of continuous compounding. So **so** far we have seen that the compounding is done for a finite number of times but continuous can be done infinite number of times, large number of times.

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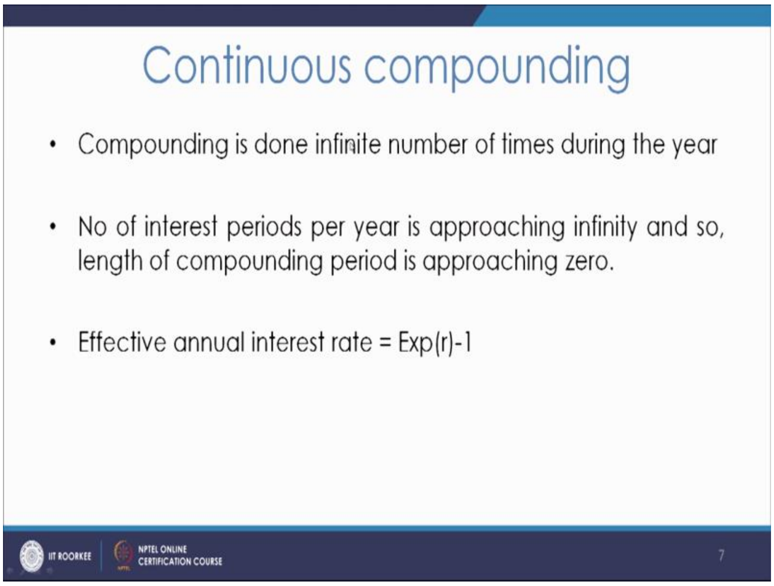

So that is known as a continuous compounding case. Now compounding is done infinite number of times during the year. So in case of continuous compounding, since it is done infinite number of times during the interval your reciprocal of the time during this the compounding occurs because this time becomes very small.

And once the time interval becomes very very small limiting to 0 then m will be tending towards infinity because it is reciprocal of that time interval. So number of interest period per year is approaching infinity and length of compounding periods is approaching 0. In that case how to calculate?

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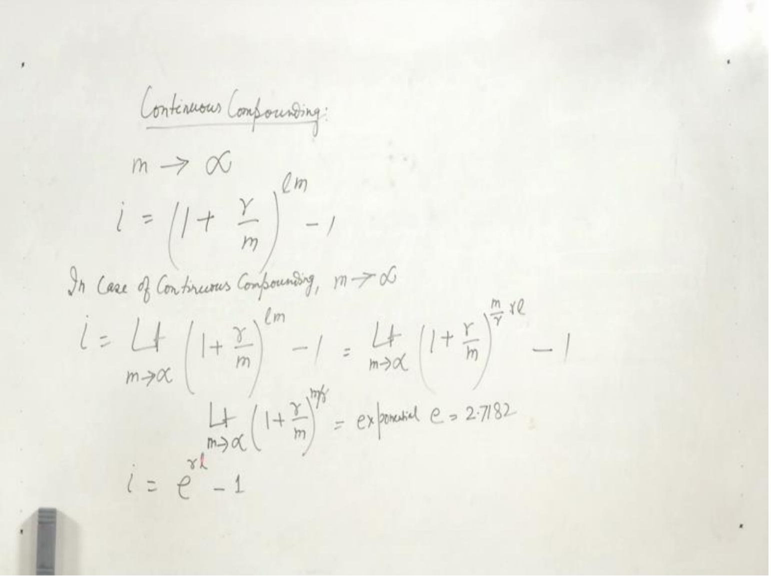

So basically we know that i is 1 + r by m raised to the power lm- 1. Now in this case, in case of continuous compounding, m is tending towards infinity. So what you can see is, i will be the limit m tends to infinity 1 + r by m lm- 1. This can be written as limit m tends to infinity 1 + r by m to the power m by r into r - 1. So you can write this expression as 1 + r by m 1 + r by m m by r into rl - 1.

Now basically if we recall our calculus principles limit of m tends to infinity, 1 + r by m raised to the power m by r becomes its value is nothing but exponential value e that is something close to 2.7182. So effective interest rate for any time period will be e to the power rl - 1. So in case of continuous compounding, this effective interest rate is calculated i is equal to e raised to the power rl exponential r into l - 1.

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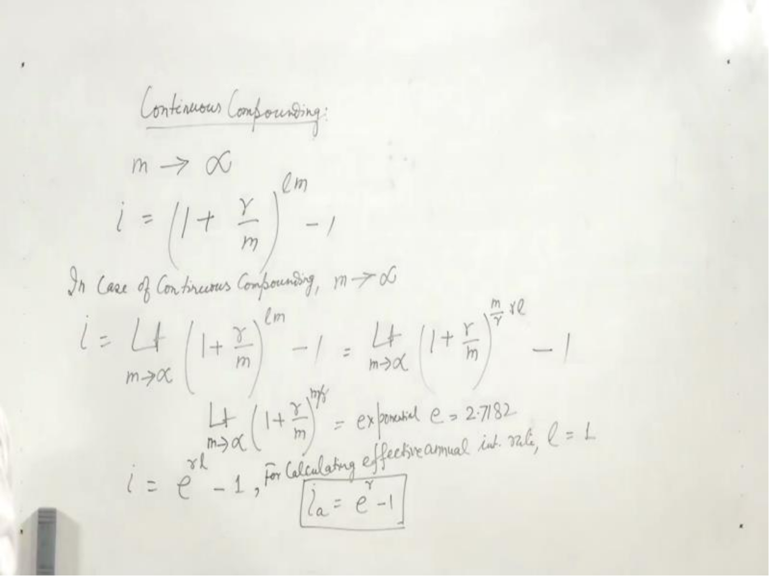

If we calculate effective annual interest rate, for calculating effective annual interest rate basically we are limiting l equal to 1. So basically the time interval which we are specifying as 1. So in that case effective annual interest rate ia is shown as this l becomes 1 so exponential r - 1. So this is how in case of continuous compounding we calculate the value of effective interest rate over any time interval or annual value.

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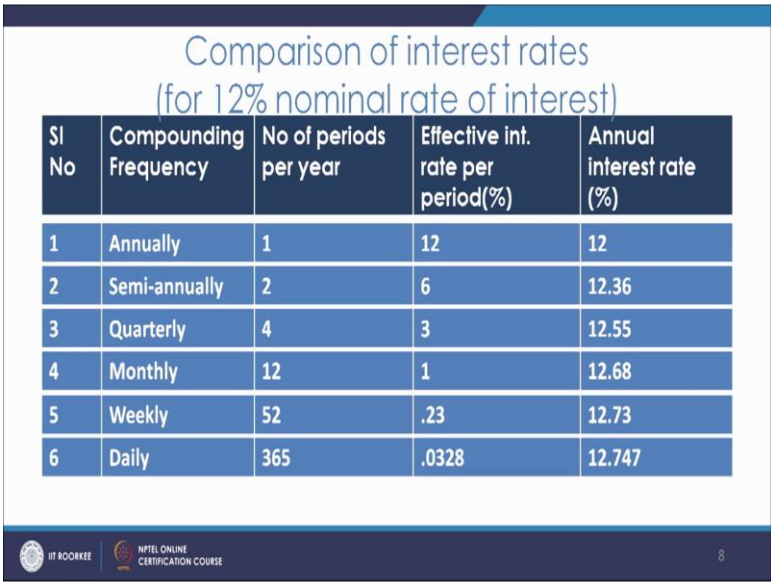

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	.23	12.73
6	Daily	365	.0328	12.747

Now we have seen that as the interest rates and its compounding changes, the effective interest rate values are changing and we can see that how these effective interest rate changes as the compounding frequency is changed, that we can see in our next lecture. Thanks.