Engineering Economic Analysis
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Lecture 08
Interest Formulas for Geometric Gradient Series

Welcome to the lecture on Interest Formula for Geometric Gradient Series. So we have earlier discuss about the interest formula for uniform gradient series factors where the amount which is paid in subsequent years they increase or decrease by a constant amount but many a times the change is as a percentage that this is known as a geometric gradient series factor.

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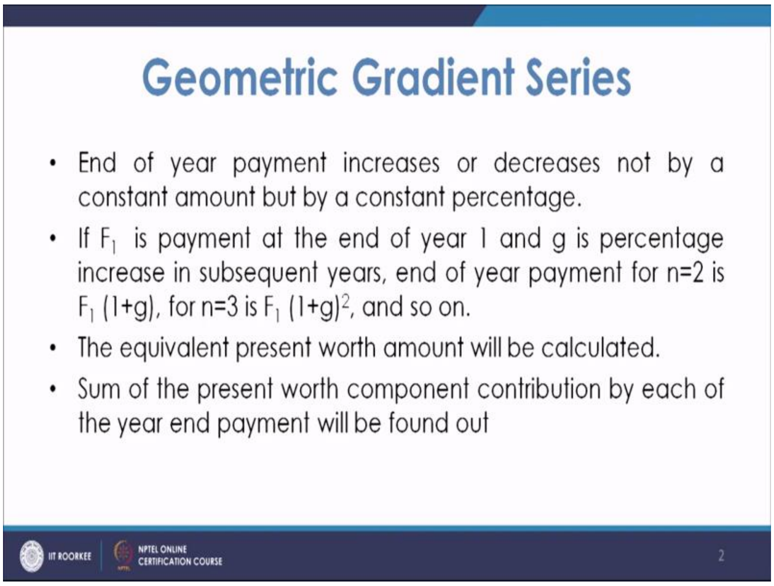

So in this case end of year payment increases or decreases not by a constant amount but by a constant percentage from what we have made in the previous year. So it means if F1 is the payment as the end of year 1, so at the end of year 2 if G is the percentage increase or decrease, in that case the second year it will be F1 into 1 + G. In the third year, F1 into 1 + G raised to the power 2. So this way it will go on.

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Now in this case we have to find a equivalent cash flow diagram and here basically we will try to find the equivalent present worth and then we will try to find the expression in terms of the gradient series. So let us see how this gradient series looks like. The cash flow diagram for a gradient series will look like this, in the 0th year, 0, 1, 2, 3, n - 1 and n.

As we have discussed in the first year you pay F1 and then there is an increase not in not in a linear form. So if it is F1, in the second year it comes as F1 into 1 + G, in the third year it becomes F1 into 1 + G2 because G percentage is the increase every year end. So in the n - 1th year it will be F1 multiplied by 1 + G raised to the power n -2. And in the nth year and you will be paying F1 into 1 + G raised to power n - 1.

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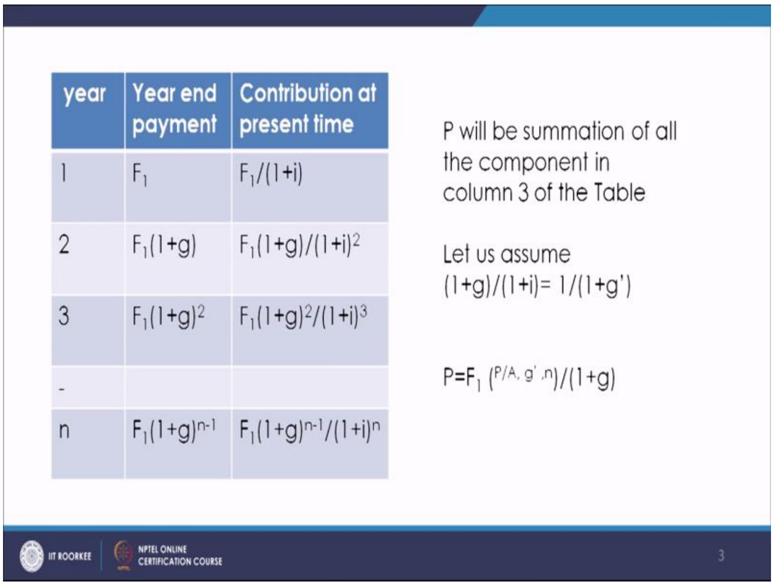

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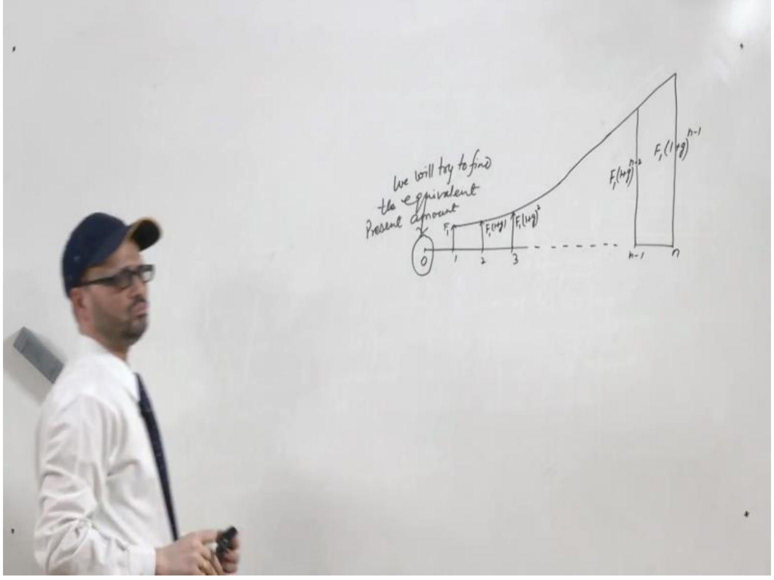

So what we will do is, for this series, we will try to have the contribution at the present time here. We will try to have the contribution at, we will try to find the equivalent present amount at this time. Now as we see the table, in the first year we are paying F1 and our interest rate is I. So for the first year, this amount which is paid F1 at the end of one year, its contribution here will F1 by 1 + I.

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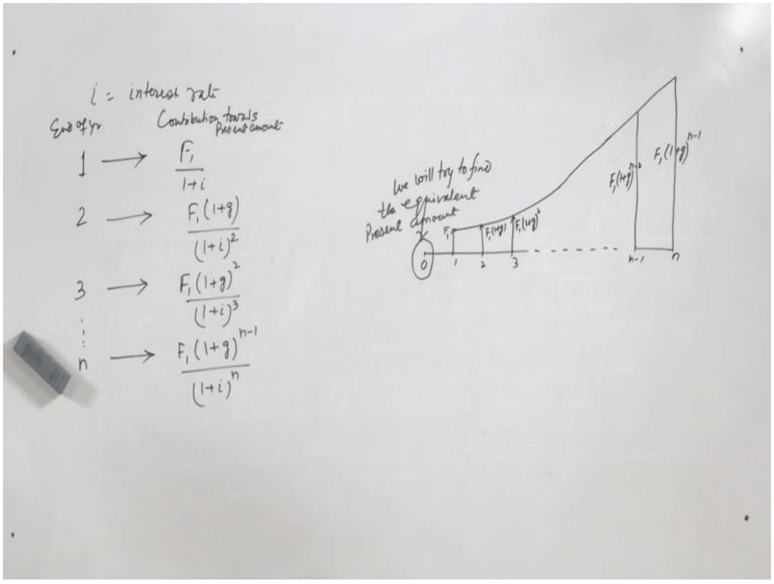

As we see for the first year payment for which we have paid F1, this F1 its contribution will be year as F1 by1 + I. So first year you get F1 upon 1 + i that is what you get here. In the second year, this is end of year and this is contribution towards present amount. So we have already discussed, your payment is F1 into 1 + G and it is to interest periods hence.

So in the second year it will be F1 into 1 + G, the amount paid but it will be multiplied by P by F i n that is 1 by 1 + i raised to the power 2. So this way in the third year as we have seen you have F1, you pay in the third year F1 into 1 + G raised to the power 2 divided by 1 + i raised to the power 3 because the amount is paid three years hence.

And its equivalent value at the present time with this time multiplied by 1 + i raised to the power - 3 or divided by 1 + i raised to the power 3 in this way we are getting a series. So you **ssh** should see that in the nth year, you are paying F1 1 + Gn - 1 but since is it paid at the nth year and, its present worth value will be 1 + i raised to the power n. This amount will be divided by 1 + i raised to the power n.

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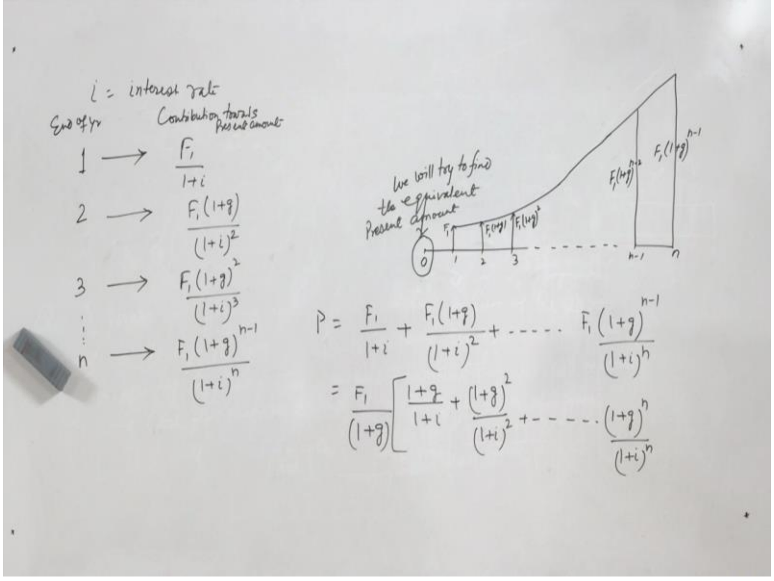

So basically all its contributions,, each of these payments contributions, if we add here, this will be the present worth of this particular cash flow diagram. So what we will do, P will be nothing but F1 by 1 + i + F1 into 1 + G by 1 + i raised to the power 2 + it will go on F1 1 + G raised to the power n - 1 divided by 1 + i raised to the power n.

This is the present amount and this is all are the individual contributions of all the year end payments which are basically increasing or decreasing in a geometrically gradient manner. So what begin write here is F1 will come out and also we will divide it by 1 + G, we will put another factor, so 1 + G will be multiplied to each term, the first becomes 1 + G by 1 + I.

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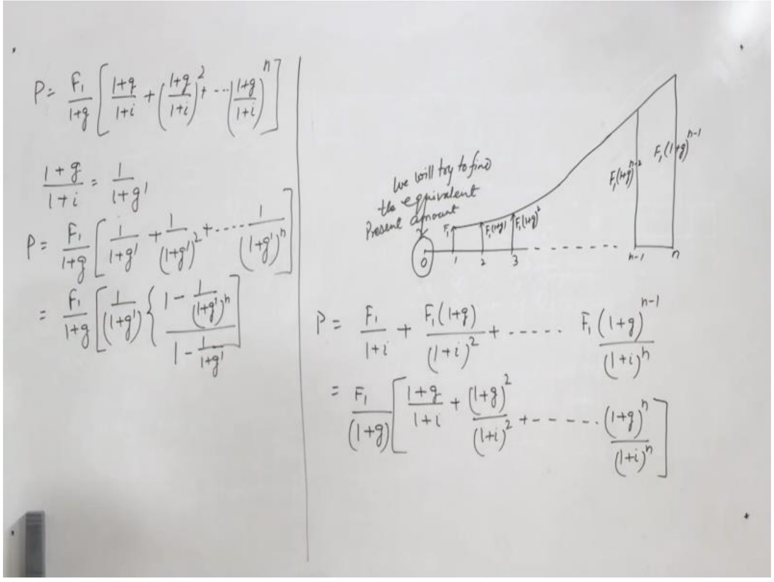

Second term becomes 1 + G square by 1 + i square, so this way the nth term will be 1 + G n by 1 + i n. Further so what we have seen so far, P equal to F1 by 1 + G1 + G by 1 + i +1 + G by 1 + i square + up to 1 + G by 1 + i raised to the power n. This is a geometric progression series, now for solving this let us assume1 + G upon 1 + i as 1 by 1 + G prime.

So you can write this series as P equal to F1 by 1 + G 1 by 1 + G prime + 1 by 1 + G prime whole square plus dash dash 1 by 1 + G prime to the power n. So we have got this series. Now this is a geometric progression series, for this F1 by 1 + G, this is a geometric progression series. So A into 1 - R n so 1 -1 + G prime 1 - 1 by 1 + G prime to the power n divided by 1 - 1 by 1 + G prime.

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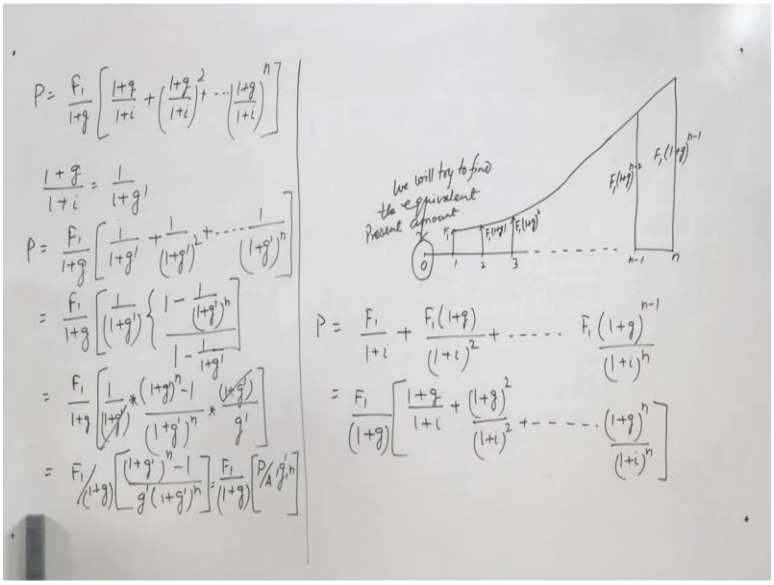

This can be written as F1 by 1 + G 1 by 1 + G prime 1 + G prime to the power n - 1 by 1 + G prime to the power n multiplied by 1 + G prime, this term will go up and divided by G prime. So this 1 + G prime and this term will cut, you are left with the term F1 by 1 + G multiplied by 1 + G prime to the power n - 1 by G prime into 1 + G prime to the power n.

And this is nothing but we have already studied about this series factor, so you can further write it as F1 by 1 + G and this will be P by A i n. So P by A, i is nothing but G prime and n. So ultimately what we see is that we knew G and from that G and i we can get G prime. Corresponding to this G prime we will calculate this factor, this factor will be multiplied with this first payment, first year end payment and divided by 1 + G.

So that will give us the present equivalent amount for this particular series. Now let us see the different aspects of the series. So there may be cases, what should be the value of G and what should be the value of i, how they are matching with each other. Whether G is more than i or G is less then i or G is equal to I. So basically that will determine the value of G prime.

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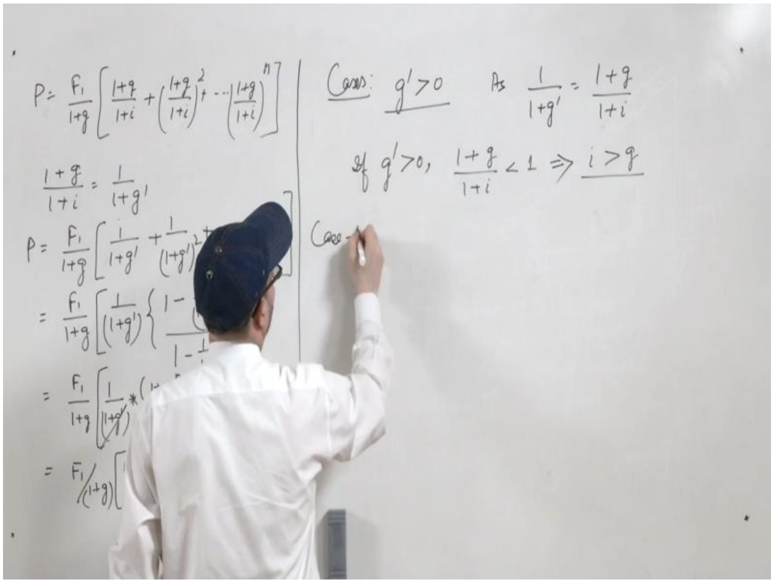

Now let us say if G prime is greater than zero we know that that 1 by 1 + G prime is 1 + G by 1 +i. So if G prime is greater than zero, means this quantity will be less than one. So if G prime is greater than 0, 1 + G by 1 + i will be less than one. This will lead to the co-relation i is greater than G.

So whenever you will have i more than equal to G, the G prime value will be a positive value and you can find its, can find the value using the normal equation.

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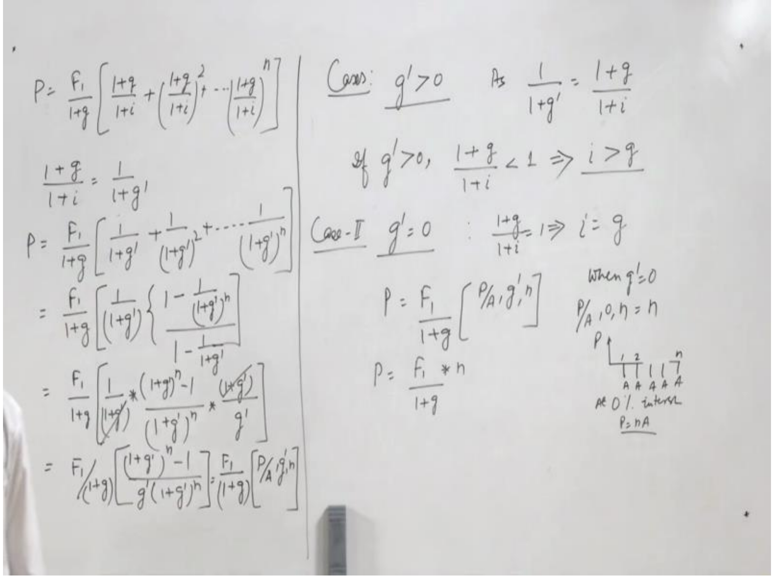

Second case will be G prime is equal to 0, when this G prime will be equal to 0, in that case 1 + G by 1 + i will be one, this implies i will be G. Now in this case, when there will be i and G equal, your G prime becomes 0. Now in this case as we see, P will be F1 and multiplied **by** by 1 + G+ P by A G prime n. And when G prime is 0, when G prime is 0, so P by A 0 n. This value will be nothing but n.

So means when there is no interest rate, this is nothing but a cash flow diagram whereequivalent P has to be has to be found out and you have A for n times. So basically in this case if 0% interest is there, P will be nA. It will be basically P equal to F1 by 1 + G into n. So this P by A G prime n, when G prime is 0, it becomes n, so P will be equal to F1 into n by 1 + G.

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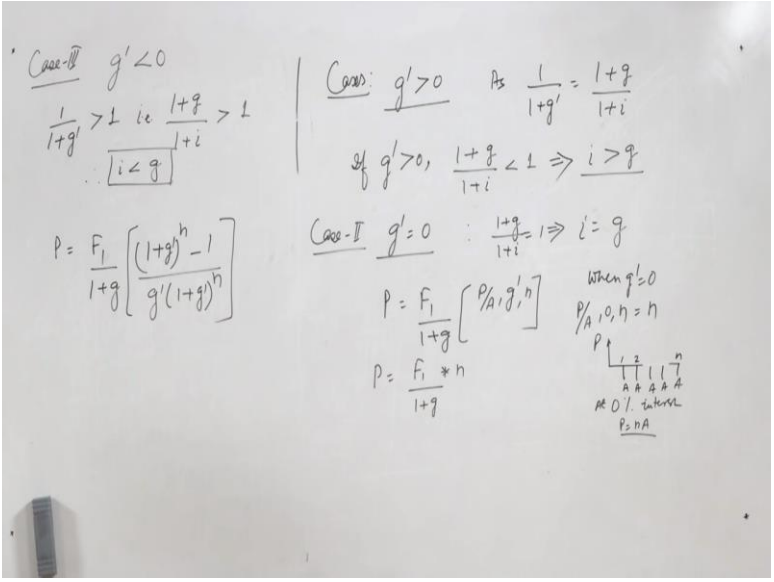

The third case is G prime is less than 0. Let us find it at the top, so case 3 is G prime less than 0. Now in this case, what happens, since G prime is less than 0, 1 by 1 + G prime will be more than 1 that is 1 + G upon 1 + i will be more than 1. So i will be less than G. So whenever you will have this condition, the constant percentage increase is more than the interest rate, in that case you G dash will come as negative.

What will happen in those cases if you look at the factor, P is F1 by 1 + G into P by A G prime n and P by A G prime n is nothing but 1 + i n - 1 by i into 1 + i n and i is replaced by G prime. Now what will happen in this case, this factor, this is negative and this i is also G prime. So this factor also become negative and this factor also become negative.

1 + G prime to the power n - 1, so this amount will be less than one and once it is subtracted from one, this quantity will be negative as well as this quantity is anyways negative so both these negative will cut up and then you will get a positive value. So you will get certainly a realistic P value. So in this case we try to find the equivalent value of P for these cases.

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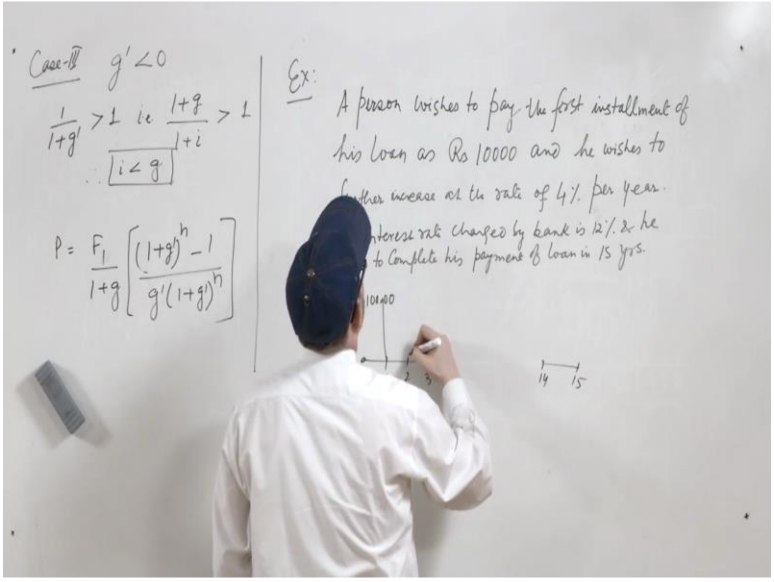

An example you can see, examples of this type, suppose a person is paying one loan amount at the end of year one, a person wishes to pay the first instalment of his loan as Rs. 10,000 and he wishes to further increase at the rate of 4% per year. Assume that he is paying the instalment at the end of first year as 10,000 and he wants to increase at the rate of 4 year percent per year.

It means in the first year, the interest rate charged by bank is 12% and he wishes to complete his payment of loan in 15 years. So basically this problem tells that in the first year, he is paying 10,000 from the banks perspective. If you look at from the banks perspective, bank is gaining Rs. 10,000 in the first year, and the second year the bank will get 4% increase, so it will be 10,000 + into 1.04 so 10,400.

Second time again 10,000 multiplied by 1.04 to the power 2, that is 1.0816. So this way he will go on increasing. So what is ultimately what is the present equivalent amount that can be found at this time using this formula. So this will be F1 by 1 + G, so F1 will be 10,000 by 1 + G, G you know, G is .04, so this will be 10,000 by 1.04.

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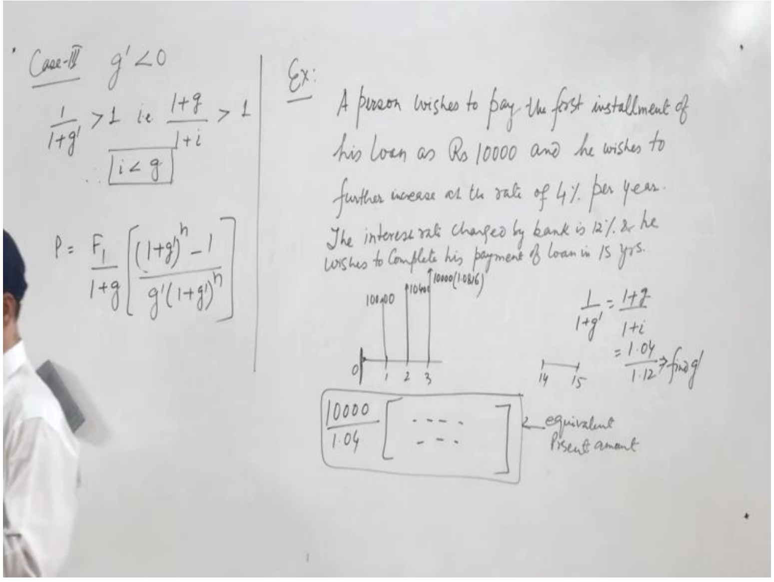

Then you have to find these factor values, this is P by A G prime n, so you have to calculate first of all G prime and we know how to calculate G prime. So G prime will be calculated as 1 by 1 + G prime is 1 + G by 1 +I, so in this case 1.04 by **1.** i is 12% so 1.12. So this way we get the values, so from here find G prime.

And once you find the G prime, you can find this by putting in this equation 1 + G prime to the power n that is 15 - 1 by G prime into 1 + G prime raised to the power 15. So this amount will be calculated and this amount will be the equivalent principal amount, present amount. These are the type of problems which can be dealt with such type of series factors.

We can discuss some problem solving on such series factors in our next lectures for then bye thank you.